

## CS140 - Sample Problems for Midterm 2

Below are some practice problems to help give you study for the upcoming midterm. Note that not all of these would necessarily be good exam problems, but are there to provide you with some additional practice on the materials.

1. A student wants to walk up a staircase with  $n$  steps. They can skip up to 2 steps at a time: in other words, they can go from step  $k$  to step  $k + 1$ , they can skip one step and go from step  $k$  directly to step  $k + 2$ , or they can skip 2 steps and go from step  $k$  directly to step  $k + 3$ . Let  $Num(i)$  be the number of ways that the student can get to step  $i$ . Write and justify a recursive formula for  $Num(i)$ . If you were to implement your solution as an iterative dynamic program, what would the pseudocode be? What are bounds on the space and time requirements of your dynamic programming algorithm?

2. Change revisited

In class we discussed the change problem and, in particular, proved that a greedy strategy was optimal for US denominations (penny=1, nickel=5, dime=10, quarter=25).

The change problem in general can be specified as: make change for an amount of money  $C$  with as few coins as possible for coin denominations with values  $v_1 > v_2 > \dots > v_n$  (all integers), where  $v_n = 1$ .

- (a) The greedy approach only works for certain coin values. Given an example of coin denominations and a target amount such that the greedy strategy does not provide the optimal solution.
- (b) Give a dynamic programming solution for the function which calculates the minimum number of coins necessary to make the amount. Make sure you give both a written description of what's stored in the dynamic programming table as well as the recursive definition.
- (c) What is the size of your dynamic programming table? What entry contains the answer? What is the running time of your algorithm?
- (d) Fill out the dynamic programming table assuming the denominations are  $v_1 = 6, v_2 = 5, v_3 = 1$  and the value  $C = 10$ .