Graphs and Connectivity

https://cs.pomona.edu/classes/cs140/

Outline

Topics and Learning Objectives

- Discuss the basics of graphs
- Introduce graph searching

Exercise

- Graph search

Extra Resources

- Introduction to Algorithms, 3rd, Chapter 22
- Algorithms Illuminated Part 2: Chapter 7

Graphs

Represent pairwise relationships

Tons of uses

- Physical connections : roads (driving directions), network routing (phone), …
- Relationship groups : social networks, similar purchases, …
- Problem solving : each vertex may represent a partial part of the problem, and each edge is a step/move (e.g., Sudoku)

Tons of algorithms

- Cuts, clustering, searching, partitioning, contracting, …

Graphs

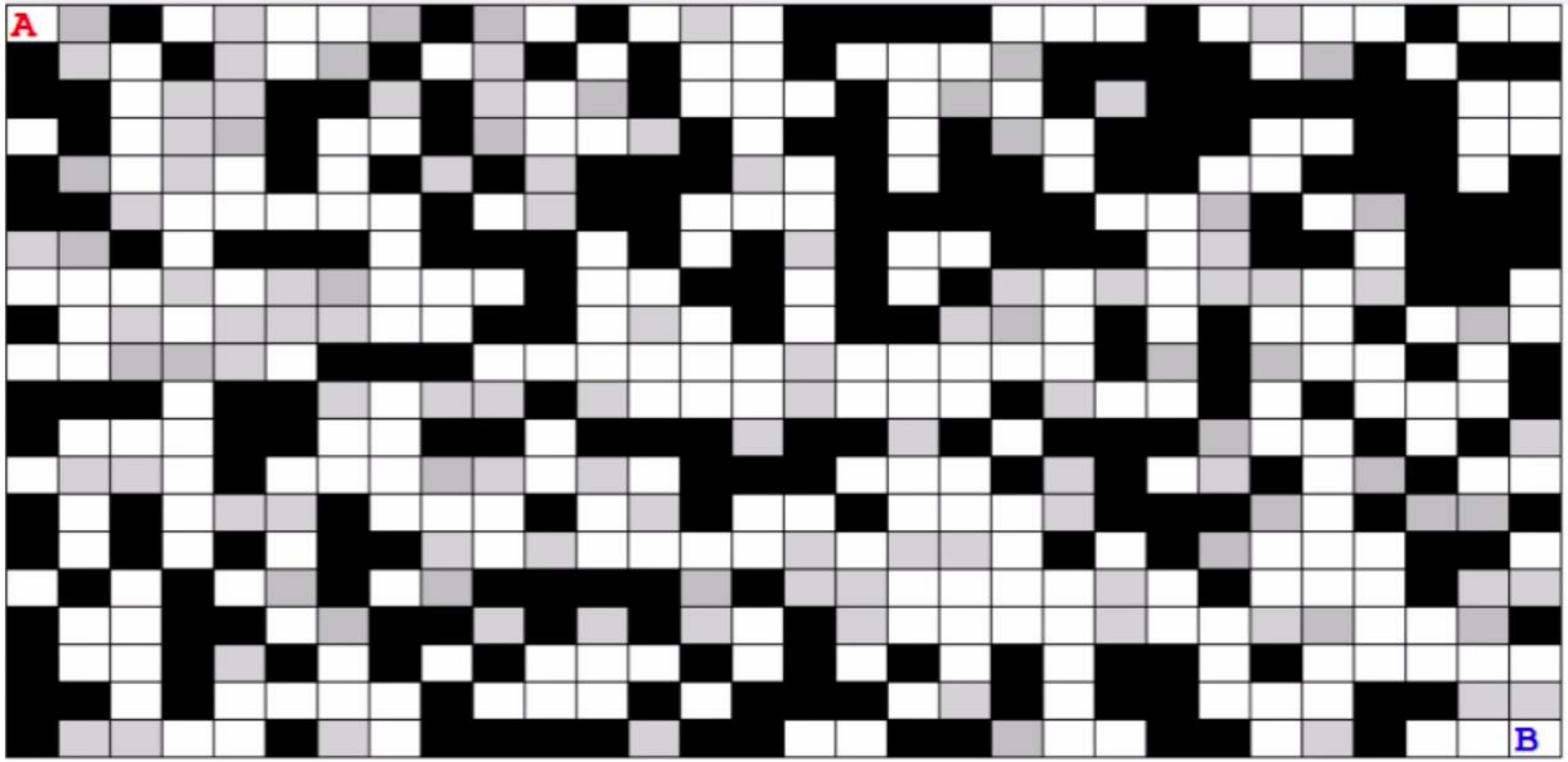
For many reasons, graph algorithms are extremely important.

They are a ubiquitous tool for solving many engineering problems

- Signal traces on a PCB
- Balancing the load on a server
- Balancing the load across cores on a computer
- Scheduling the delivery of packages via drone
- Scheduling the path of an automated robot that is grabbing your Amazon purchase from shelves in a warehouse
- Topological networks
- Data mirroring across a network
- Modeling an ecology
- Modeling the nervous system
- The list goes on and on

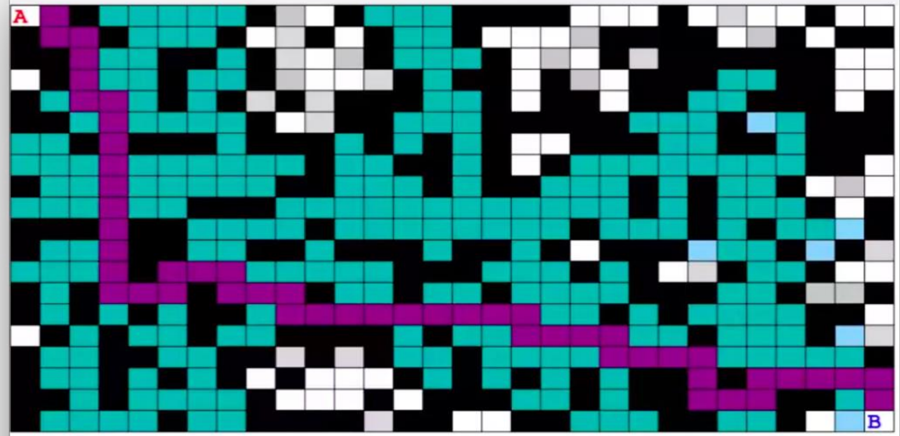
For this reason, you will often be asked graph-related questions during interviews

Algorithm



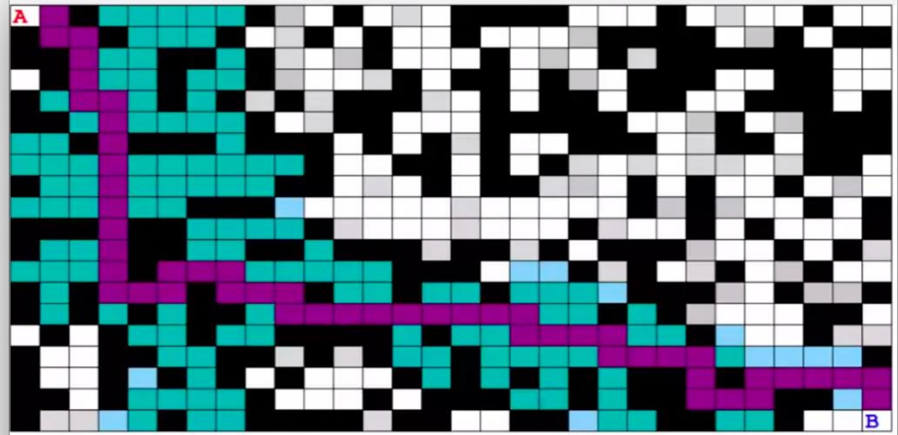
	Operations	Unexplored	Open	Closed	Path Length/Cost
Dijkstra:	0	0	0	0	?/?
A*:	0	0	0	0	?/?
Breadth-First:	0	0	0	0	?/?
Depth-First:	0	0	0	0	?/?

Algorithm: Dijkstra's



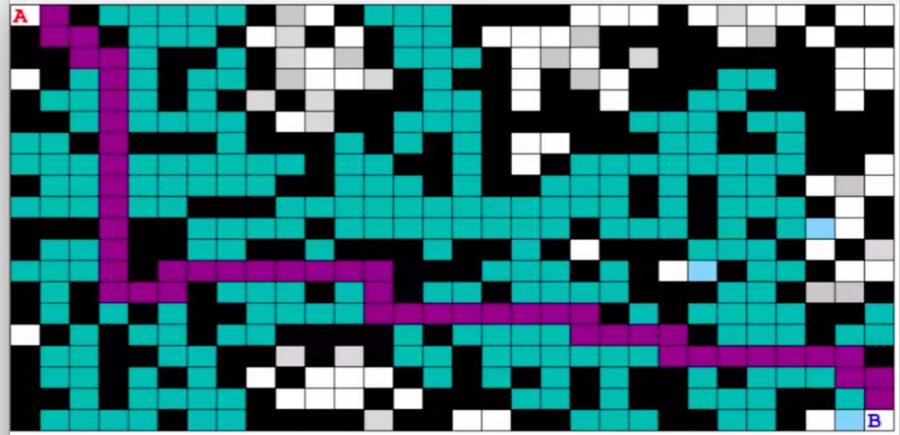
	Operations	Unexplored	Open	Closed	Path Length/Cost
Dijkstra:	732	80	6	283	53/65
A*:	0	0	0	0	?/?
Breadth-First:	0	0	0	0	?/?
Depth-First:	0	0	0	0	?/?

Algorithm: A*



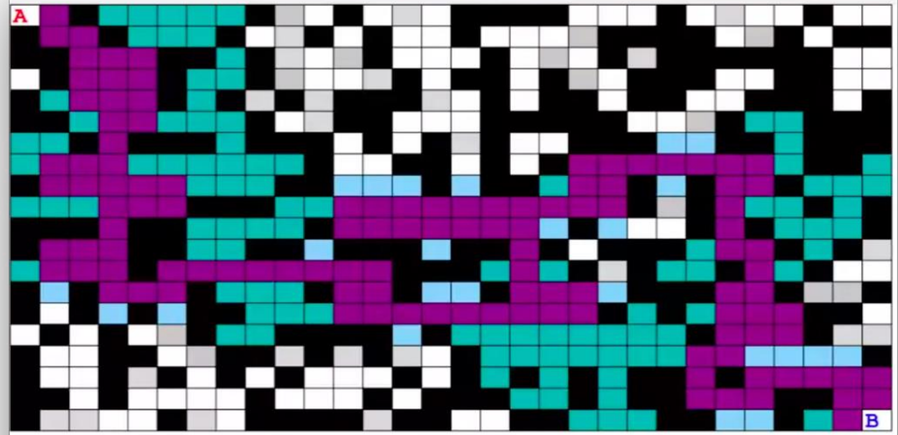
	Operations	Unexplored	Open	Closed	Path Length/Cost
Dijkstra:	732	80	6	283	53/65
A*:	441	188	13	168	53/65
Breadth-First:	0	0	0	0	?/?
Depth-First:	0	0	0	0	?/?

Algorithm: Breadth-First

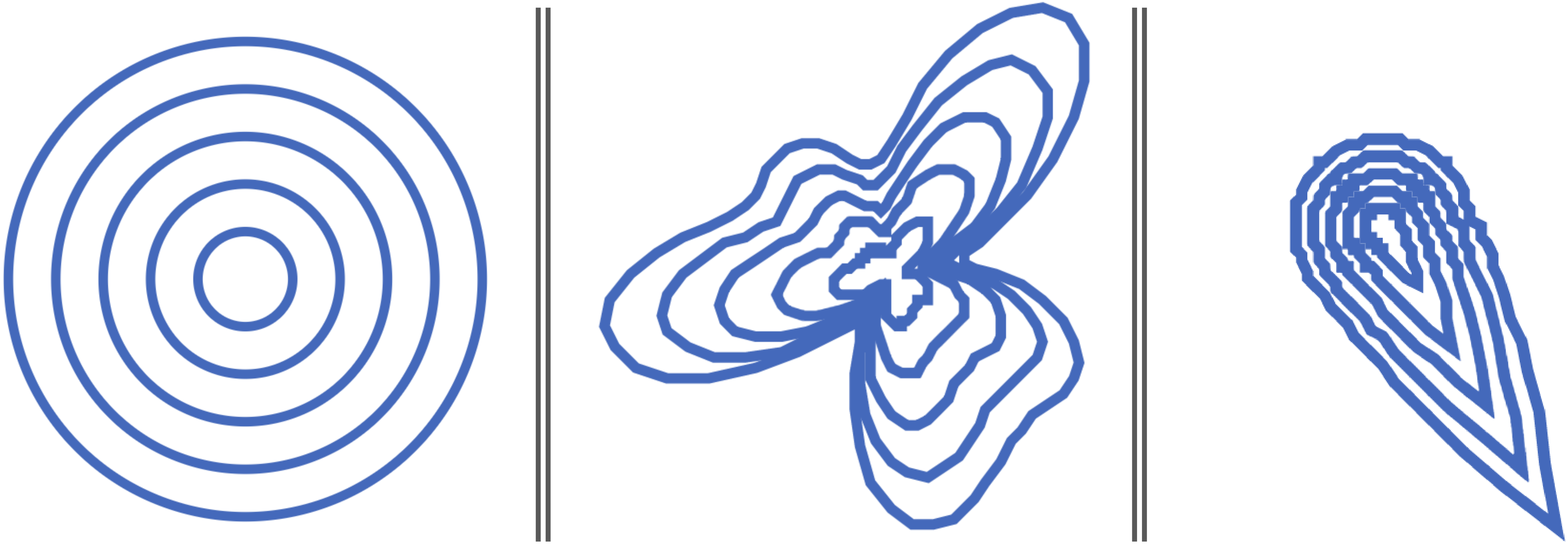


	Operations	Unexplored	Open	Closed	Path Length/Cost
Dijkstra:	732	80	6	283	53/65
A*:	441	188	13	168	53/65
Breadth-First:	367	79	5	288	51/75
Depth-First:	0	0	0	0	?/?

Algorithm: Depth-First



	Operations	Unexplored	Open	Closed	Path Length/Cost
Dijkstra:	732	80	6	283	53/65
A*:	441	188	13	168	53/65
Breadth-First:	367	79	5	288	51/75
Depth-First:	314	130	117	99	117/160



BFS vs Dijkstra’s vs A*

https://www.redblobgames.com/pathfinding/a-star/introduction.html

$G = (V, E)$

G is the standard symbol representing a graph

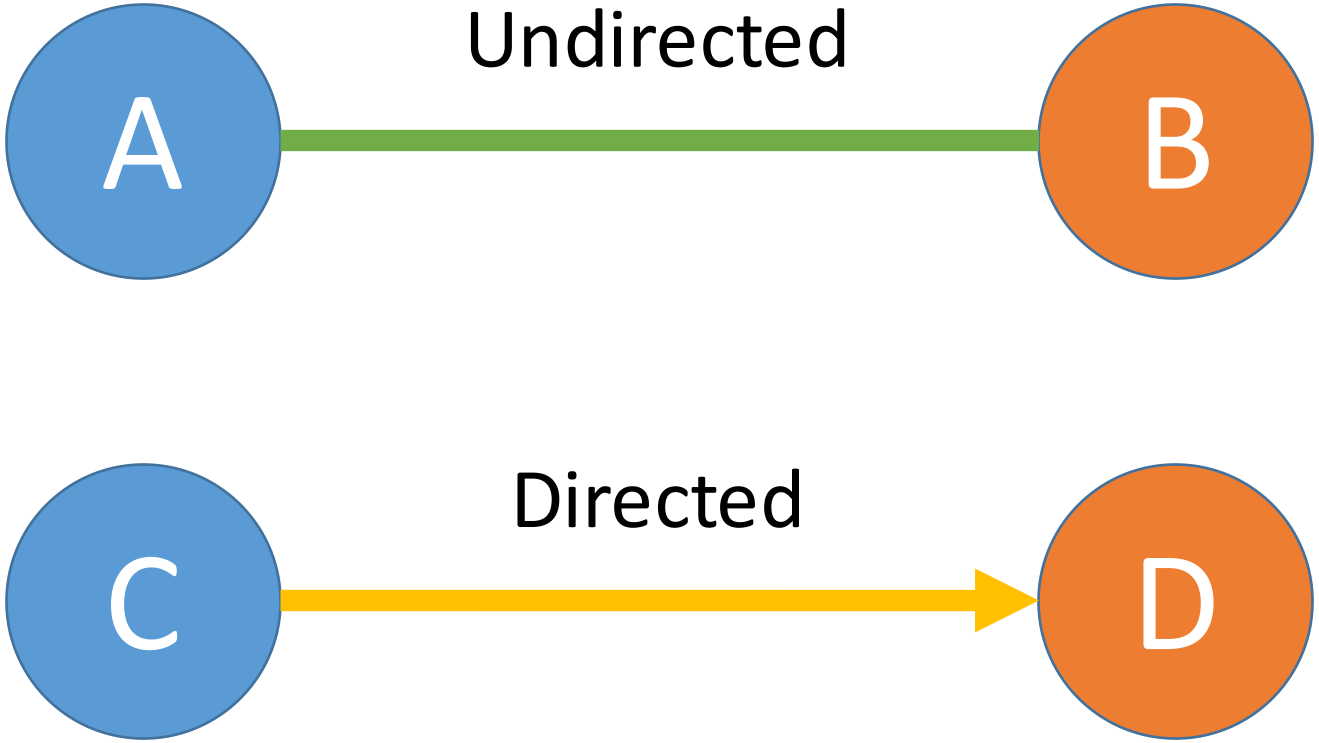
V is the standard symbol representing a set of graph vertices ($|V| = n$)

- Vertices are also sometimes referred to as nodes

E is the standard symbol representing a set of graph edges ($|E| = m$)

- Each edge contains pointers to two vertices, for example: ($v1$, $v2$)
- The order of the vertices may or may not matter

Directed and Undirected



Notation for Edges

(A, B) or (B, A)

(C, D)

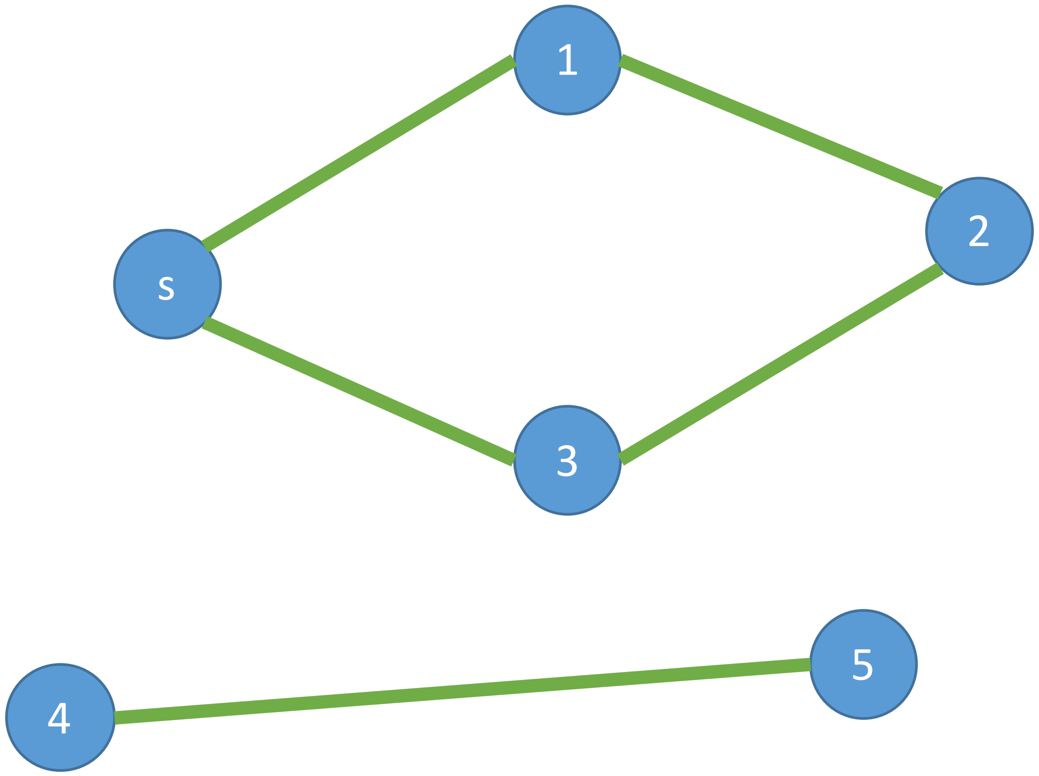
Graph Search and Connectivity

Goals:

- Find everything that is findable (a "path" from the start node exists)
- Don't explore anything twice (don't waste time)

- These operations are done in linear time,
- Note: it is often useful to consider O(n) algorithms as being "free"
  - (when compared to more complex tasks)

Findable



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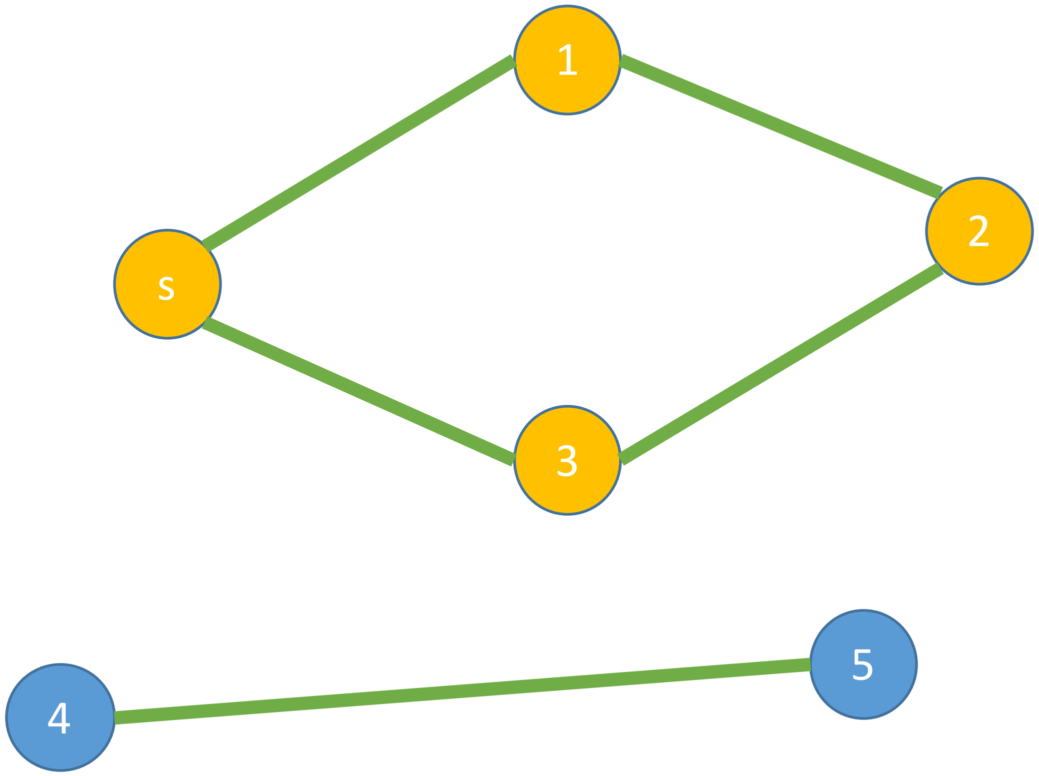

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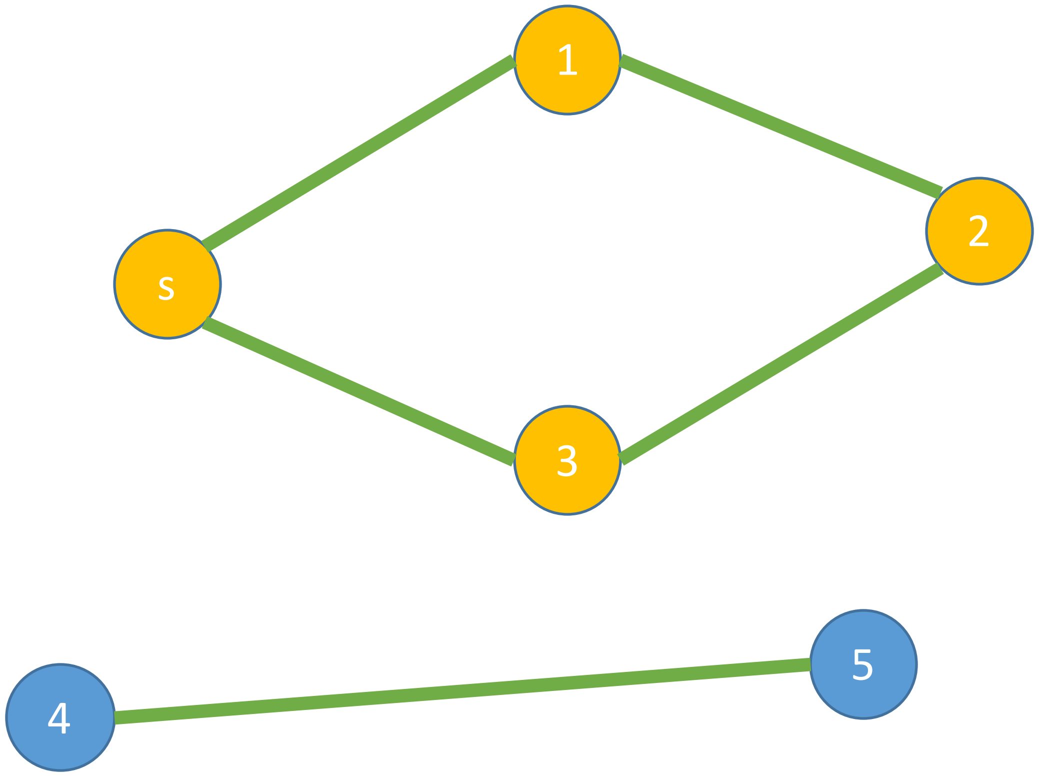

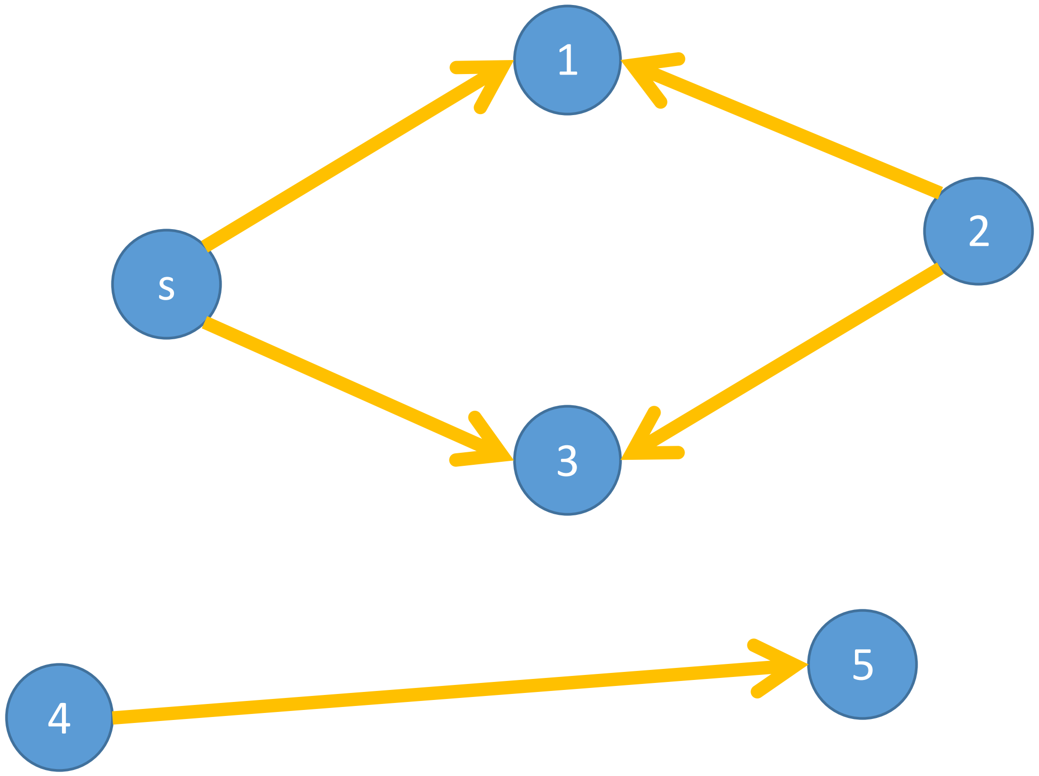

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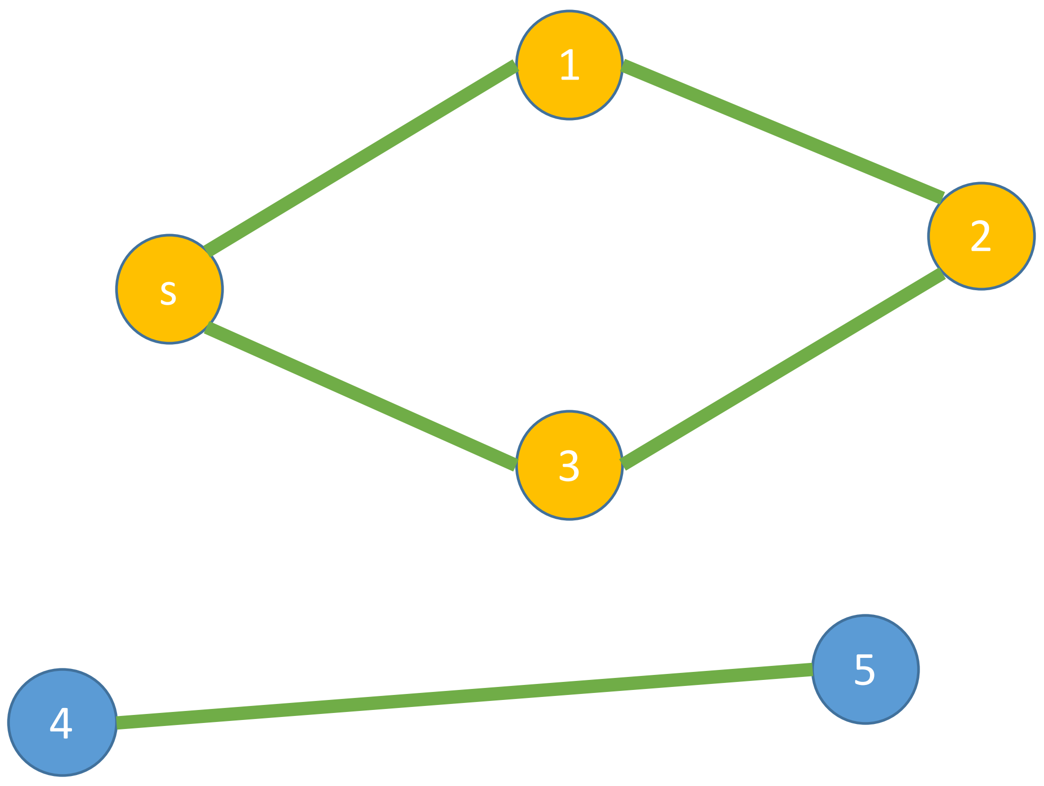
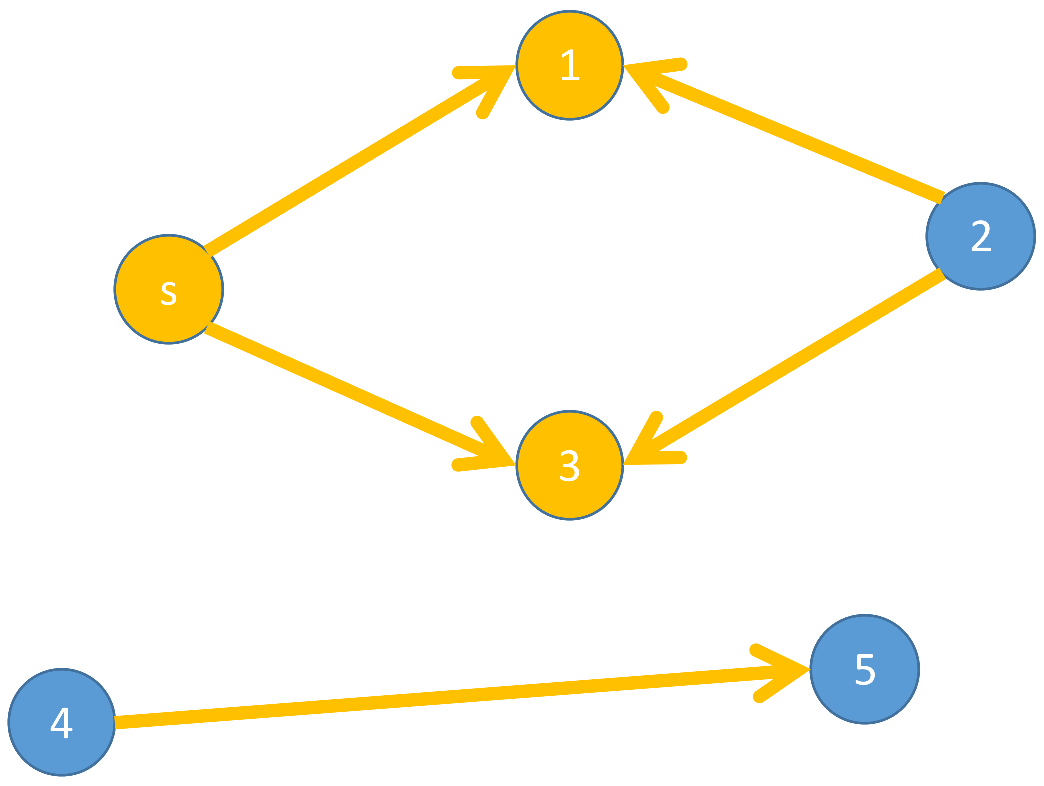
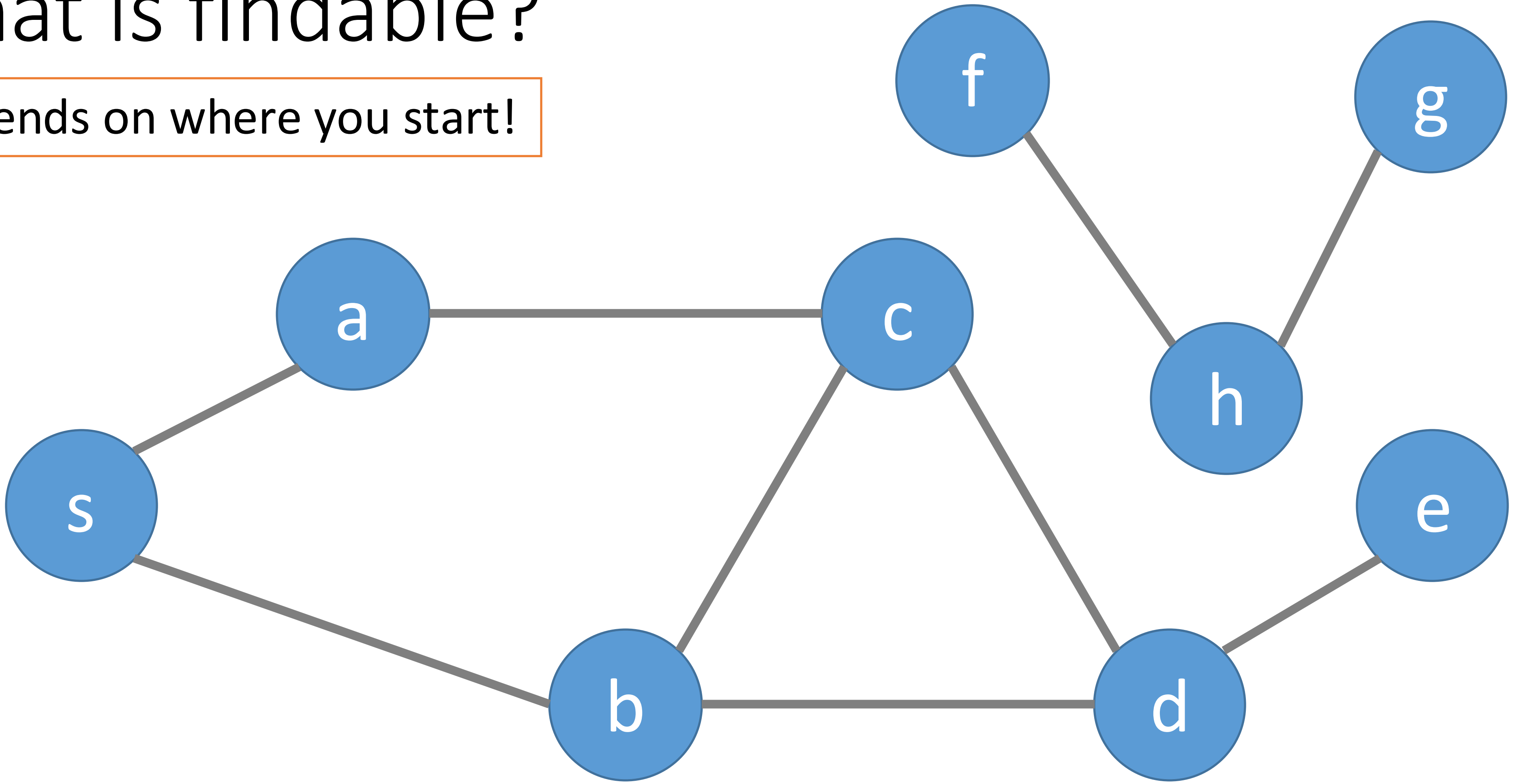




What is findable?

Depends on where you start!



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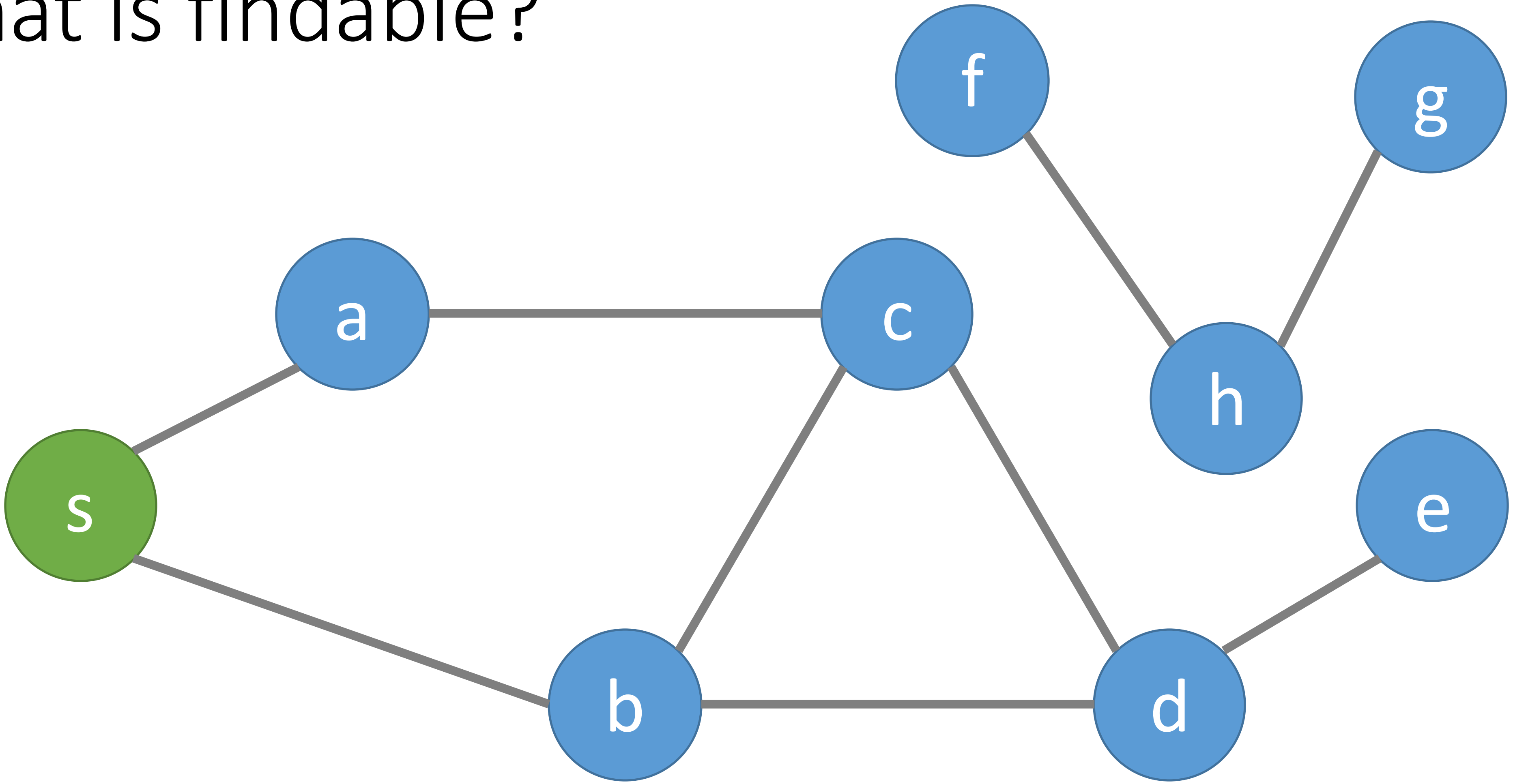

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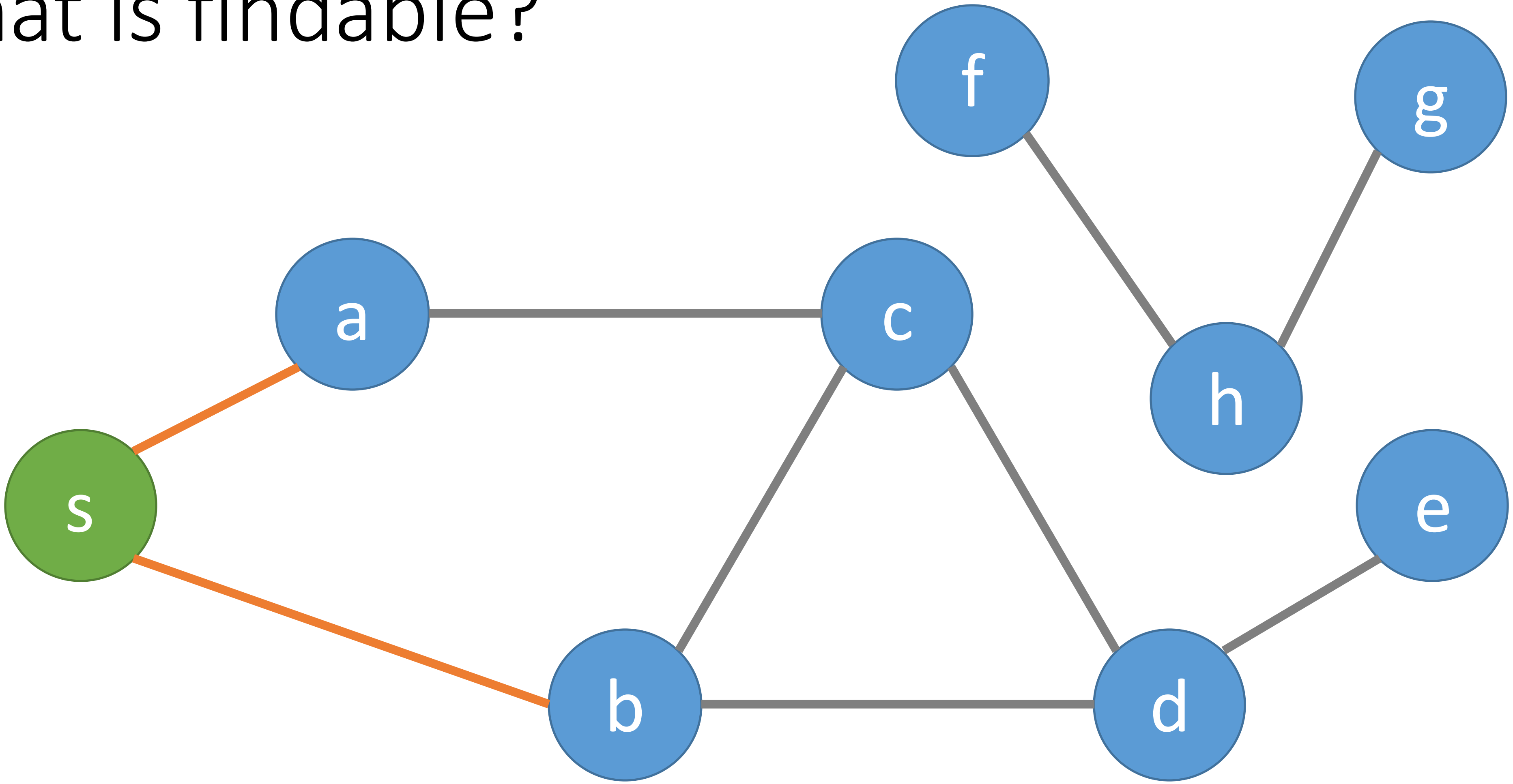

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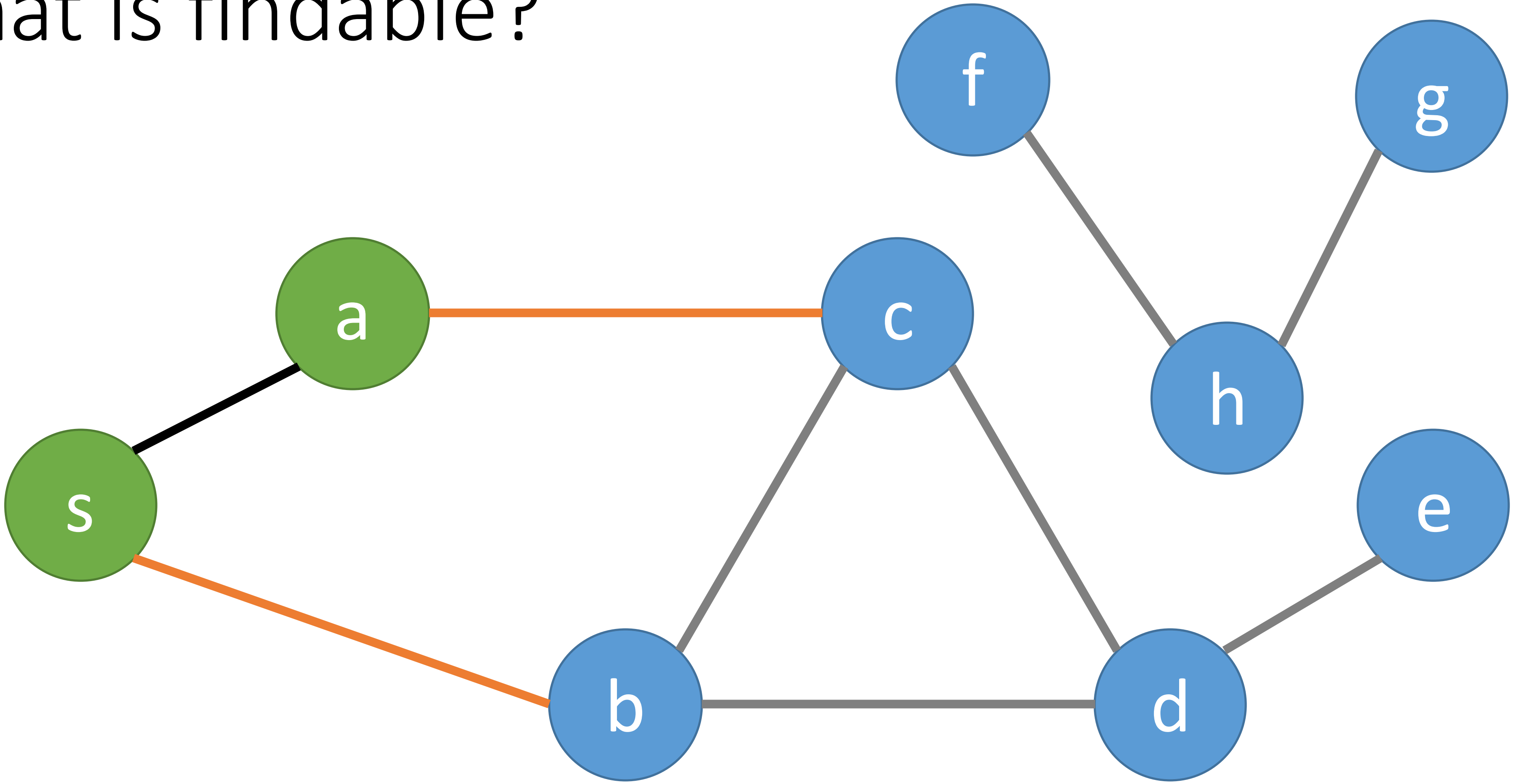

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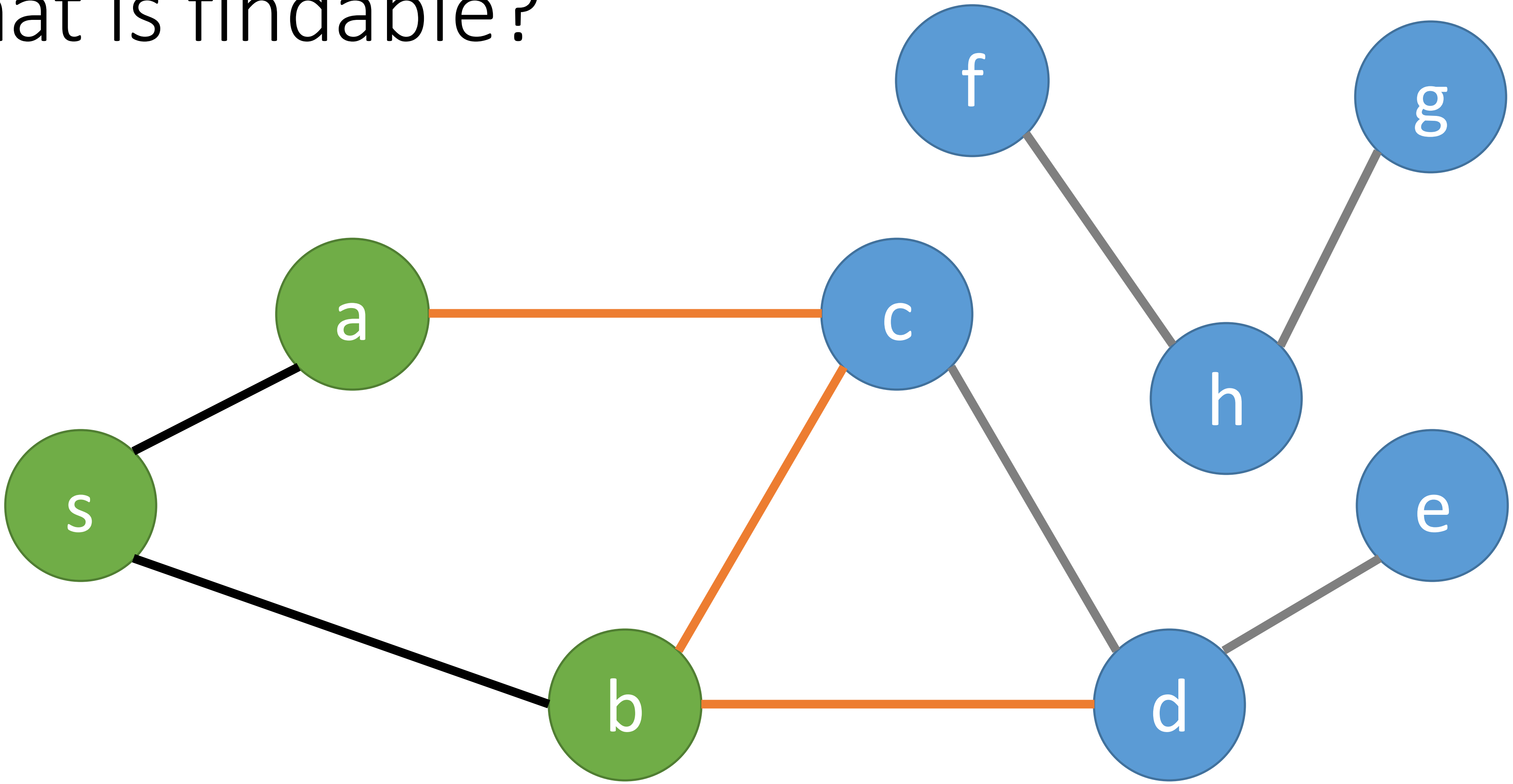

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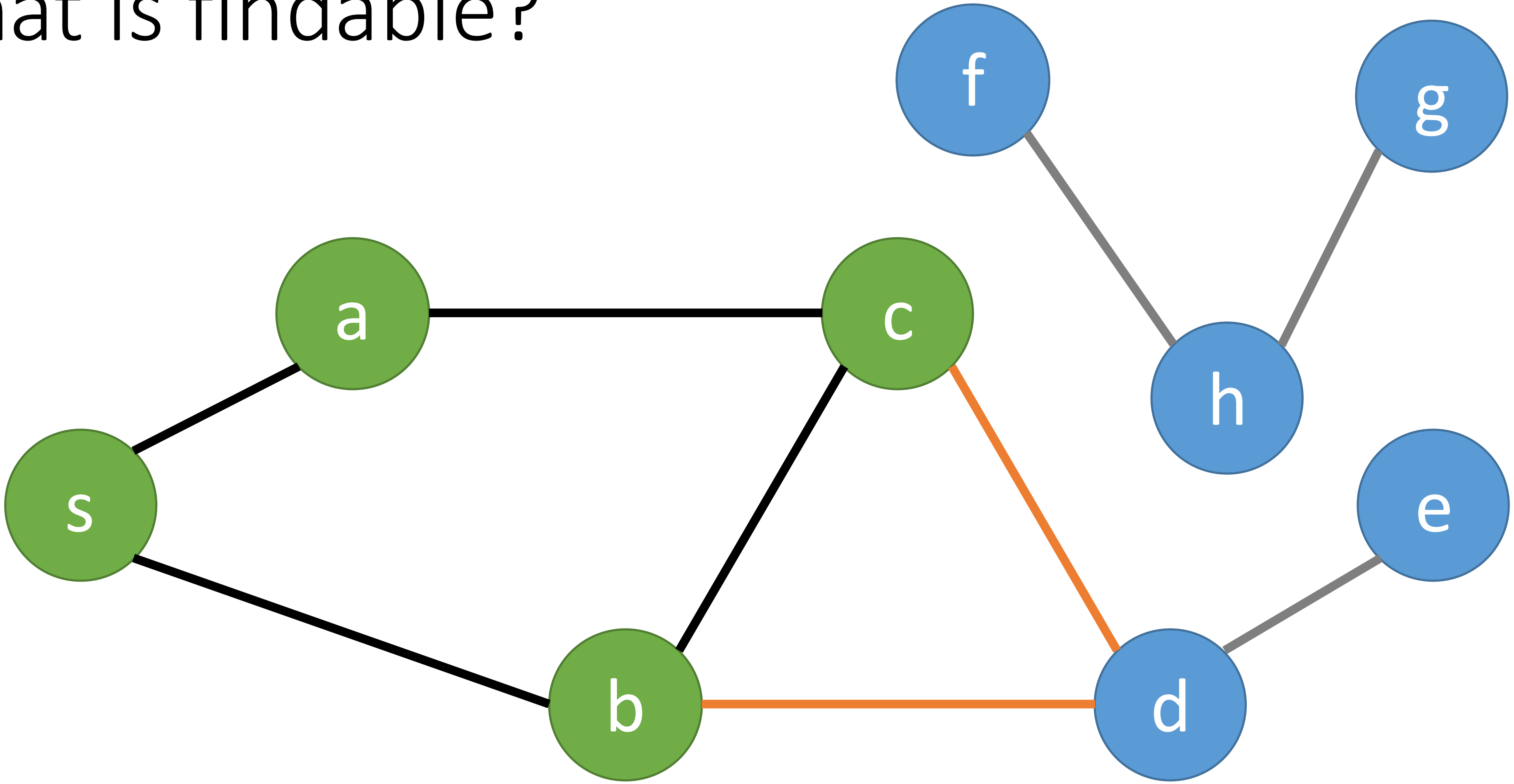

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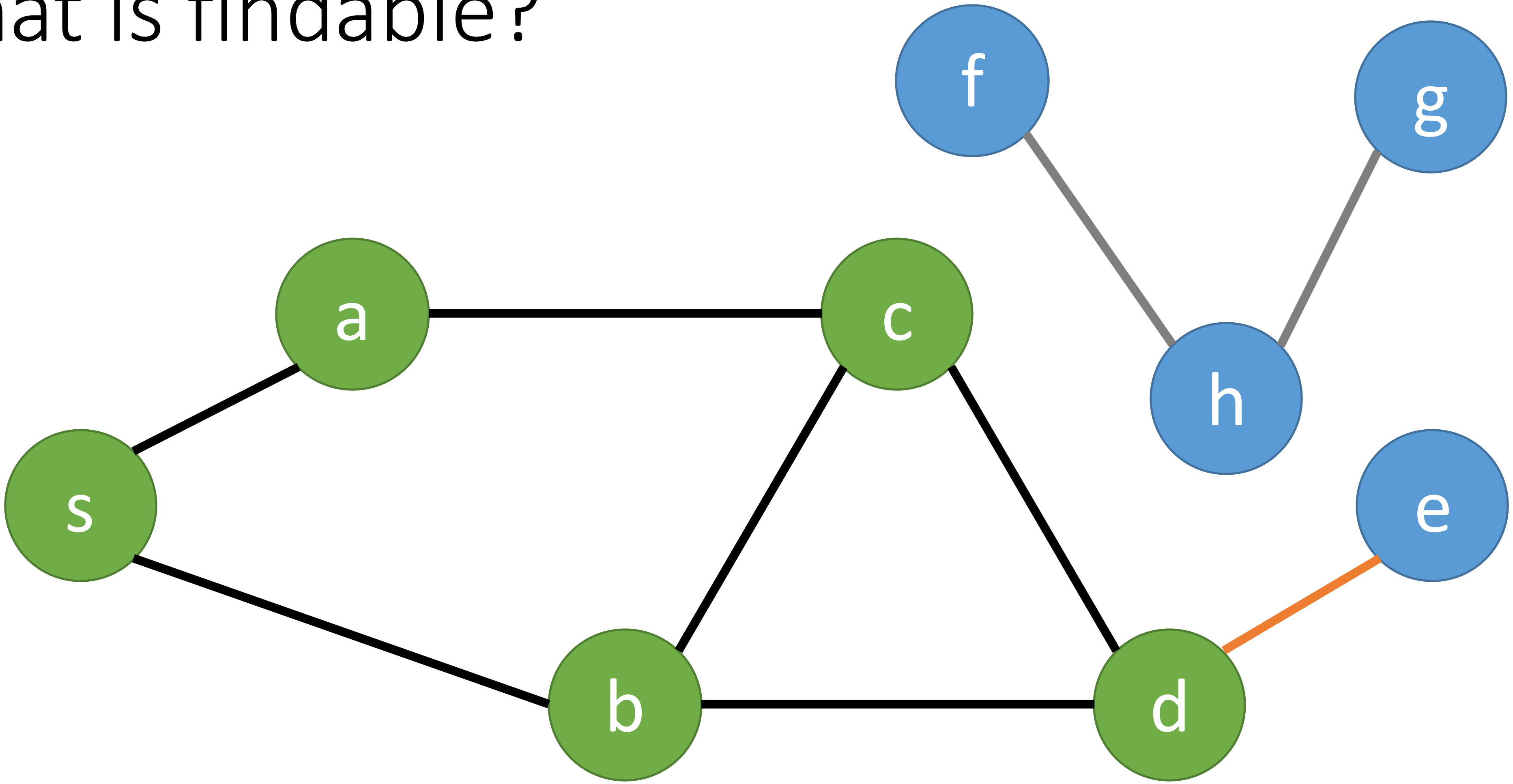

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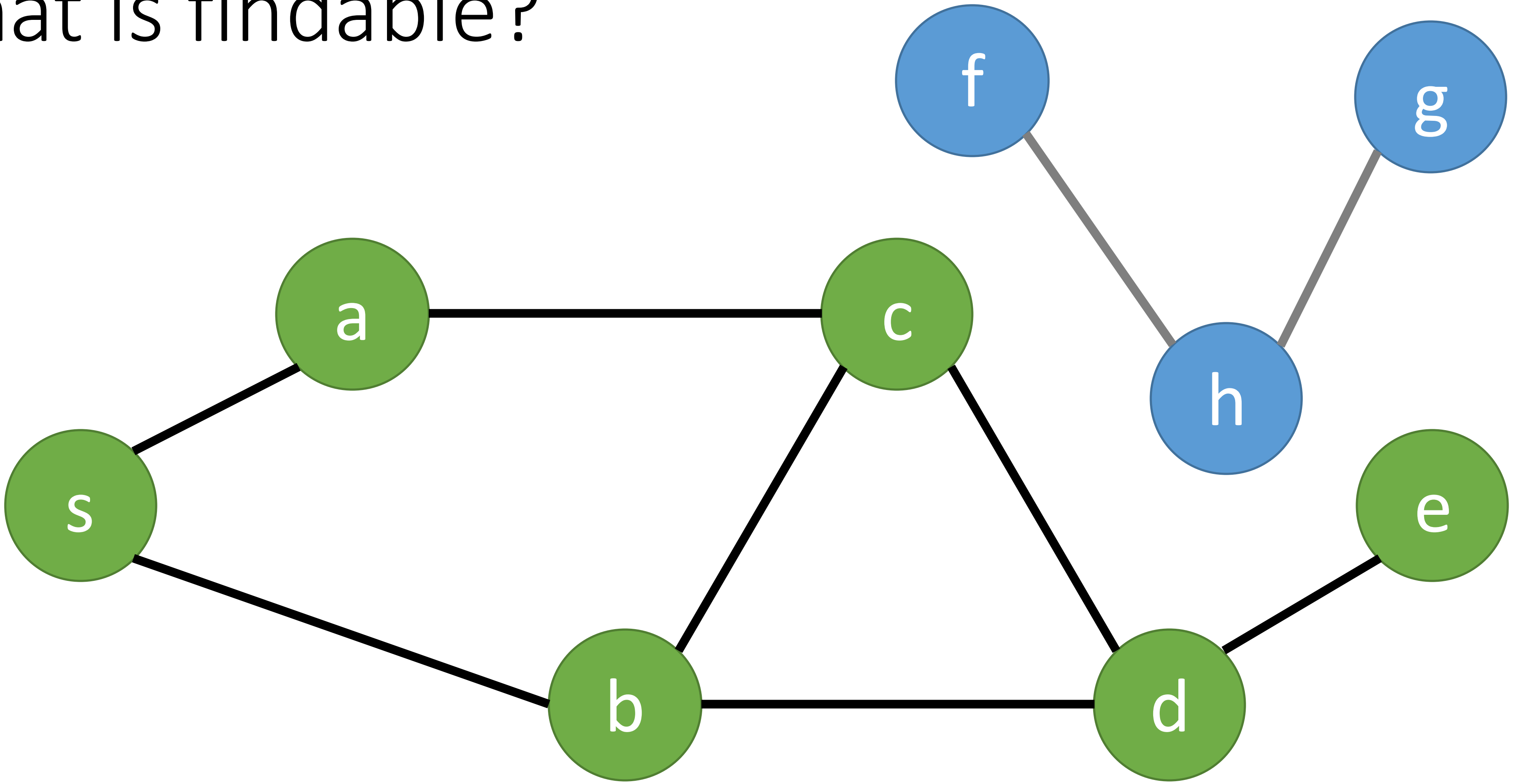
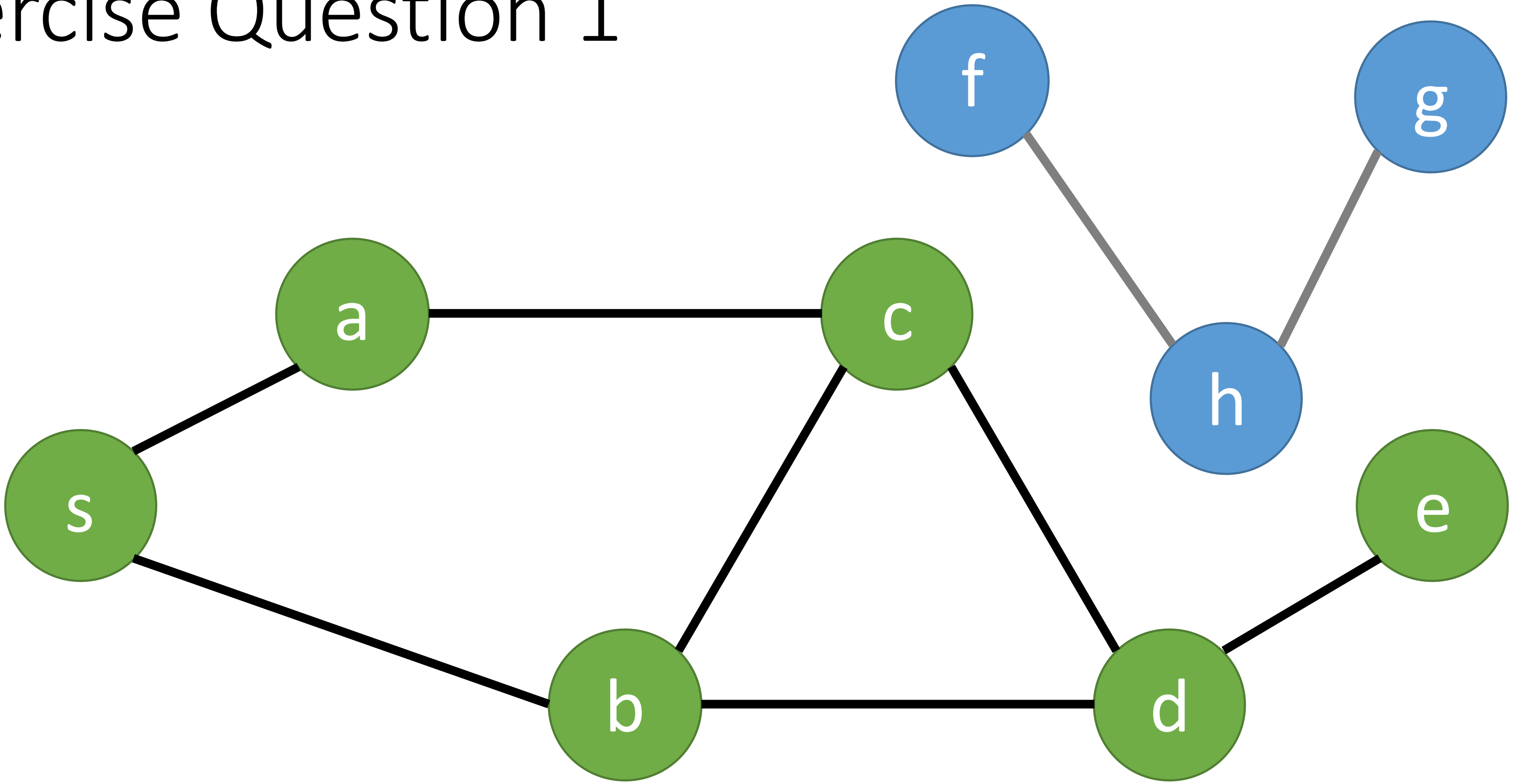


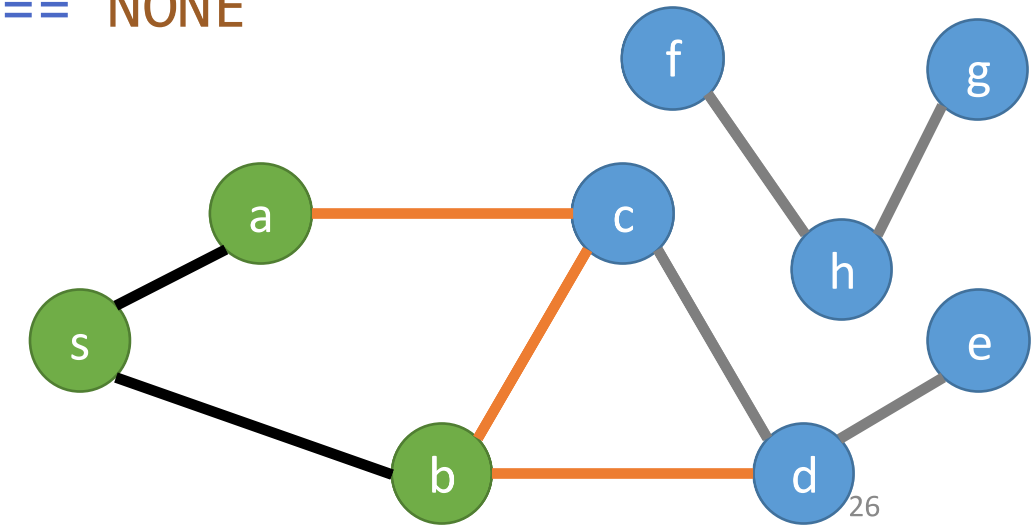
Exercise Question 1



General Algorithm

```
FUNCTION Connectivity(G, start_vertex)
    found = {v: FALSE FOR v IN G.vertices}
    found[start_vertex] = TRUE
    LOOP
        (vFound, vNotFound) = get_valid_edge(G.edges, found)
        IF vFound == NONE || vNotFound == NONE
            BREAK
        ELSE
            found[vNotFound] = TRUE
    RETURN found
```

Find an edge where one vertex has been found and the other vertex has not been found.



General Algorithm Outline

Claim: at the end of this algorithm

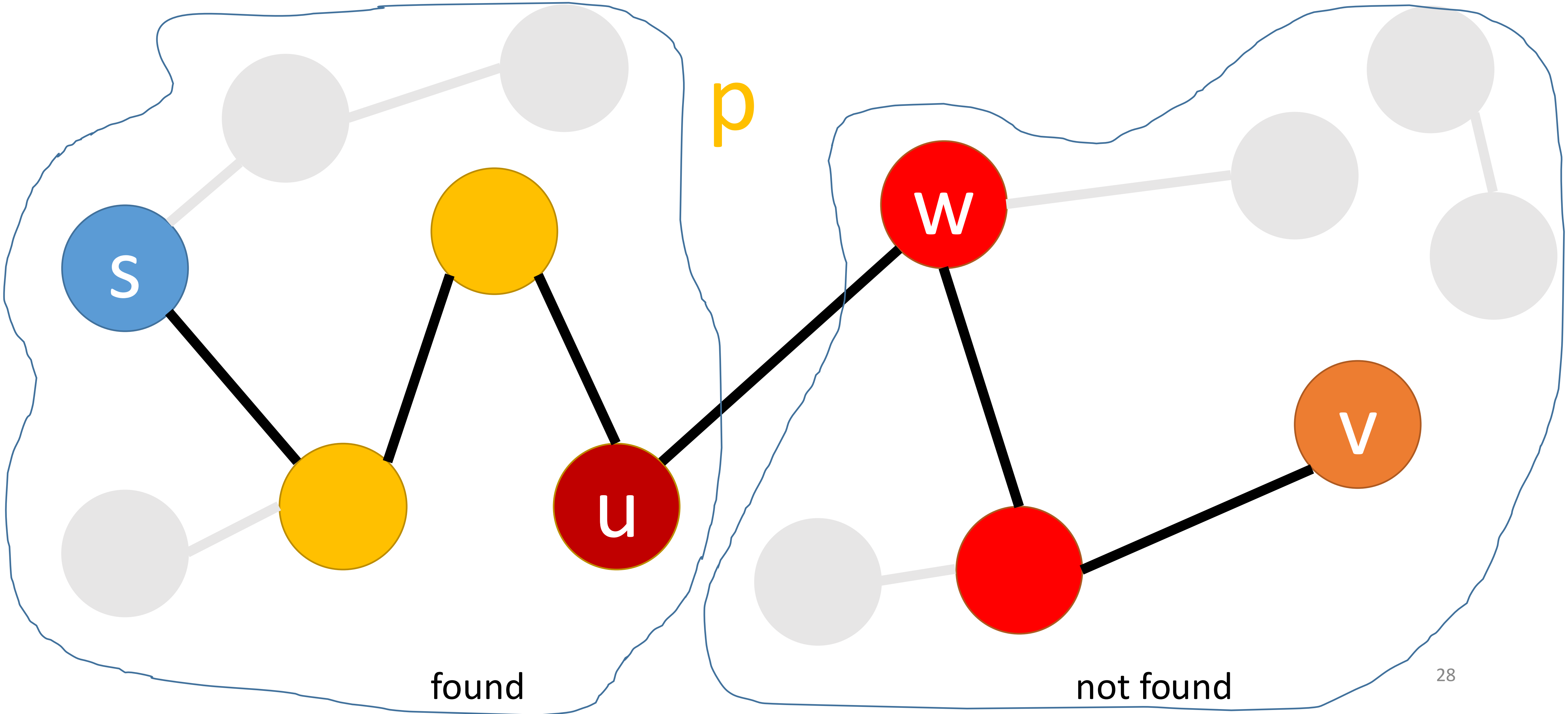
- if v is found
- Then there exists a path from s to v

Proof by contradiction

- Suppose the graph G has a path p from the vertex s to the vertexs v
- Also suppose that upon completion of the algorithm v was not found
- Thus, we have an edge (u, w) such that u is found, and w is not found
- This is **contradictory** to the termination condition of the algorithm

Contradiction

Suppose G has a path p from s to v
Also suppose that upon completion of the algorithm v was not found
Thus we have an edge (u, w) such that u is found and w is not found
This is **contradictory** to the termination condition of the algorithm



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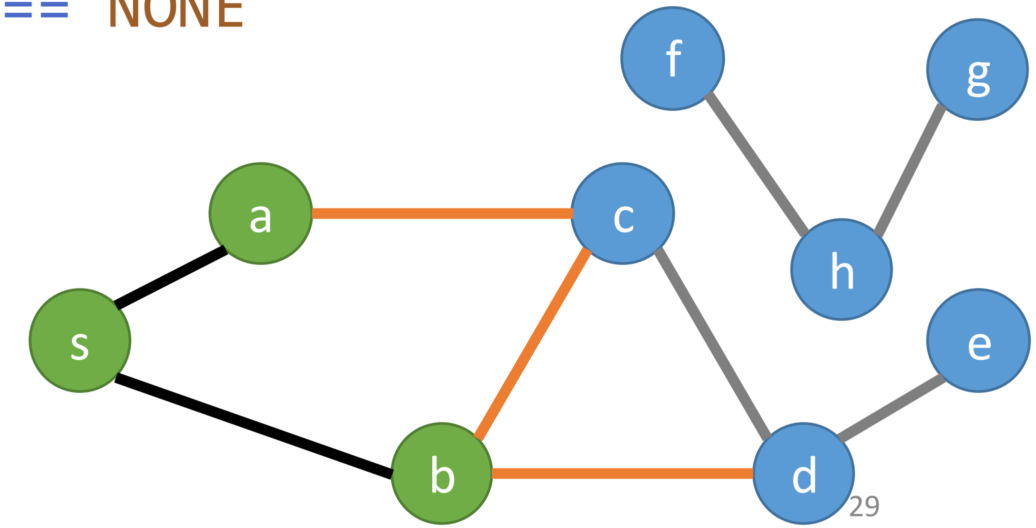
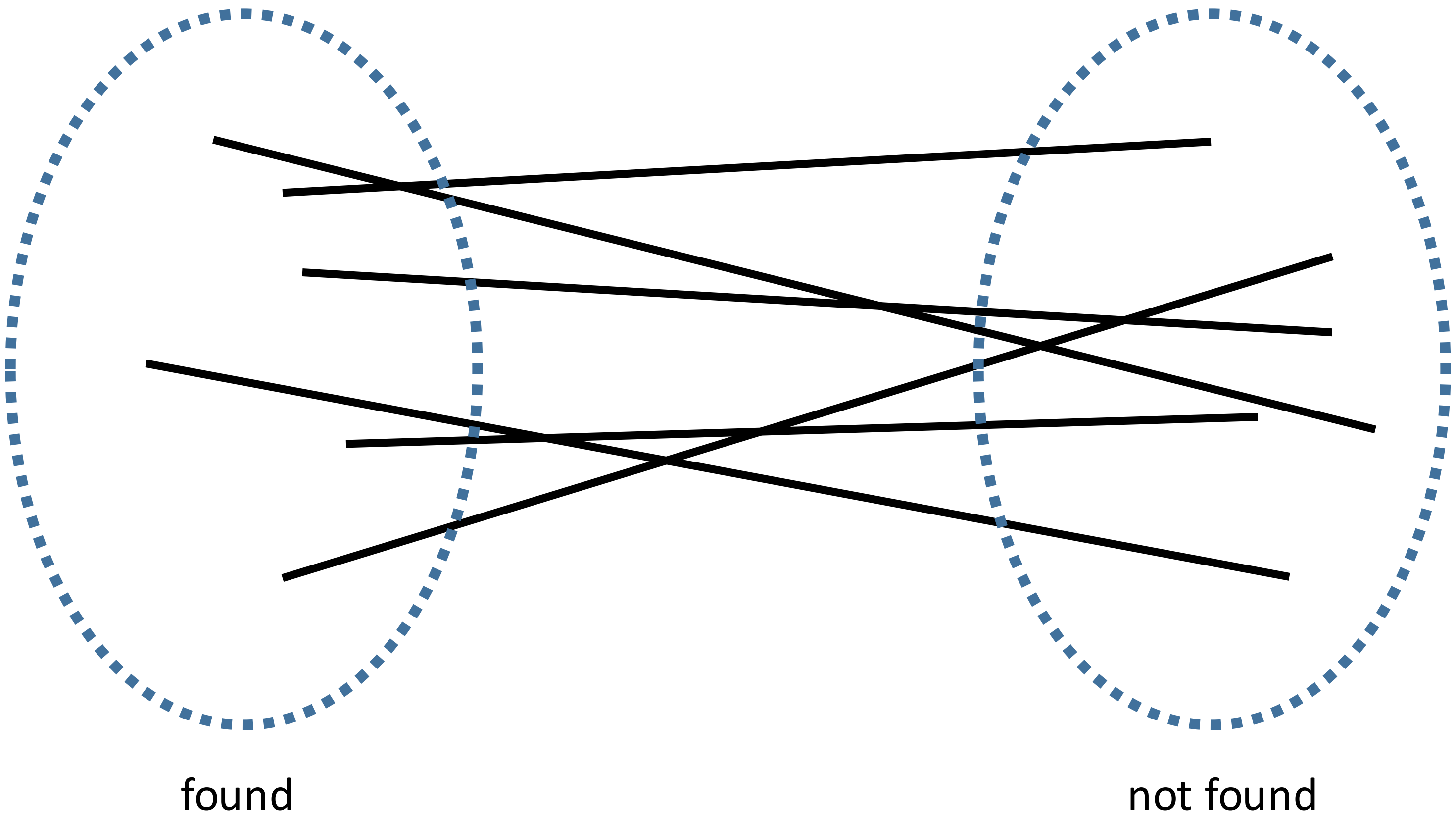


How do we choose the next edge?



Two common (and well studied) options

Breadth-First Search

- Explore the graph in layers
- *"Cautious"* exploration
- Use a FIFO data structure (can you think of an example?)

Depth-First Search

- Explore recursively
- A more *"aggressive"* exploration (we backtrack if necessary)
- Use a LIFO data structure (or recursion)