Breadth First Search

https://cs.pomona.edu/classes/cs140/

Outline

Topics and Learning Objectives

- Discuss breadth first search for graphs

Exercises

- Continued from previous lecture slides
- Compute distance with Breadth-first search

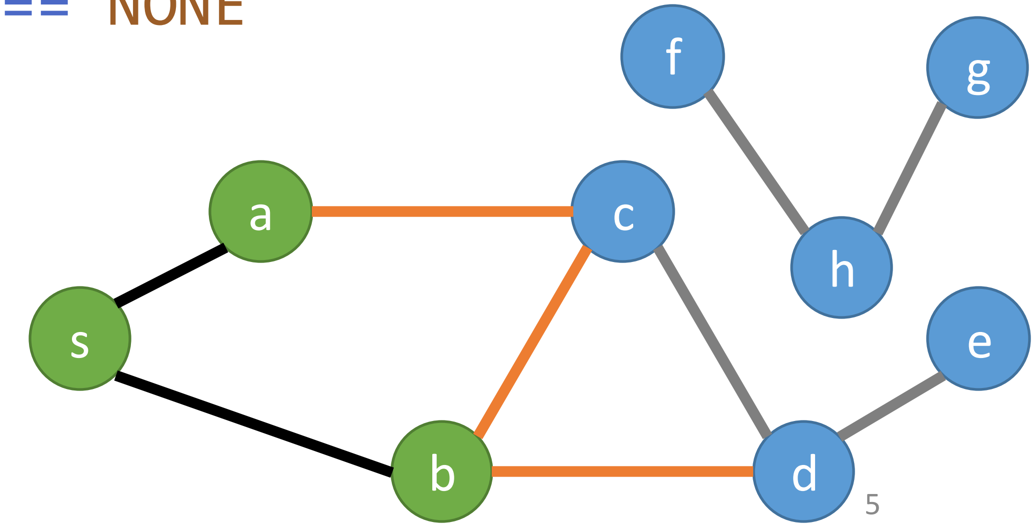
Extra Resources

- Introduction to Algorithms, 3rd, Chapter 22
- Algorithms Illuminated Part 2: Chapter 8

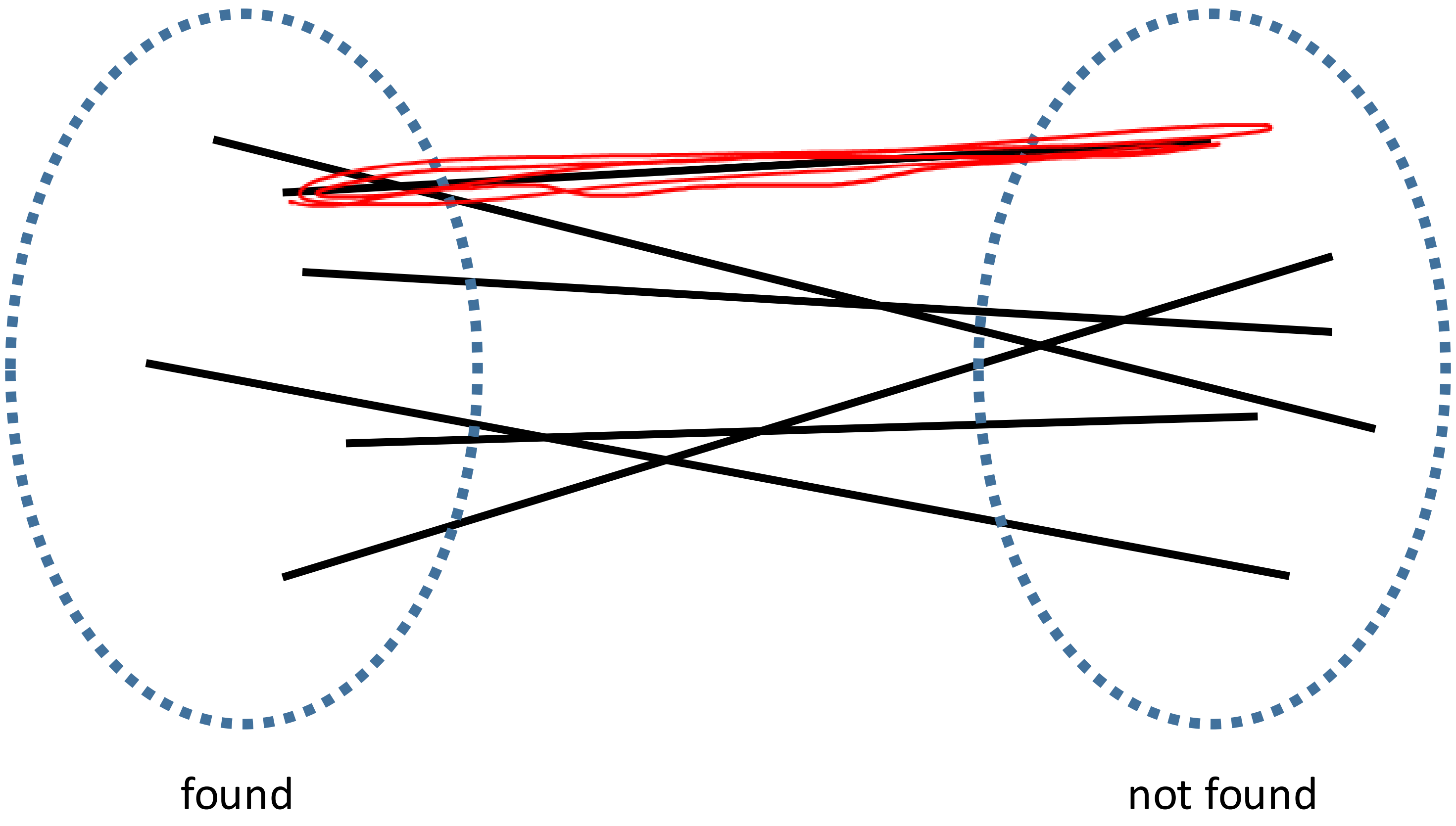
General Algorithm

```
FUNCTION Connectivity(G, start_vertex)
    found = {v: FALSE FOR v IN G.vertices}
    found[start_vertex] = TRUE
    LOOP
        (vFound, vNotFound) = get_valid_edge(G.edges, found)
        IF vFound == NONE || vNotFound == NONE
            BREAK
        ELSE
            found[vNotFound] = TRUE
    RETURN found
```

Find an edge where one vertex has been found and the other vertex has not been found.



How do we choose the <u>next</u> edge?



Two common (and well studied) options

Breadth-First Search

- Explore the graph in layers
- *"Cautious"* exploration
- Use a FIFO data structure (can you think of an example?)

Depth-First Search

- Explore recursively
- A more *"aggressive"* exploration (we backtrack if necessary)
- Use a LIFO data structure (or recursion)

```
FUNCTION BFS(G, start_vertex)
    found = {v: FALSE FOR v IN G.vertices}
    found[start_vertex] = TRUE
    visit_queue = [start_vertex]

    WHILE visit_queue.length != 0
        vFound = visit_queue.pop()
        FOR vOther IN G.edges[vFound]
            IF found[vOther] == FALSE
                found[vOther] = TRUE
                visit_queue.add(vOther)

    RETURN found
```

```
FUNCTION Connectivity(G, start_vertex)
    found = {v: FALSE FOR v IN G.vertices}
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    LOOP
        (vFound, vNotFound) =
            get_valid_edge(G.edges, found)
        IF vFound == NONE || vNotFound == NONE
            BREAK
        ELSE
            found[vNotFound] = TRUE
    RETURN found
```
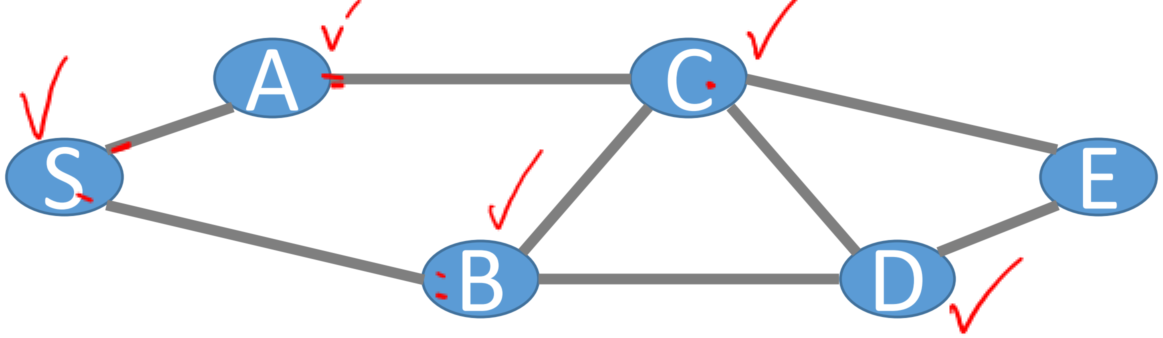

Exercise questions 2 and 3




```
FUNCTION BFS(G, start_vertex)
    found = {v: FALSE FOR v IN G.vertices}
    found[start_vertex] = TRUE
    visit_queue = [start_vertex]

    WHILE visit_queue.length != 0
        vFound = visit_queue.pop()
        FOR vOther IN G.edges[vFound]
            IF found[vOther] == FALSE
                found[vOther] = TRUE
                visit_queue.add(vOther)

    RETURN found
```

= [S A B C D E

vFound = S
vO = A
vO = B
vF = A
vO = C
vO = S
vF = B
vO = C
vO = D
vO = S
vF = C
vO = A
vO = B
vO = D
vO = E
vF = D

S A B C D E

Given a tie, visit edges are in alphabetical order

Running Time

What is the running time?


```
FUNCTION BFS(G, start_vertex)
    found = {v: FALSE FOR v IN G.vertices}
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    WHILE visit_queue.length != 0
        vFound = visit_queue.pop()
        FOR vOther IN G.edges[vFound]
            IF found[vOther] == FALSE
                found[vOther] = TRUE
                visit_queue.add(vOther)

    RETURN found
```

x, y

How many times to we consider each edge?

twice

~~$O(m) \cdot O(n)$~~

$O(n) + O(m) = O(n+m)$

Running Time

What is the running time?

```
FUNCTION BFS(G, start_vertex)
    found = {v: FALSE FOR v IN G.vertices}
    found[start_vertex] = TRUE
    visit_queue = [start_vertex]

    WHILE visit_queue.length != 0
        vFound = visit_queue.pop()
        FOR vOther IN G.edges[vFound]
            IF found[vOther] == FALSE
                found[vOther] = TRUE
                visit_queue.add(vOther)

    RETURN found
```

How many times to we consider each edge?

$$T_{BFS}(n, m) = O(n_s + m_s)$$

where n_s and m_s are the nodes and edges **findable/connected** from/to the start vertex

Proof: BFS

Claim: BFS finds all nodes connected to the start node.

At the end of the BFS algorithm, v is marked found if there exists a path from s to v

- Note: this is just a special case of the general algorithm that we proved by contradiction

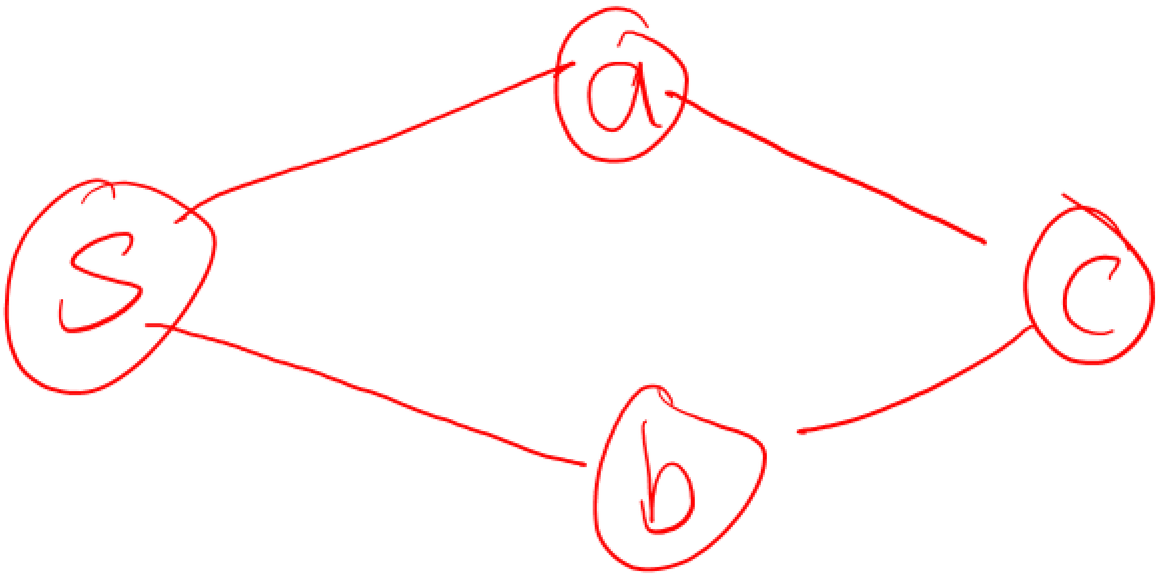
Practice for a loop invariant

Homework question

Question

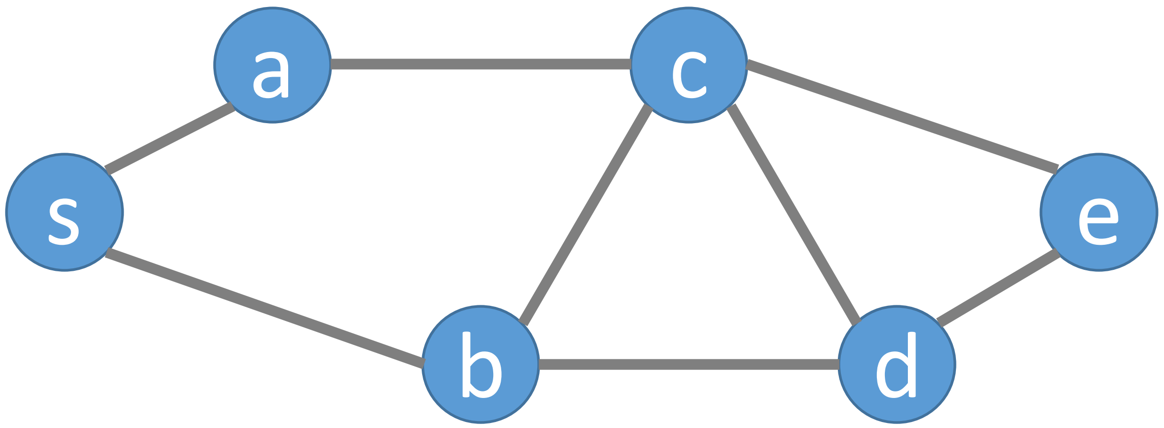
The Shortest Path Problem

- How can we determine the fewest number of hops between the start vertex and all other connected vertices?



BFS Exercise Question 1

How can we determine the fewest number of hops between the start vertex and all other connected vertices?




```
FUNCTION BFS(G, start_vertex)
    found = {v: FALSE FOR v IN G.vertices}
    found[start_vertex] = TRUE
    visit_queue = [start_vertex]

    WHILE visit_queue.length != 0
        vFound = visit_queue.pop()
        FOR vOther IN G.edges[vFound]
            IF found[vOther] == FALSE
                found[vOther] = TRUE
                visit_queue.add(vOther)

    RETURN found
```

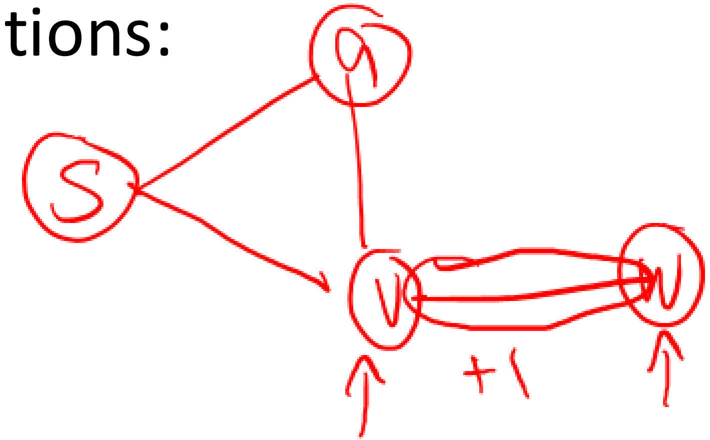
Given a tie, visit edges are in alphabetical order

The Shortest Path Problem

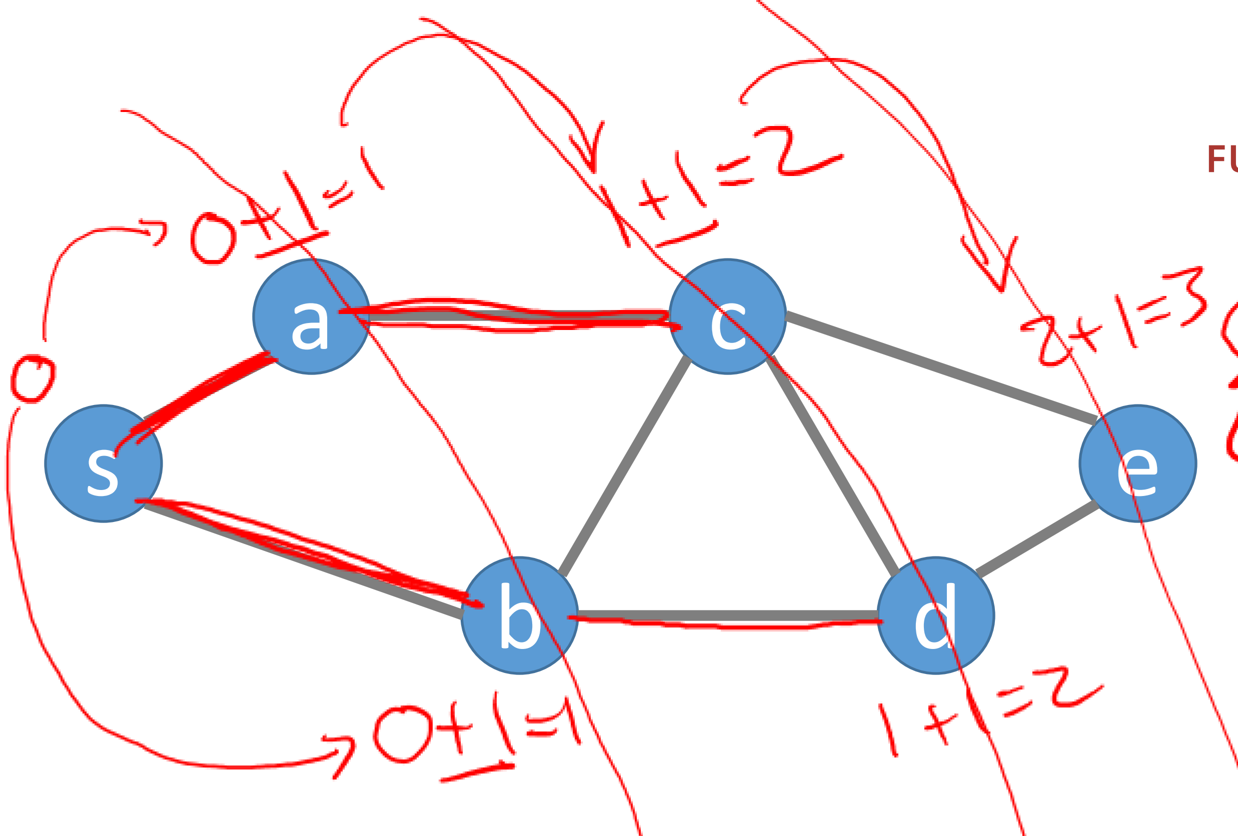
Determine the fewest number of hops between the start vertex and all other vertices

Same algorithm as before with the following additions:

- Initialize the distances[s] as 0
- Initialize all other distances to **infinity**
- When considering an edge (v, w)
    - If w is not found, then set dist(w) to dist(v) + 1



The Shortest Path Problem

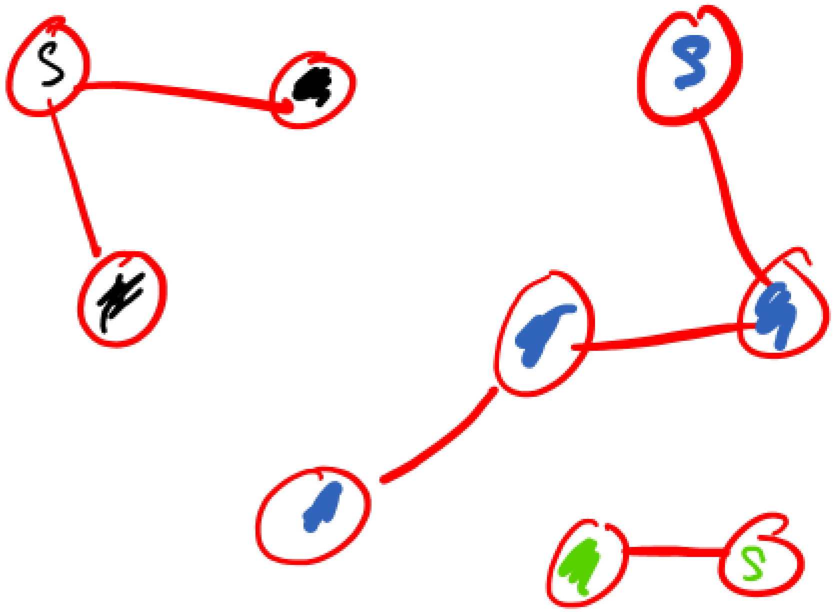



```
FUNCTION DistanceBFS(G, start_vertex)
   found = {v: FALSE FOR v IN G.vertices}
   found[start_vertex] = TRUE

   distances = {v: INFINITY FOR v IN G.vertices}
   distances[start_vertex] = 0

   visit_queue = [start_vertex]
   WHILE visit_queue.length != 0
      vFound = visit_queue.pop()
      FOR vOther IN G.edges[vFound]
         IF found[vOther] == FALSE
            found[vOther] = TRUE
            visit_queue.add(vOther)
            distances[vOther] = distances[vFound] + 1

   RETURN distances
```

After we terminate, distances[v] = ”the layer that v is in”

Given a tie, visit edges are in alphabetical order

Connected Components



Let's only consider undirected graphs for now

Let G = (V,E) be an undirected graph

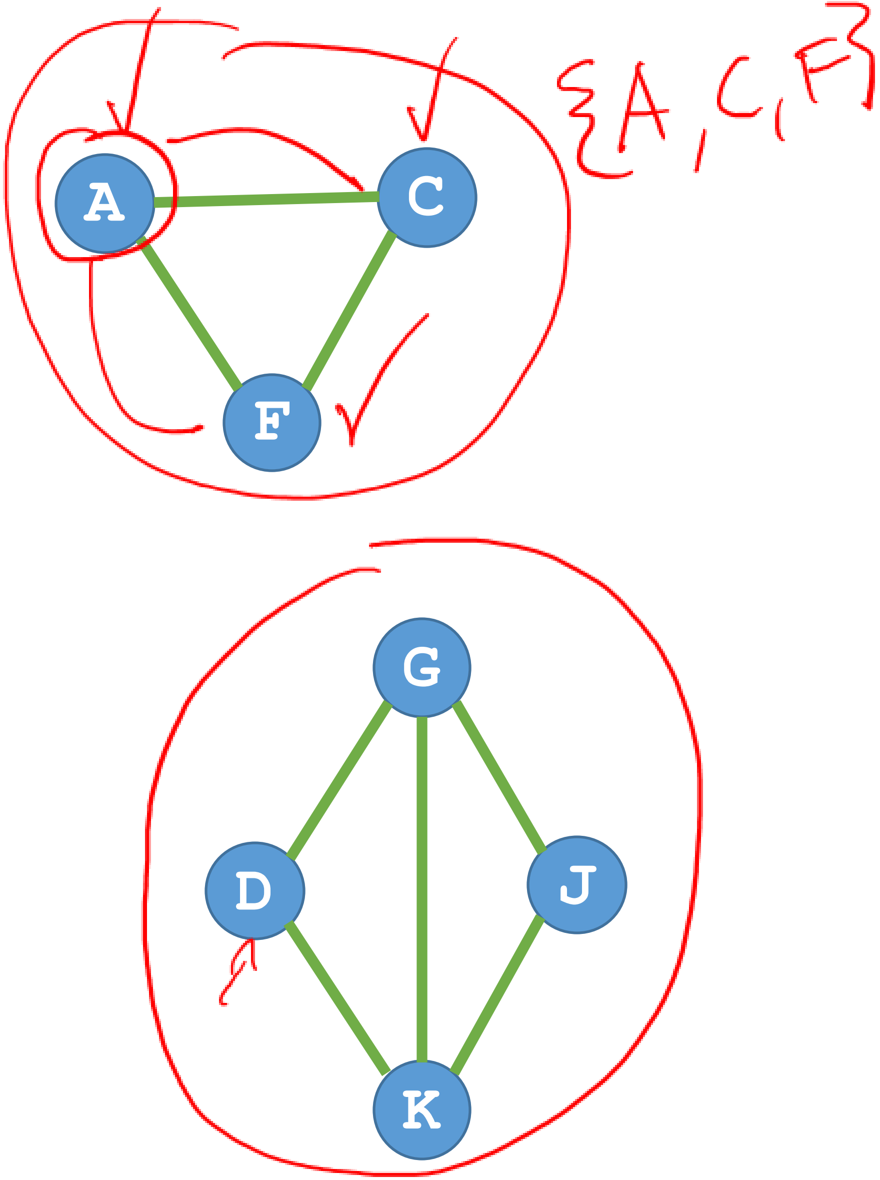
Goal: compute all connected components in O(m + n)

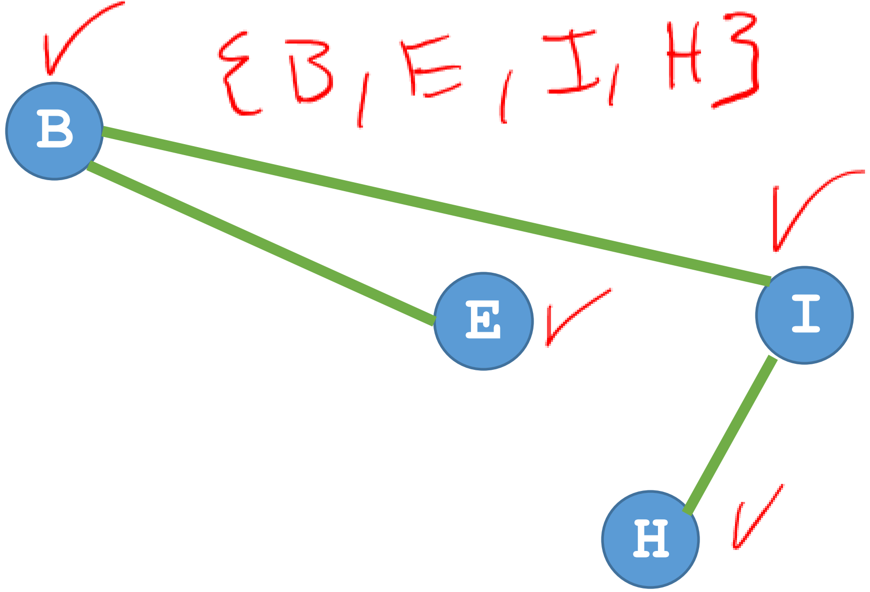
- A component is any group of vertices that can reach one another
- For example, if we are trying to see if a network has become disconnected

Exercise question 2:

How would you do this using our BFS procedure from before?

BFS Exercise Question 2






```
FUNCTION FindComponents(G)
    components = []
    found = {v: FALSE FOR v IN G.vertices}
    FOR v IN G.vertices
        IF NOT found[v]
            newly_found = BFS(G, v)
            new_component = {
                w FOR w, w_is_found IN newly_found
                IF w_is_found
            }
            component.append(new_component)
            FOR w IN new_component:
                found[w] = TRUE
    RETURN components
```