LINEAR PROGRAMMING

David Kauchak
CS 140 – Spring 2023

1

Admin

Assignment 11 (last one)

LCs

Monday: review

Wednesday: No class - office hours

2

Final logistics

Available on Monday via Gradescope

Graduating seniors: must take by Thursday (4/4) at noon

Everyone else: must take by end of day on Wednesday (4/10)

Open notes, book, assignments

3 hours to take the exam

Review slides posted for Monday which includes overview of topics

3

Linear programming

A linear function is a function of n variables defined by

$$f(x_1, x_2, \ldots, x_n) = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$$

A linear equality is a linear function with an equality constraint

$$f(x_1, x_2, \ldots, x_n) = a_1 x_1 + a_2 x_2 + \ldots + a_n x_n = b$$

A linear inequality is a linear function with an inequality constraint

$$f(x_1, x_2, \ldots, x_n) = a_1 x_1 + a_2 x_2 + \ldots + a_n x_n \leq b$$
$$f(x_1, x_2, \ldots, x_n) = a_1 x_1 + a_2 x_2 + \ldots + a_n x_n \geq b$$

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Linear programming

A linear programming problem consists of two parts

1. a linear function to maximize or minimize

$$\text{maximize } c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

2. subject to a set of linear constraints

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b$$

$$\ldots$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \geq b$$

5

For example

$$\text{maximize } x_1 + x_2$$ objective function

subject to

$$\begin{aligned} 4x_1 - x_2 &\leq 8 \\ 2x_1 + x_2 &\leq 10 \\ 5x_1 - 2x_2 &\geq -2 \\ x_1, x_2 &\geq 0 \end{aligned}$$

constraints

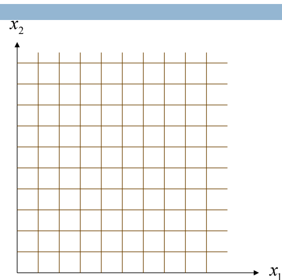
6

For example

$$\text{maximize } x_1 + x_2$$

subject to

$$\begin{aligned} 4x_1 - x_2 &\leq 8 \\ 2x_1 + x_2 &\leq 10 \\ 5x_1 - 2x_2 &\geq -2 \\ x_1, x_2 &\geq 0 \end{aligned}$$



7

For example

$$\text{maximize } x_1 + x_2$$

subject to

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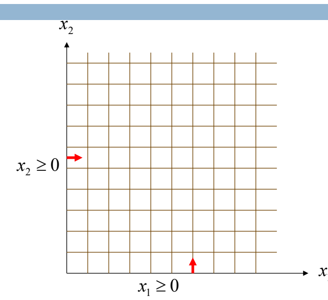


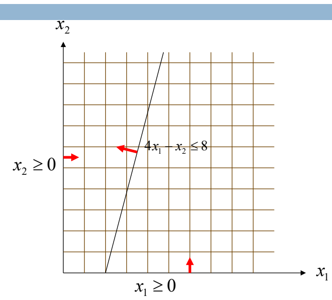
8

For example

maximize $x_1 + x_2$

subject to

$$\begin{array}{rcr} 4x_1 - x_2 & \le & 8 \\ 2x_1 + x_2 & \le & 10 \\ 5x_1 - 2x_2 & \ge & -2 \\ x_1, x_2 & \ge & 0 \end{array}$$



$x_2 \ge 4x_1 - 8$

9

For example

maximize $x_1 + x_2$

subject to

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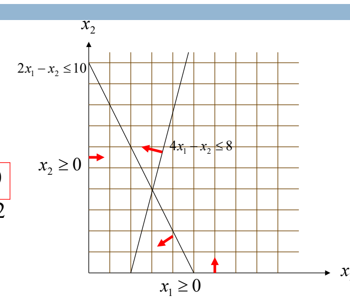


$x_2 \le -2x_1 + 10$

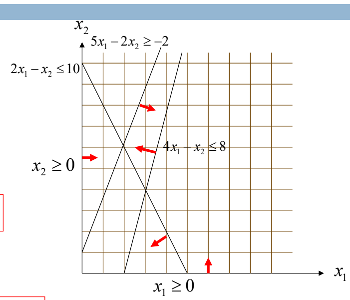
10

For example

maximize $x_1 + x_2$

subject to

$$\begin{array}{rcr} 4x_1 - x_2 & \le & 8 \\ 2x_1 + x_2 & \le & 10 \\ 5x_1 - 2x_2 & \ge & -2 \\ x_1, x_2 & \ge & 0 \end{array}$$



$x_2 \le \frac{5}{2}x_1 + 1$

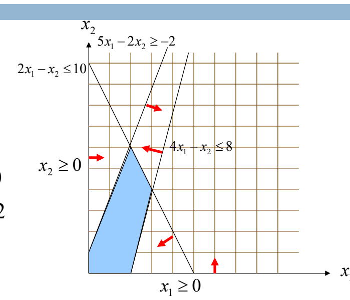
11

For example

maximize $x_1 + x_2$

subject to

$$\begin{array}{rcr} 4x_1 - x_2 & \le & 8 \\ 2x_1 + x_2 & \le & 10 \\ 5x_1 - 2x_2 & \ge & -2 \\ x_1, x_2 & \ge & 0 \end{array}$$



the constraints define
the **feasible region**

12

For example

maximize $x_1 + x_2$

subject to

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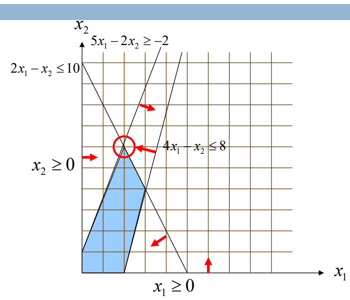

What are the values of x_i that maximize the function within the feasible region?

13

Another example

A chocolatier has two products: a basic product and a deluxe. The company makes x_1 boxes of the basic per day at a profit of $1 each and x_2 boxes of the deluxe at a profit of $6 each. The daily demand is 200 of the basic and 300 of the deluxe. The workforce can create 400 boxes of chocolate per day. How much of each should they create to maximize profits?

14

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How many variables do we need to model the problem?

15

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x_1 = number of boxes per day of basic

x_2 = number of boxes per day of deluxe

16

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What are the constraints?

17

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$$\begin{aligned} x_1 &\leq 200 \\ x_2 &\leq 300 \end{aligned}$$

18

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$$\begin{aligned} x_1 &\leq 200 \\ x_2 &\leq 300 \\ x_1 + x_2 &\leq 400 \end{aligned}$$

any others?

19

Another example

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$$\begin{aligned} x_1 &\leq 200 \\ x_2 &\leq 300 \\ x_1 + x_2 &\leq 400 \\ x_1, x_2 &\geq 0 \end{aligned}$$

What function are we trying to maximize/minimize?

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Another example

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$$\text{maximize } x_1 + 6x_2$$

subject to

$$\begin{aligned} x_1 &\le 200 \\ x_2 &\le 300 \\ x_1 + x_2 &\le 400 \\ x_1, x_2 &\ge 0 \end{aligned}$$

21

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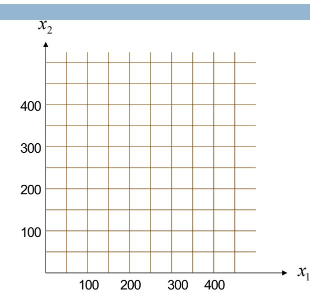

22

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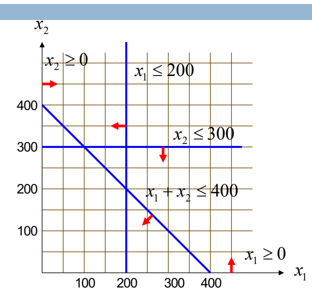

Where is the feasibility region?

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Another example

$$\text{maximize } x_1 + 6x_2$$

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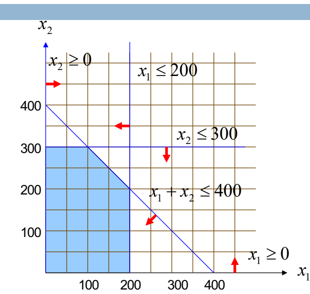

Where is the maximum within the feasibility region?

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Another example

maximize $x_1 + 6x_2$

subject to

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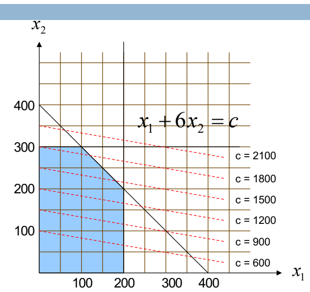

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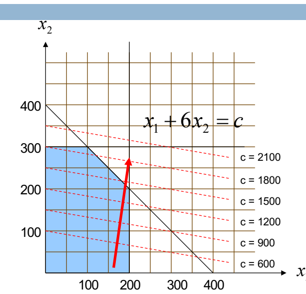

to maximize, move as far in this direction as the constraints allow

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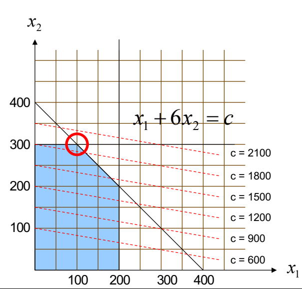
Solutions to LPs

A solution to an LP is a **vertex** on the feasibility polygon

- the very last feasible point in the direction of improving objective function



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Except?

maximize $x_1 + x_2$

subject to

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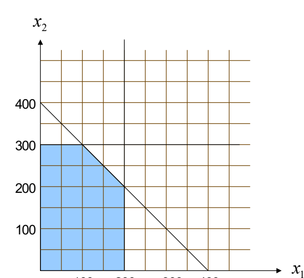

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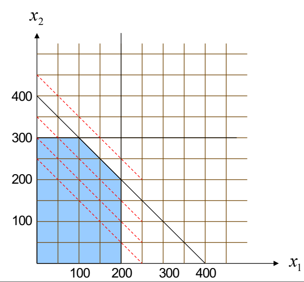

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Solutions to LPs

A solution to an LP is a **vertex** on the feasibility polygon

- the very last feasible point in the direction of improving objective function

Except?

$$\text{maximize } x_1 + 6x_2$$

subject to

$$\begin{aligned} x_1 &\le 1 \\ x_1 &\ge 2 \\ &\ldots \end{aligned}$$

linear program is **infeasible**

30

Solutions to LPs

A solution to an LP is a **vertex** on the feasibility polygon

- the very last feasible point in the direction of improving objective function

Except?

$$\text{maximize } x_1 + x_2$$

subject to

$$x_1, x_2 \ge 0$$

linear program is **unbounded**

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More products

The chocolatier decides to introduce a third product line called premium with a profit of $13. We still can only produce 400 units per day. The deluxe and premium products require the same packaging machinery. Deluxe uses one unit and premium uses 3 units. We have a total of 600 units a day of this packaging material.

$$\text{maximize } x_1 + 6x_2$$

subject to

$$\begin{aligned} x_1 &\le 200 \\ x_2 &\le 300 \\ x_1 + x_2 &\le 400 \\ x_1, x_2 &\ge 0 \end{aligned}$$

what changes?

32

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Introduce a new variable x_3

$$\begin{aligned} &\text{maximize } x_1 + 6x_2 \\ &\text{subject to} \\ &x_1 \leq 200 \\ &x_2 \leq 300 \\ &x_1 + x_2 \leq 400 \\ &x_1, x_2, x_3 \geq 0 \end{aligned}$$

33

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Introduce a new constraint

$$\begin{aligned} &\text{maximize } x_1 + 6x_2 \\ &\text{subject to} \\ &x_1 \leq 200 \\ &x_2 \leq 300 \\ &x_1 + x_2 \leq 400 \\ &x_1, x_2, x_3 \geq 0 \end{aligned}$$

34

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35

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36

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modify existing constraints

$$\begin{aligned} &\text{maximize } x_1 + 6x_2 \\ &\text{subject to} \\ &x_1 \le 200 \\ &x_2 \le 300 \\ &x_1 + x_2 + x_3 \le 400 \\ &x_2 + 3x_3 \le 600 \\ &x_1, x_2, x_3 \ge 0 \end{aligned}$$

37

More products

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Anything else?

$$\begin{aligned} &\text{maximize } x_1 + 6x_2 \\ &\text{subject to} \\ &x_1 \le 200 \\ &x_2 \le 300 \\ &x_1 + x_2 + x_3 \le 400 \\ &x_2 + 3x_3 \le 600 \\ &x_1, x_2, x_3 \ge 0 \end{aligned}$$

38

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modify the objective function

$$\begin{aligned} &\text{maximize } x_1 + 6x_2 + 13x_3 \\ &\text{subject to} \\ &x_1 \le 200 \\ &x_2 \le 300 \\ &x_1 + x_2 + x_3 \le 400 \\ &x_2 + 3x_3 \le 600 \\ &x_1, x_2, x_3 \ge 0 \end{aligned}$$

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What does the feasibility region look like?

$$\begin{aligned} &\text{maximize } x_1 + 6x_2 + 13x_3 \\ &\text{subject to} \\ &x_1 \le 200 \\ &x_2 \le 300 \\ &x_1 + x_2 + x_3 \le 400 \\ &x_2 + 3x_3 \le 600 \\ &x_1, x_2, x_3 \ge 0 \end{aligned}$$

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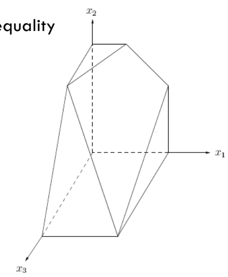
Feasibility region

For n variables, the feasibility region is a polyhedron in R^n (i.e. n-dimensional space)

Each constraint defines a R^{n-1} plane and the inequality a half-space on one side of the plane

$$\begin{array}{l} \text{maximize } x_1 + 6x_2 + 13x_3 \\ \text{subject to} \\ \begin{array}{rcl} x_1 & \leq & 200 \\ x_2 & \leq & 300 \\ x_1 + x_2 + x_3 & \leq & 400 \\ x_2 + 3x_3 & \leq & 600 \\ x_1, x_2, x_3 & \geq & 0 \end{array} \end{array}$$



adapted from figure 7.2 of [1]

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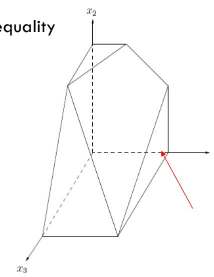

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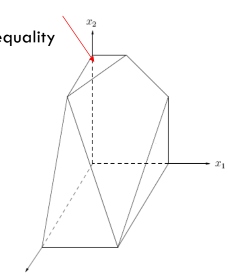

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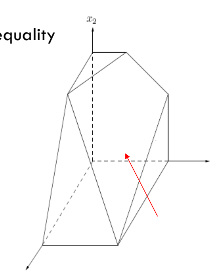

adapted from figure 7.2 of [1]

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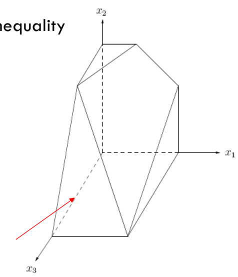

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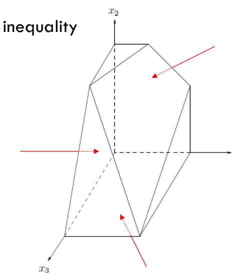

adapted from figure 7.2 of [1]

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Solving linear programs

If a solution exists, a vertex of the feasibility space is an optimal solution

Because the objective function is linear, we can use a *hill-climbing* or greedy approach by starting at one vertex and moving to an adjacent vertex that improves the object function

When we're on top of the "hill", i.e. we cannot move to a better neighboring vertex, we're done

The simplex method does exactly this

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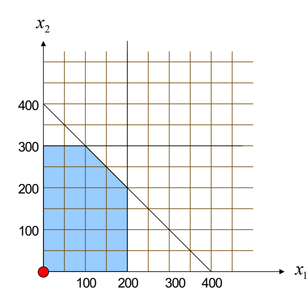
Simplex method

Start: [0,0]

Objective score:

$$\text{maximize } x_1 + x_2$$
subject to
$$\begin{aligned} x_1 &\leq 200 \\ x_2 &\leq 300 \\ x_1 + x_2 &\leq 400 \\ x_1, x_2 &\geq 0 \end{aligned}$$



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Simplex method

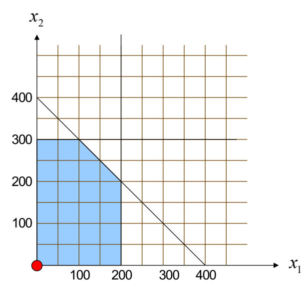
Start: [0,0]

Objective score: 0

maximize $x_1 + x_2$

subject to

$$\begin{aligned} x_1 &\leq 200 \\ x_2 &\leq 300 \\ x_1 + x_2 &\leq 400 \\ x_1, x_2 &\geq 0 \end{aligned}$$



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Simplex method

Adjacent vertices

[0, 300]

Objective score:

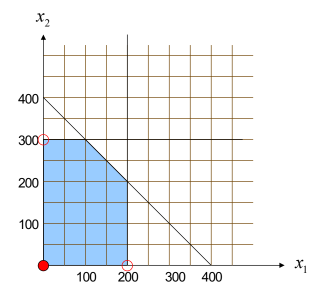
[200, 0]

Objective score:

maximize $x_1 + x_2$

subject to

$$\begin{aligned} x_1 &\leq 200 \\ x_2 &\leq 300 \\ x_1 + x_2 &\leq 400 \\ x_1, x_2 &\geq 0 \end{aligned}$$



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Simplex method

Adjacent vertices

[0, 300]

Objective score: 300

[200, 0]

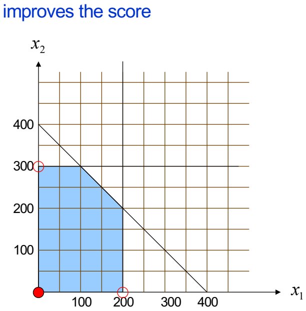
Objective score: 200

either option improves the score

maximize $x_1 + x_2$

subject to

$$\begin{aligned} x_1 &\leq 200 \\ x_2 &\leq 300 \\ x_1 + x_2 &\leq 400 \\ x_1, x_2 &\geq 0 \end{aligned}$$



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Simplex method

Adjacent vertices

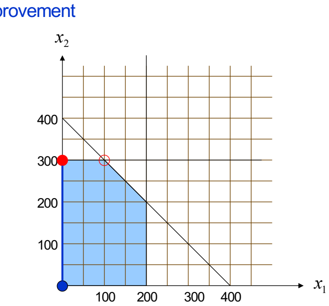
[100, 300]

Objective score: 400 improvement

maximize $x_1 + x_2$

subject to

$$\begin{aligned} x_1 &\leq 200 \\ x_2 &\leq 300 \\ x_1 + x_2 &\leq 400 \\ x_1, x_2 &\geq 0 \end{aligned}$$



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Simplex method

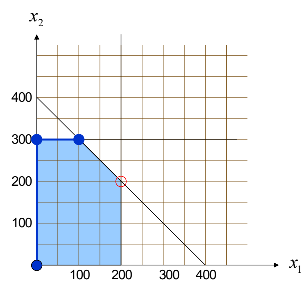
Adjacent vertices

[200, 200]

Objective score: 400

No improvement, so we' re done

$$\text{maximize } x_1 + x_2$$
$$\text{subject to}$$
$$\begin{aligned} x_1 &\leq 200 \\ x_2 &\leq 300 \\ x_1 + x_2 &\leq 400 \\ x_1, x_2 &\geq 0 \end{aligned}$$



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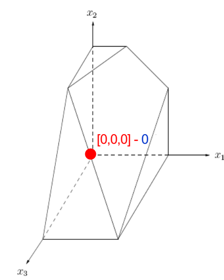
Another simplex example

Start at [0,0,0]

Adjacent vertices

$$\text{maximize } x_1 + 6x_2 + 13x_3$$
$$\text{subject to}$$
$$\begin{aligned} x_1 &\leq 200 \\ x_2 &\leq 300 \\ x_1 + x_2 + x_3 &\leq 400 \\ x_2 + 3x_3 &\leq 600 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$



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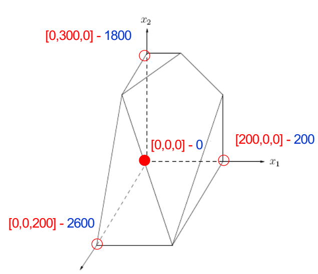
Another simplex example

Start at [0,0,0]

Adjacent vertices

$$\text{maximize } x_1 + 6x_2 + 13x_3$$
$$\text{subject to}$$
$$\begin{aligned} x_1 &\leq 200 \\ x_2 &\leq 300 \\ x_1 + x_2 + x_3 &\leq 400 \\ x_2 + 3x_3 &\leq 600 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$



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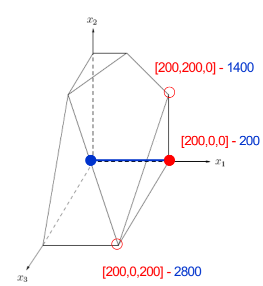
Another simplex example

[200,0,0]

Adjacent vertices

$$\text{maximize } x_1 + 6x_2 + 13x_3$$
$$\text{subject to}$$
$$\begin{aligned} x_1 &\leq 200 \\ x_2 &\leq 300 \\ x_1 + x_2 + x_3 &\leq 400 \\ x_2 + 3x_3 &\leq 600 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$



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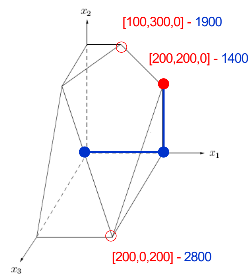
Another simplex example

[200,200,0]

Adjacent vertices

$$\begin{array}{lcl} \text{maximize } x_1 + 6x_2 + 13x_3 & & \\ \text{subject to} & & \\ x_1 & \le & 200 \\ x_2 & \le & 300 \\ x_1 + x_2 + x_3 & \le & 400 \\ x_2 + 3x_3 & \le & 600 \\ x_1, x_2, x_3 & \ge & 0 \end{array}$$



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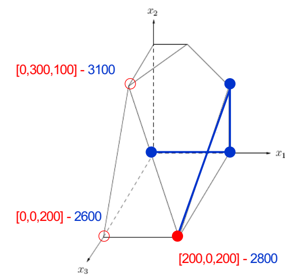
Another simplex example

[200,0,200]

Adjacent vertices

$$\begin{array}{lcl} \text{maximize } x_1 + 6x_2 + 13x_3 & & \\ \text{subject to} & & \\ x_1 & \le & 200 \\ x_2 & \le & 300 \\ x_1 + x_2 + x_3 & \le & 400 \\ x_2 + 3x_3 & \le & 600 \\ x_1, x_2, x_3 & \ge & 0 \end{array}$$



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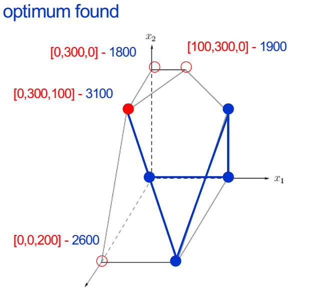
Another simplex example

[0,300,100]

Adjacent vertices

All others are lower, optimum found

$$\begin{array}{lcl} \text{maximize } x_1 + 6x_2 + 13x_3 & & \\ \text{subject to} & & \\ x_1 & \le & 200 \\ x_2 & \le & 300 \\ x_1 + x_2 + x_3 & \le & 400 \\ x_2 + 3x_3 & \le & 600 \\ x_1, x_2, x_3 & \ge & 0 \end{array}$$



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Running time of simplex

Worst case is exponential

There are theoretical examples that can be created where the running time is exponential

In practice, the algorithm is much faster

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Other LP solvers

Many other LP algorithms exist
- ellipsoid algorithm
- interior point methods (many of these)

Both of the above algorithms are O(polynomial time)

State of the art run-times are a toss up between interior point methods and simplex solvers

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Another example

Carpet company: From our analysts, we're given carpet demand estimates for the next calendar year: $d_1, d_2, \ldots, d_n$. The company has 30 employees, each of which makes 20 carpets per month and gets paid $2000/mo.

The demand fluctuates from month to month. We can handle this demand in three ways:
- Overtime: Overtime costs 80% more than regular pay. Workers can put in at most 30% overtime
- Hiring and firing, at a cost of $320 and $400 respectively per worker
- Store extra carpets at a cost of $8 per carpet per month. We must end the year without any stored carpets.

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Uses

- operational problems
- network flow
- planning
- microeconomics

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References

- [1] Algorithms (2008). Sanjoy Dasgupta, Christos Papadimitiou and Umesh Vazirani.

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