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Algorithms, Assignment 5: Greedy Algorithms

1. Consider an undirected graph $G = (V, E)$ where every edge e has a given cost c_e. Assume that all edge costs are positive and distinct. Let T be a minimum spanning tree of G and P be a shortest path from the vertex s to the vertex t. Now suppose that the cost of every edge e of G is increased by 1 (for each edge: $c_e = c_e + 1$). Call this new graph G'. Which of the following is true about G'?

   Is T a minimum spanning tree of G'? Please provide a proof for your answer.

   Is P a shortest s-t path in G'? Please provide a proof for your answer.

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2. You work at a ski rental place and you would like to design an algorithm that matches skiers to skis. Ideally every skier should get a pair of skis whose length matches their height. Unfortunately, in general this is not possible. So, to measure how good a particular match of ski to skier is, we'll use the disparity between them, which we'll define as the absolute value of the difference between the length of the skis and the height of the skier. Now we want a way to assign skis to skiers that minimizes the sum of the disparities.

   The input to this problem is a list of n skiers (each skier is a pair comprising their name and height) and a list of $m \geq n$ skis (we know the length of each pair of skis). These arrays are given in sorted order from shortest to tallest (shortest to longest for the skis).

   a. Describe a brute-force algorithm and explain how long it would take to execute this algorithm on a computer that performs 1 billion operations per second if there were 20 skiers and 20 pairs of skis.

   b. Now consider the following sub-optimal greedy algorithm:
      i. Consider the sorted skiers and skis from shortest to tallest.
      ii. For each skier under consideration, assign that skier the pair of skis that most closely match that skier's height.
      iii. Then remove that skier and pair of skis from consideration and repeat the process.

      Find a small example where this gives a solution that is worse than optimal. You'll need to provide a particular small set of skier heights, a small set of ski heights, show the solution produced by the greedy "algorithm", and then show a solution that is better.

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c. In search of something better, you realize that if there is a short person and a tall person, it is never better to give the shorter person a taller pair of skis than were given to the tall person. In other words, there exists an optimal solution to the ski problem in which there are no “inversions” in which persons x and y, with x shorter than y, are assigned skis such that x has longer skis than y. It turns out that this conjecture is not hard to prove, particularly if you perform a detailed case analysis.

   Now assume there are n skiers and n pairs of skis. Describe (in English sentences) a fast algorithm for assigning skis to skiers, briefly explain how the proof of correctness would work, and give the running time of your algorithm.

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3. Consider the problem of handing back change (in US currency denominations of 25¢ quarters, 10¢ dimes, 5¢ nickels, and 1¢ pennies). Develop a greedy algorithm for handing back change that uses a minimal number of coins total. Argue that your solution is optimal, or prove that it is not optimal.