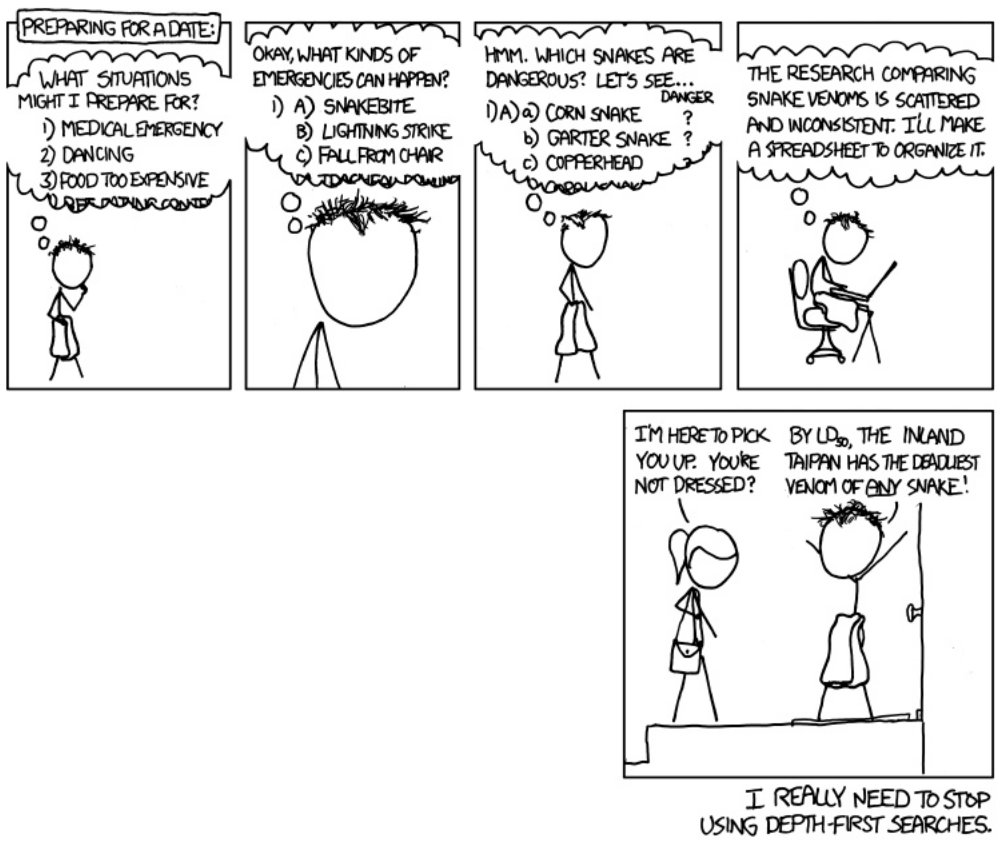
CS140 - Assignment 8

Due: Sunday, Mar. 31st at 11:59pm



http://xkcd.com/761/

Notes:

- Many of the algorithms below can be accomplished by either modifying the graph and applying a known algorithm or slightly modifying a known algorithm. Try thinking of these *first* as they will save you a lot of work, and writing :) I don't expect long answers, but be precise.
- You will be graded on efficiency!
- If not specified in the problem, you may assume whatever graph representation makes your algorithm more efficient (adjacency list or adjacency matrix). State which one you are using.

1. **[8 points]** State whether the following statements about a graph G that is undirected and connected are true or false and justify your answer.

   (a) Prim's algorithm works correctly if G has negative edge weights.

   (b) The shortest path between two nodes is always part of some MST.

2. **[5 points]** Write pseudocode for an algorithm which, given an undirected graph G and a particular edge e in it, determines whether G has a cycle containing e. What is the runtime of this algorithm?

3. **[8 points]** Often there are multiple shortest paths between nodes of a graph. Write pseudocode for an algorithm that given an undirected, unweighted graph G and nodes $u, v \in V$, outputs the number of distinct shortest paths from u to v. What is the running time?

4. **[5 points]** Given a directed graph $G = (V, E)$ with positive edge weights and a particular node $v_i \in V$, give an efficient algorithm for finding the shortest paths between **all pairs of nodes**, with the one restriction that these paths must all pass through v_i. Give the runtime of your algorithm. Points will be deducted for an inefficient algorithm.

   Hints:

   - Any path in this problem can be seen as two parts, the part to v_i and the part from v_i.
   - Look at how we determined if a graph was strongly connected.

5. **[5 points]** If a graph does not have a negative cycle, when calculating the shortest paths from a given vertex using the Bellman-Ford algorithm, we can stop early and do not need to do all $|V| - 1$ iterations and will still have a correct answer for all the shortest paths from that vertex. Describe how to modify the Bellman-Ford algorithm to stop early when all of the distances are already correct.

6. **[6 points]** Given an undirected graph G with nonnegative edge weights $w_e \geq 0$. Suppose you have calculated the minimum spanning tree of G and also the shortest paths to all nodes from a particular node $s \in V$. Now, suppose that each edge weight is increased by 1, i.e. the new weights are $w'_e = w_e + 1$.

   (a) (3 points) Does the minimum spanning tree change? Give an example where it does or prove that it cannot change.

   (b) (3 points) Do the shortest paths from s change? Given an example where it does or prove that it cannot change.