CS140 - Assignment 11

Due: Sunday, Apr. 28th at 11:59pm

"I can't find an efficient algorithm, but neither can all these famous people."

http://asfarian.wordpress.com/2011/01/11/np-completeness/

For any NP-completeness reduction you need to:

- Show that the decision problem is in NP
- Describe a reduction, making sure to
  - Argue that the reduction can be done in polynomial time.
  - Explain why the reduction "works" (i.e., why a "yes" $\rightarrow$ "yes" and a "no" $\rightarrow$ "no" or, more often, why a "yes" $\rightarrow$ "yes" and a "yes" $\leftarrow$ "yes")

1. **[4 points]** Most of the NP-Complete problems we looked at in class were *decision* problems, in that we were trying to decide if a condition was true or not (e.g. "does the graph have a clique of size k?"). However, for real problems we often would want to solve the max/min problem (often called the *search* problem), for example "what is the size of the largest clique in the graph?".

   I claim that in most situations if you can solve the decision problem in polynomial time then you can solve the search problem in polynomial time. Prove that this is true for the CLIQUE problem, i.e., assume you have a polynomial-time solver for the decision problem and show how you can use this to solve the search problem.

2. **[10 points]** Zero sum

   ZERO-SUM is the following problem: Given a set of integers S is there a subset that sums to 0?

   Prove that ZERO-SUM is **NP-Complete**.

   (a) Show ZERO-SUM is in NP.

   (b) Prove that ZERO-SUM is NP-complete.

3. **[10 points]** Getting along

   In this problem you are given a set $E = \{e_1, \ldots, e_n\}$ of n employees and a set H which consists of pairs of employees (e_i, e_j) such that employees e_i and e_j do not get along. Does a subset $S \subseteq E$ where $|S| = m$ such that for every pair of employees $e_k, e_\ell \in S$, $(e_k, e_\ell) \notin H$.

   (a) Show that this problem is in NP.

   (b) Use CLIQUE to prove that the problem is NP-complete.

4. **[11 points]** Hitting Set

   Assume you are given a set S, and a collection C of subsets of S. A hitting set for C is a subset $S' \subseteq S$ such that S' contains at least one element from each subset in C. The optimization problem asks for the size of the smallest hitting set.

   You can think of S as being a set of students, and of the C as a collection of subsets of S where all the students in a subset share some skill (eg, "good writer", "entertaining", "can code for 3 days straight", "has access to free pizza", and so on). The hitting set, then, would be a study group whose members collectively possess every skill. The search problem asks how to minimize the size of the group.

   (a) Formulate the HITTING-SET decision problem corresponding to the search problem described above.

   (b) Show that HITTING-SET is in NP.

   (c) Prove that HITTING-SET is NP-complete.