HASHTABLES

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CS 140 – Spring 2023

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Admin

Assignment 4 and 5 back soon

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Hashtables

What do you know/remember?

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Hashtables

Constant time insertion and search (and deletion in some cases) for a large space of keys

Applications

- Does *x* belong to *S*?
- I've found them very useful (go by many names, maps, dictionaries, …)
- compilers
- databases
- search engines
- storing and retrieving non-sequential data
- save memory over an array

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Hashtables

Constant time insertion and search (and deletion in some cases) for a large space of keys

For this class, we'll just think of them as a collection of keys

For many applications/implementations, there is a value associated with the key, i.e., key/value pair (though lookup is still exclusively based on the key)

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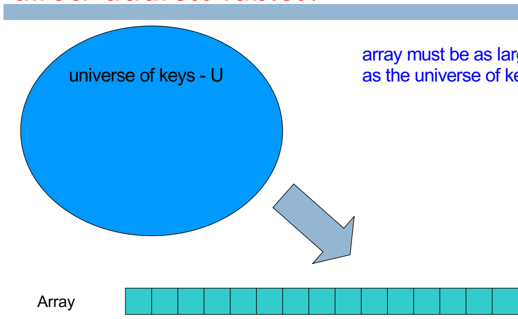
Why not just arrays aka direct-address tables?



array must be as large as the universe of keys

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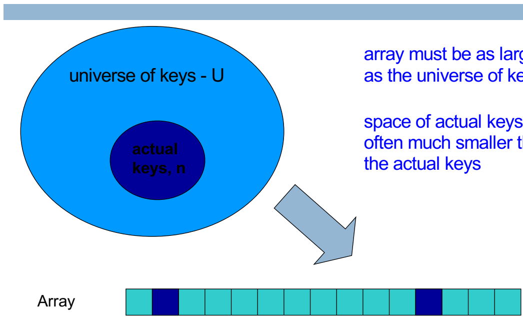
Why not just arrays?



array must be as large as the universe of keys

space of actual keys is often much smaller than the actual keys

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Why not arrays?

Think of indexing all last names < 10 characters

- Census listing of all last names
  http://www.census.gov/genealogy/names/dist.all.last
  - 88,799 last names
- What is the size of our space of keys?
  - 26^{10} = a big number
- Not feasible!
- Even if it were, not space efficient

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The load of a table/hashtable

m = number of possible entries in the table

n = number of keys stored in the table

α = n/m is the **load factor** of the hashtable

What is the load factor of the last example?

- $\alpha = 88{,}799 / 26^{10}$ would be the load factor of last names using direct-addressing

The smaller α, the more wasteful the table

The load also helps us talk about run time

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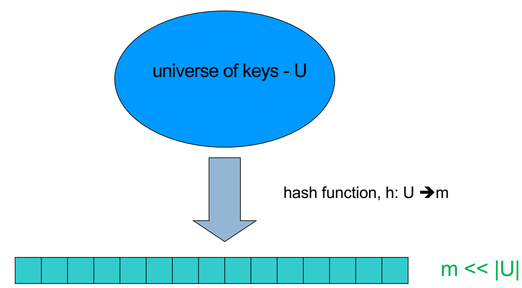
Hash function, *h*

A hash function is a function that maps the universe of keys to the slots in the hashtable



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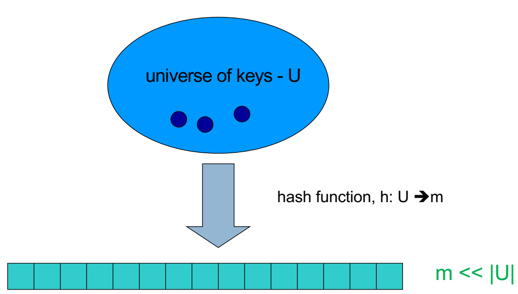

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Hash function, *h*

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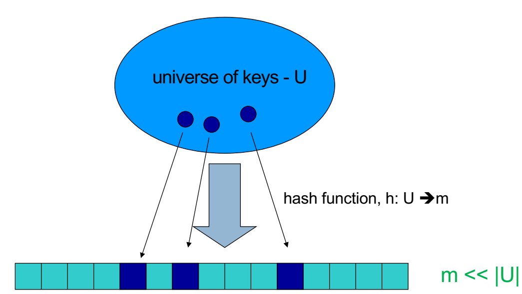

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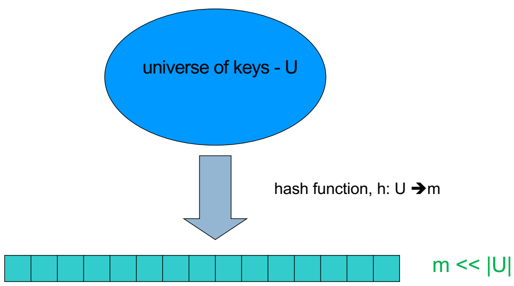
Hash function, *h*

What can happen if m < |U|?



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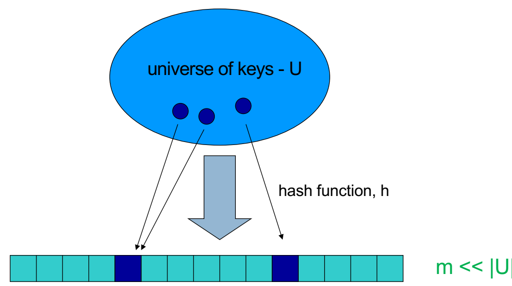
Collisions

If m < |U|, then two keys can map to the same position in the hashtable (pidgeonhole principle)



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Collisions

A collision occurs when h(x) = h(y), but x ≠ y

A good hash function will minimize the number of collisions

Because the number of hashtable entries is less than the possible keys (i.e. m < |U|) collisions are inevitable!

Collision resolution techniques?

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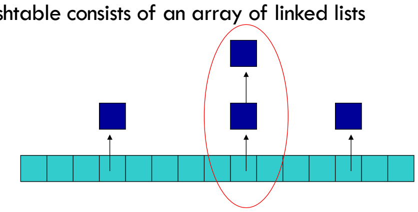
Collision resolution by chaining

Hashtable consists of an array of linked lists



When a collision occurs, the element is added to linked list at that location

If two entries x ≠ y have the same hash value h(x) = h(y), then T(h(x)) will contain a linked list with both values

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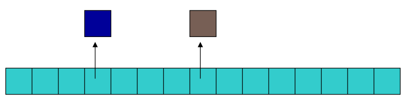
Insertion

ChainedInsert(x):
  entry = h(x)
  insert x at the head of T[entry]

ChainedHashInsert( )



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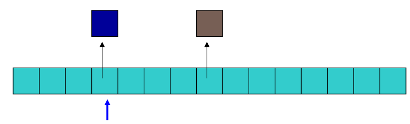
Insertion

ChainedInsert(x):
  entry = h(x)
  insert x at the head of T[entry]

h( )

hash function is a mapping from the key to some value < m



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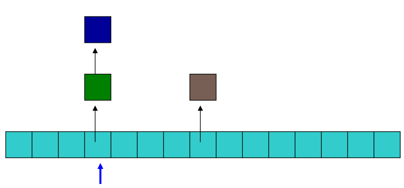

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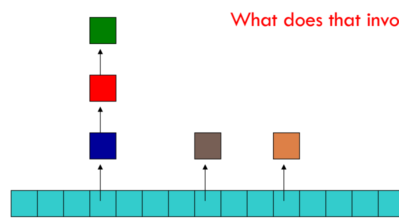
Deletion

ChainedDelete(x):
  entry = h(x)
  delete x at the list at T[entry]

What does that involve?



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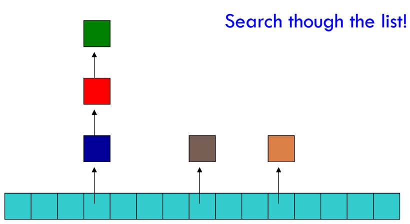

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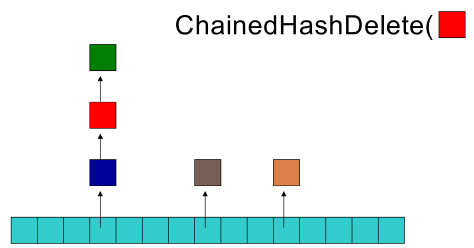

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Deletion

ChainedDelete(x):
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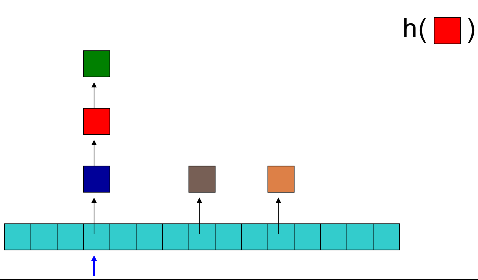

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Deletion

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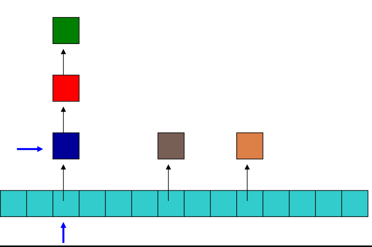

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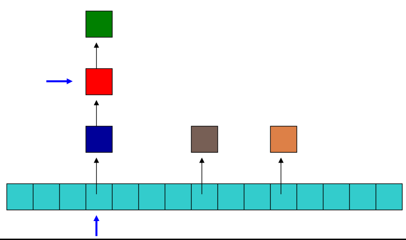

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Deletion

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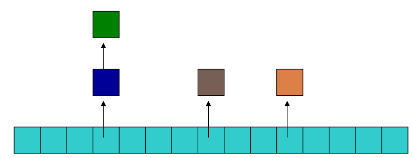

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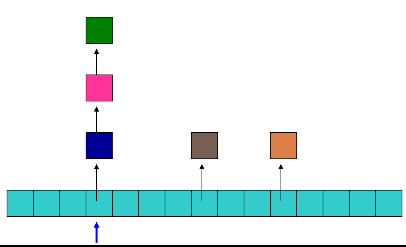
Search

ChainedSearch(x):
  entry = h(x)
  search for x in list T[entry]

ChainedHashSearch( )



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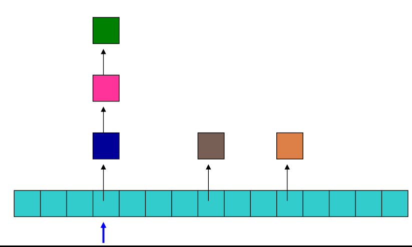
Search

ChainedSearch(x):
  entry = h(x)
  search for x in list T[entry]

h( )



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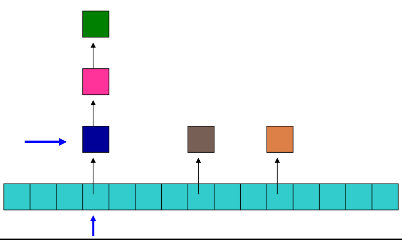

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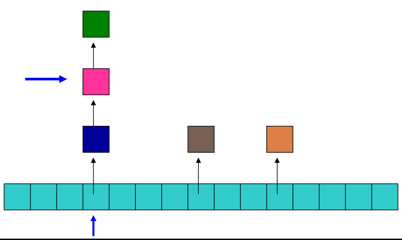

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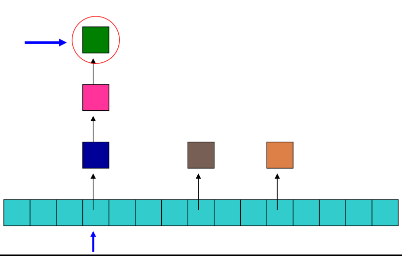

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Running time

ChainedInsert(x):
entry = h(x)
insert x at the head of T[entry]

$\Theta(1)$

ChainedDelete(x):
entry = h(x)
delete x at the list at T[entry]

O(length of the chain)

ChainedSearch(x):
entry = h(x)
search for x in list T[entry]

O(length of the chain)

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Length of the chain

Worst case?

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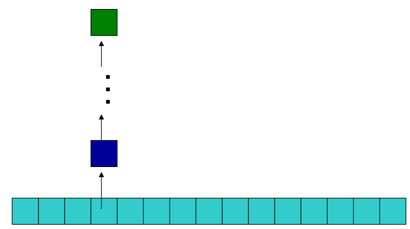
Length of the chain

Worst case?

- All elements hash to the same location
- h(k) = 4
- **n**



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Length of the chain

Average case:

Depends on how well the hash function distributes the keys

What is the best we could hope for a hash function?

- simple uniform hashing: an element is equally likely to end up in any of the *m* slots

Under simple uniform hashing what is the average length of a chain in the table?

- *n* keys over *m* slots = $n / m = \alpha$

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Average chain length

If you roll a fair *m* sided die *n* times, how many times are we likely to see a given value?

For example, 10 sided die:

1 time
- 1/10

100 times
- 100/10 = 10

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Search average running time

Two cases:

- Key is **not** in the table
  - must search all entries
  - Θ(1 + α)
- Key **is** in the table
  - on average search half of the entries
  - O(1 + α)

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Hash functions

What makes a good hash function?

- Approximates the assumption of simple uniform hashing
- Deterministic – h(x) should always return the same value
- Low cost – if it is expensive to calculate the hash value (e.g. log n) then we don't gain anything by using a table

Challenge: we don't generally know the distribution of the keys

- Frequently data tend to be clustered (e.g. similar strings, run-times, SSNs). A good hash function should spread these out across the table

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Hash functions

What are some hash functions you've heard of before?

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Division method

h(k) = k mod m

m	k	h(k)
11	25	
11	1	
11	17	
13	133	
13	7	
13	25	

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Division method

h(k) = k mod m

m	k	h(k)
11	25	3
11	1	1
11	17	6
13	133	3
13	7	7
13	25	12

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Division method

Don't use a power of two. Why?

m	k	bin(k)	h(k)
8	25	11001	
8	1	00001	
8	17	10001	

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Division method

Don't use a power of two. Why?

m	k	bin(k)	h(k)
8	25	11001	1
8	1	00001	1
8	17	10001	1

if h(k) = k mod 2^p, the hash function is just the lower *p* bits of the value

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Division method

Good rule of thumb for *m* is a prime number not too close to a power of 2

Pros:

- quick to calculate
- easy to understand

Cons:

- keys close to each other will end up close in the hashtable

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Multiplication method

Multiply the key by a constant 0 < A < 1 and extract the fractional part of *kA*, then scale by *m* to get the index

$$h(k) = \lfloor m(kA - \lfloor kA \rfloor) \rfloor$$

extracts the fractional portion of *kA*

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Multiplication method

$$h(k) = \lfloor m(kA - \lfloor kA \rfloor) \rfloor$$

Common choice is for *m* as a power of 2 and

$$A = (\sqrt{5} - 1)/2 = 0.6180339887$$

Why a power of 2?

Book has other heuristics

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Multiplication method

m	k	A	kA	h(k)
8	15	0.618		
8	23	0.618		
8	100	0.618		

$$h(k) = \lfloor m(kA - \lfloor kA \rfloor) \rfloor$$

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Multiplication method

m	k	A	kA	h(k)
8	15	0.618	9.27	floor(0.27*8) = 2
8	23	0.618	14.214	floor(0.214*8) = 1
8	100	0.618	61.8	floor(0.8*8) = 6

$$h(k) = \lfloor m(kA - \lfloor kA \rfloor) \rfloor$$

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Other hash functions

http://en.wikipedia.org/wiki/List_of_hash_functions

cyclic redundancy checks (i.e. disks, cds, dvds)

Checksums (i.e. networking, file transfers)

Cryptographic (i.e. MD5, SHA)

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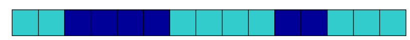
Open addressing

Keeping around an array of linked lists can be inefficient and a hassle

Like to keep the hashtable as just an array of elements (no pointers)

How do we deal with collisions?

- compute another slot in the hashtable to examine



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Hash functions with open addressing

Hash function must define a **probe sequence** which is the list of slots to examine when searching or inserting

The hash function takes an additional parameter *i* which is the number of collisions that have already occurred

The probe sequence **must** be a permutation of every hashtable entry. Why?

{ h(k,0), h(k,1), h(k,2), …, h(k, m-1) } is a permutation of { 0, 1, 2, 3, …, m-1 }

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Hash functions with open addressing

Hash function must define a **probe sequence** which is the list of slots to examine when searching or inserting

The hash function takes an additional parameter *i* which is the number of collisions that have already occurred

The probe sequence **must** be a permutation of every hashtable entry. Why?

If not, we wouldn't explore all the possible location in the table!

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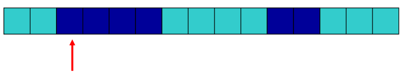
Probe sequence

h(k, 0)



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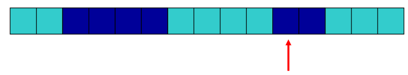
Probe sequence

h(k, 1)



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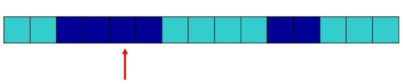
Probe sequence

h(k, 2)



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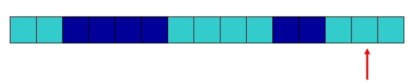
Probe sequence

h(k, 3)



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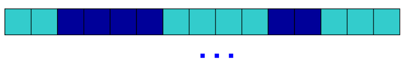
Probe sequence

h(k, …)

must visit all locations



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Open addressing: Insert

```
Hash-Insert(T, k)
i ← 0
j ← h(k, i)
while i < m − 1 and T[j] ≠ null
        i ← i + 1
        j ← h(k, i)
if T[j] = null
        return j
else
        error "hash is full"
```

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Open addressing: Insert

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```

get the first hashtable entry to look in

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Open addressing: Insert

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```

follow the probe sequence until we find an open entry

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Open addressing: Insert

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```

return the open entry

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Open addressing: Insert

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        return j
else
        error "hash is full"
```

hashtable can fill up

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Open addressing: search

```
HASH-SEARCH(T, k)
i ← 0
j ← h(k, i)
while i < m − 1 and T[j] ≠ null and T[j] ≠ k
        i ← i + 1
        j ← h(k, i)
if T[j] = k
        return j
else
        return null
```

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Open addressing: search

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```

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Open addressing: search

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i ← 0
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HASH-INSERT(T, k)
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        i ← i + 1
        j ← h(k, i)
if T[j] = null
        return j
else
        error "hash is full"
```

"breaks" the probe sequence

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Open addressing: delete

Two options:

- mark node as "deleted" (rather than null)
  - modify search procedure to continue looking if a "deleted" node is seen
  - modify insert procedure to fill in "deleted" entries
  - increases search times
- if a lot of deleting will happen, use chaining

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Probing schemes

Linear probing – if a collision occurs, go to the next slot

- h(k,i) = (h(k) + i) mod m
- Does it meet our requirement that it visits every slot?
- for example, m = 7 and h(k) = 4

h(k,0) = 4
h(k,1) = 5
h(k,2) = 6
h(k,3) = 0
h(k,3) = 1

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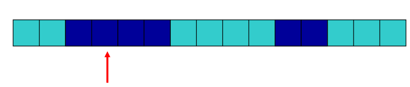
Linear probing: search

h(, 0)



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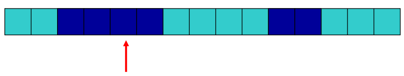
Linear probing: search

h(■, 1)



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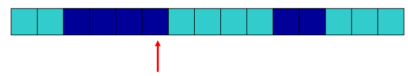
Linear probing: search

h(■, 2)



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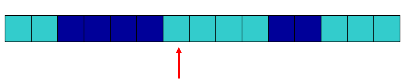
Linear probing: search

h(■, 3)



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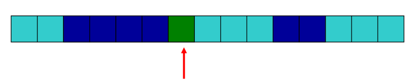
Linear probing: search

h(, 3)



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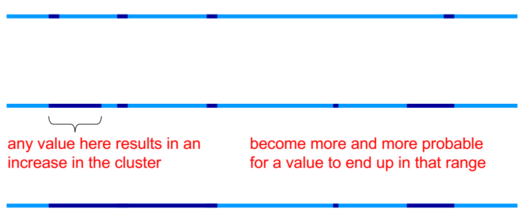
Linear probing

Problem:

primary clustering – long runs of occupied slots tend to build up and these tend to grow



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Quadratic probing

$h(k,i) = (h(k) + c_1 i + c_2 i^2) \bmod m$

Rather than a linear sequence, we probe based on a quadratic function

Problems:

- must pick constants and *m* so that we have a proper probe sequence
- if $h(x) = h(y)$, then $h(x,i) = h(y,i)$ for all i
- secondary clustering

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Double hashing

Probe sequence is determined by a second hash function

$h(k,i) = (h_1(k) + i(h_2(k))) \bmod m$

Problem:

- $h_2(k)$ must visit all possible positions in the table

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Running time of insert and search for open addressing

Depends on the hash function/probe sequence

Worst case?

- O(n) – probe sequence visits every full entry first before finding an empty

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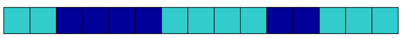
Running time of insert and search for open addressing

Average case?

We have to make at least one probe



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Running time of insert and search for open addressing

Average case?

What is the probability that the first probe will **not** be successful (assume uniform hashing function)?

α

83

Running time of insert and search for open addressing

Average case?

What is the probability that the first **two** probed slots will **not** be successful?

why '~' ? → $\sim\alpha^2$

84

Running time of insert and search for open addressing

Average case?

What is the probability that the first **two** probed slots will **not** be successful

Technically, second probe is: $\frac{n-1}{m-1}$ $\sim\alpha^2$

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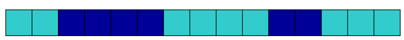
Running time of insert and search for open addressing

Average case?

What is the probability that the first **three** probed slots will **not** be successful?

$\sim\alpha^3$



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Running time of insert and search for open addressing

Average case: expected number of probes

sum of the probability of making 1 probe, 2 probes, 3 probes, …

$$E[probes] = 1 + \alpha + \alpha^2 + \alpha^3 + \ldots$$
$$= \sum_{i=0}^{m} \alpha^i$$
$$< \sum_{i=0}^{\infty} \alpha^i$$
$$= \frac{1}{1-\alpha}$$

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Average number of probes

$$E[probes] = \frac{1}{1-\alpha}$$

α	Average number of searches
0.1	$1/(1-.1) = 1.11$
0.25	$1/(1-.25) = 1.33$
0.5	$1/(1-.5) = 2$
0.75	$1/(1-.75) = 4$
0.9	$1/(1-.9) = 10$
0.95	$1/(1-.95) = 20$
0.99	$1/(1-.99) = 100$

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How big should a hashtable be?

A good rule of thumb is the hashtable should be around half full

What happens when the hashtable gets full?

Copy: Create a new table and copy the values over
- results in one expensive insert
- simple to implement

Amortized copy: When a certain ratio is hit, grow the table, but copy the entries over a few at a time with every insert
- no single insert is expensive and can guarantee per insert performance
- more complicated to implement

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Checkpoint 1

Induction on trees

$T(n) = T(\sqrt{n}) + c$

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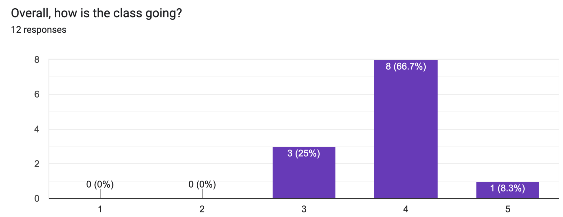
Course feedback



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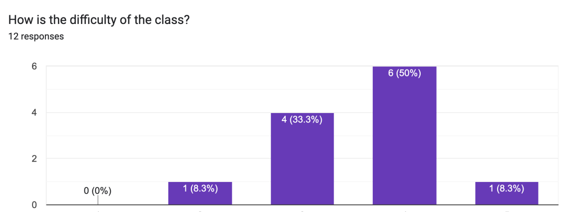
Course feedback



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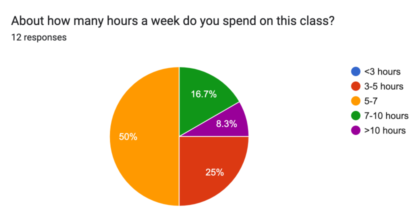
Course feedback



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Course feedback

I love proving things and looking at the Math behind the concepts from CS62.

the group assignments

Honestly I just really like the little comics at the start of every homework

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Course feedback

lectures are wayyy too fast, barely enough time to process things so it feels pointless to take notes; current course content is comprehensive and makes sense but it feels disorganized, like different content stitched together sort of so…

Having more examples, or going through the slides a bit slower

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Course feedback

The homeworks are a lot of work and the mentors are super helpful but someone's even they don't have the solutions and that wastes hours of our time. I think homeworks can have more straight forward problems that show we understand things rather than problems that we always have to scavenge the internet and bug mentors for understandings.

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Course feedback

During Class, could we have some more exercises along with the lecture contents?

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