1. [7.5 points] T/F - State whether the statements below are true or false AND give a brief justification for your answer.

_____ $2^{c\sqrt{n}} = O(2^{\sqrt{n}})$ for any constant $c > 0$

_____ $f(n) + g(n) = O(\max(f(n), g(n)))$ assuming $f(n)$ and $g(n)$ are positive functions.

_____ You are given two algorithms $A_1$ and $A_2$ for solving a problem. $A_1$ runs in time $O(n^3)$ and $A_2$ runs in time $O(\log n)$. It is possible for $A_1$ to take less time to run than $A_2$ on all possible inputs.

_____ A $k$-sorted array is an array where any value is no more than $k$ positions from its correct location. The worst case running time of Insertion-Sort on a $k$-sorted array is $O(n^2)$.

_____ If $f$ is $O(g)$, then $2^f$ is $O(2^g)$
2. **[6 points]** You’re given an array of $n$ elements and would like to print the $k$ largest in sorted, *decreasing* order. For example, if $n = 8$ and $k = 3$ and the input were:

$$8 \ 10 \ 2 \ 1 \ 4 \ 6 \ 2 \ 15$$

Then the output would be: 15 10 8

For each of the methods below, describe the *most efficient*, worst-case run-time for the method described. Note your run-times should be in terms of $n$ and $k$.

(a) Sort all $n$ numbers and then print the largest $k$.

(b) Find the largest value. Remove it from the array and print it. Repeat until you’ve found the $k$ largest values.

(c) Find the $k$th largest number, partition around it, then sort the $k$ largest numbers.

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3. **[6 points]** Suppose you are given an array $A[1...n]$ of sorted integers that has been rotated $k$ positions to the right. For example, $[35, 42, 5, 15, 27, 29]$ is a sorted array that has been circularly rotated $k = 2$ positions, while $[27, 29, 35, 42, 5, 15]$ has been rotated $k = 4$ positions. Describe an algorithm to find the largest value in a $k$-shifted array in $O(\log n)$ time.
4. [6 points] If possible, solve the following recurrences and prove that your answer is correct (using the master method is fine as proof):

(a) \( T(n) = 3T\left(\frac{n}{3}\right) + \log n \)

(b) \( T(n) = T(n - 1) + n^d \log n, \text{ for } d \geq 1 \)