ALGORITHMS

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CS 140 – Spring 2024

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Introductions

Dr. | Prof | Professor

Dave | Kauchak

Pronouns: he/him/his

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Meet your neighbors

What's your name?

What year?

What has been your favorite CS class?

What's been your least favorite CS class?

What do you hope to learn (or get out of) this class?

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Algorithms

"For me, great algorithms are the poetry of computation. Just like verse, they can be terse, allusive, dense and even mysterious. But once unlocked, they cast a brilliant new light on some aspect of computing."
– Francis Sullivan

What is an algorithm?

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Example algorithms

sort a list of numbers
find a route from one place to another (cars, packet routing, phone routing, ...)
find the longest common substring between two strings
add two numbers
microchip wiring/design (VLSI)
solve sudoku
cryptography
compression (file, audio, video)
spell checking
pagerank
classify a web page

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To the course webpage…

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Log properties

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Log properties

$x = a^b$

What is b?

$b = \log_a x$

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Log properties

$\log_a x$ $x = a^b$

a raised to what exponent is *x*?

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Log properties

$\log_a a = ?$

a raised to what exponent is *x*?

$\log_a x = ?$ *if* $x > a$

$\log_a x = ?$ *if* $x < a$

greater than 1 less than 1 exactly 1

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Log properties

$\log_a a = 1$

a raised to what exponent is *x*?

$\log_a x > 1$ *if* $x > a$

$\log_a x < 1$ *if* $x < a$

greater than 1 less than 1 exactly 1

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Log properties

Which is bigger?

1) $\log_3 2$
2) $\log_4 2$

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Log properties

Which is bigger?

1) $\log_3 2 = x$ ➡ $2 = 3^x$

2) $\log_4 2 = x$ ➡ $2 = 4^x$

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Log properties

$\log(ab) = \log a + \log b$

Which is bigger?

1) $\log_3 27$

2) $\log_4 36$

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Log properties

$\log(ab) = \log a + \log b$

Which is bigger?

1) $\log_3 27 = \log_3 3 + \log_3 3 + \log_3 3$

2) $\log_4 36 = \log_4 4 + \log_4 3 + \log_4 3$

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Log properties

$\log(ab) = \log a + \log b$

Which is bigger?

1) $\log_3 27 = \overset{1}{\log_3 3} + \overset{1}{\log_3 3} + \overset{1}{\log_3 3}$

2) $\log_4 36 = \overset{1}{\log_4 4} + \overset{<1}{\log_4 3} + \overset{<1}{\log_4 3}$

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Log properties

log (a/b) = log a - log b

Which is bigger?

1) $\log_3 4.5$

2) $\log_4 8$

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Log properties

log (a/b) = log a - log b

Which is bigger?

1) $\log_3 4.5 = \log_3 9 - \log_3 2$

2) $\log_4 8 = \log_4 16 - \log_4 2$

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Log properties

log (a/b) = log a - log b

Which is bigger?

1) $\log_3 4.5 = \overset{2}{\log_3 9} - \log_3 2$

2) $\log_4 8 = \overset{2}{\log_4 16} - \log_4 2$

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Log properties

$\log b^x = x \log b$

$\log b^x = \underbrace{\log b + \log b + \ldots + \log b}_{x \text{ times}}$

$\log b^x = \sum_{i=1}^{x} \log b$

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Log properties

$\log_a b = \dfrac{\log b}{\log a}$

allows you to change bases!

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Log properties

$\log_a a = ?$

$\log_a b = \dfrac{\log b}{\log a}$

$\log_a x = ?$ *if* $x > a$

$\log_a x = \dfrac{\log x}{\log a}$

$\log_a x = ?$ *if* $x < a$

greater than 1 less than 1 exactly 1

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Log properties

$\log_a a = ?$

$\log_a b = \dfrac{\log b}{\log a}$

$\log_a x > 1$ *if* $x > a$

$\log_a x = \dfrac{\log x}{\log a}$

$\log_a x < 1$ *if* $x < a$

greater than 1 less than 1 exactly 1

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Log properties

Which is bigger?

$\log_a b = \dfrac{\log b}{\log a}$

1) $\log_3 2$

2) $\log_4 2$

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Log properties

Which is bigger?

$$\log_a b = \frac{\log b}{\log a}$$

1) $\log_3 2 = \frac{\log 2}{\log 3}$

2) $\log_4 2 = \frac{\log 2}{\log 4}$

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Exponent properties

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Exponent properties

$a^b \cdot a^c =$

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Exponent properties

$a^b \cdot a^c = a^{b+c}$

$$a^b \cdot a^c = \underbrace{a \cdot a \ldots \cdot a}_{\text{b times}} \cdot \underbrace{a \ldots \cdot a}_{\text{c times}}$$

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Exponent properties

$a^b \cdot a^c = a^{b+c}$

Which is bigger?

1) x^2

2) $x^{2.1}$

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Exponent properties

$a^b \cdot a^c = a^{b+c}$

Which is bigger (x > 1)?

1) x^2

2) $x^{2.1} = x^2 x^{0.1}$

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Exponent properties

$a^b \cdot a^c = a^{b+c}$

Which is bigger (for x > 1)?

1) 2^{x+1}

2) $2^x + 2$

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Exponent properties

$a^b \cdot a^c = a^{b+c}$

Which is bigger (for x > 1)?

1) $2^{x+1} = 2^x 2$

2) $2^x + 2$

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Exponent properties

$(ab)^c =$

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Exponent properties

$(ab)^c = a^c b^c$

$(ab)^c = \underbrace{ab \cdot ab \cdot \ldots \cdot ab}_{\text{c times}}$

$= \underbrace{a \cdot a \cdot \ldots \cdot a}_{\text{c times}} \cdot \underbrace{b \cdot b \cdot \ldots \cdot b}_{\text{c times}}$

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Exponent properties

$(ab)^c = a^c b^c$

Which is bigger?

1) 12^3
2) 4^6

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Exponent properties

$(ab)^c = a^c b^c$

Which is bigger?

1) $12^3 = (4*3)^3 = 4^3 3^3$
2) $4^6 = 4^3 4^3$

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Exponent properties

$(a^b)^c =$

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Exponent properties

$(a^b)^c = a^{bc}$

$$(a^b)^c = \underbrace{\underbrace{(a \cdot a \cdot \ldots \cdot a)}_{\text{b times}} \cdot \underbrace{(a \cdot a \cdot \ldots \cdot a)}_{\text{b times}} \cdot \ldots \cdot \underbrace{(a \cdot a \cdot \ldots \cdot a)}_{\text{b times}}}_{\text{c times}}$$

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Exponent properties

$(a^b)^c = a^{bc}$

Which is bigger ($x > 1$)?

1) 2^{2x}

2) 4^x

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Exponent properties

$(a^b)^c = a^{bc}$

Which is bigger ($x > 1$)?

1) $2^{2x} = (2^2)^x = 4^x$

2) 4^x

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Pseudocode

A way to discuss how an algorithm works that is language agnostic and without being encumbered with actual implementation details.

Should give enough detail for a person to undersand, analyze and implement the algorithm.

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Pseudocode examples

```
MYSTERY1(A)
x ← −∞
for i ← 1 to length[A]
        if A[i] > x
                x ← A[i]
return x

MYSTERY2(A)
for i ← 1 to ⌊length(A)/2⌋
        swap A[i] and A[length(A) − (i − 1)]
```

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Pseudocode convections

array indices start at 1 not 0

we may use notation such as ∞, which, when translated to code, would be something like Integer.MAX VALUE

use shortcuts for simple function (e.g. swap) to make pseudocode simpler

we'll often use ← instead of = to avoid ambiguity

indentation specifies scope

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Proofs

What is a proof?

- A deductive argument showing a statement is true based on previous knowledge (axioms)

Why are they important/useful?

- Allows us to be sure that something is true
- In algs: allow us to prove properties of algorithms

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Proof techniques?

example/counterexample

enumeration

by cases

by inference (aka direct proof)

trivially

contrapositive

contradiction

induction (strong and weak)

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