

ALGORITHMS

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CS 140 – Spring 2023

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Introductions

Dr. | Prof | Professor
Dave | Kauchak

Pronouns: he/him/his

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Meet your neighbors

- What's your name?
- What year?
- What has been your favorite CS class?
- What's been your least favorite CS class?
- Why are you taking this class?

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Algorithms

“For me, great algorithms are the poetry of computation. Just like verse, they can be terse, allusive, dense and even mysterious. But once unlocked, they cast a brilliant new light on some aspect of computing.”
– Francis Sullivan

What is an algorithm?

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Example algorithms

- sort a list of numbers
- find a route from one place to another (cars, packet routing, phone routing, ...)
- find the longest common substring between two strings
- add two numbers
- microchip wiring/design (VLSI)
- solve sudoku
- cryptography
- compression (file, audio, video)
- spell checking
- pagerank
- classify a web page

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To the course webpage...

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Log properties

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Log properties

$$x = a^b$$

What is b ?

$$b = \log_a x$$

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Log properties

$$\log_a x \quad x = a^b$$

a raised to what exponent is x ?

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Log properties

$$\log_a a = ? \quad a \text{ raised to what exponent is } x?$$

$$\log_a x = ? \text{ if } x > a$$

$$\log_a x = ? \text{ if } x < a$$

greater than 1 less than 1 exactly 1

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Log properties

$$\log_a a = 1 \quad a \text{ raised to what exponent is } x?$$

$$\log_a x > 1 \text{ if } x > a$$

$$\log_a x < 1 \text{ if } x < a$$

greater than 1 less than 1 exactly 1

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Log properties

Which is bigger?

1) $\log_3 2$

2) $\log_4 2$

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Log properties

Which is bigger?

1) $\log_3 2 = x \rightarrow 2 = 3^x$

2) $\log_4 2 = x \rightarrow 2 = 4^x$

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Log properties

$$\log(ab) = \log a + \log b$$

Which is bigger?

1) $\log_3 27$

2) $\log_4 36$

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Log properties

$$\log(ab) = \log a + \log b$$

Which is bigger?

1) $\log_3 27 = \log_3 3 + \log_3 3 + \log_3 3$

2) $\log_4 36 = \log_4 4 + \log_4 3 + \log_4 3$

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Log properties

$$\log(ab) = \log a + \log b$$

Which is bigger?

1) $\log_3 27 = \overset{1}{\log_3 3} + \overset{1}{\log_3 3} + \overset{1}{\log_3 3}$

2) $\log_4 36 = \overset{1}{\log_4 4} + \overset{<1}{\log_4 3} + \overset{<1}{\log_4 3}$

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Log properties

$$\log(a/b) = \log a - \log b$$

Which is bigger?

1) $\log_3 4.5$

2) $\log_4 8$

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Log properties

$$\log(a/b) = \log a - \log b$$

Which is bigger?

1) $\log_3 4.5 = \log_3 9 - \log_3 2$

2) $\log_4 8 = \log_4 16 - \log_4 2$

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Log properties

$$\log(a/b) = \log a - \log b$$

Which is bigger?

1) $\log_3 4.5 = \log_3 9 - \log_3 2$

2) $\log_4 8 = \log_4 16 - \log_4 2$

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Log properties

$$\log b^x = x \log b$$

$$\log b^x = \underbrace{\log b + \log b + \dots + \log b}_{x \text{ times}}$$

$$\log b^x = \sum_{i=1}^x \log b$$

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Log properties

$$\log_a b = \frac{\log b}{\log a}$$

allows you to change bases!

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Log properties

$$\log_a a = ?$$

$$\log_a b = \frac{\log b}{\log a}$$

$$\log_a x = ? \text{ if } x > a$$

$$\log_a x = ? \text{ if } x < a$$

$$\log_a x = \frac{\log x}{\log a}$$

greater than 1 less than 1 exactly 1

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Log properties

$$\log_a a = ?$$

$$\log_a b = \frac{\log b}{\log a}$$

$$\log_a x > 1 \text{ if } x > a$$

$$\log_a x < 1 \text{ if } x < a$$

$$\log_a x = \frac{\log x}{\log a}$$

greater than 1 less than 1 exactly 1

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Log properties

Which is bigger?

$$\log_a b = \frac{\log b}{\log a}$$

1) $\log_3 2$

2) $\log_4 2$

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Log properties

Which is bigger?

$$\log_a b = \frac{\log b}{\log a}$$

1) $\log_3 2 = \frac{\log 2}{\log 3}$

2) $\log_4 2 = \frac{\log 2}{\log 4}$

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Exponent properties

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Exponent properties

$$a^b \cdot a^c =$$

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Exponent properties

$$a^b \cdot a^c = a^{b+c}$$

$$a^b \cdot a^c = \underbrace{a \cdot a \dots a}_{b \text{ times}} \cdot \underbrace{a \cdot a \dots a}_{c \text{ times}}$$

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Exponent properties

$$a^b \cdot a^c = a^{b+c}$$

Which is bigger?

1) x^2

2) $x^{2.1}$

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Exponent properties

$$a^b \cdot a^c = a^{b+c}$$

Which is bigger?

1) x^2

2) $x^{2.1} = x^2 x^{0.1}$

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Exponent properties

$$a^b \cdot a^c = a^{b+c}$$

Which is bigger (for $x > 1$)?

1) 2^{x+1}

2) $2^x + 2$

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Exponent properties

$$a^b \cdot a^c = a^{b+c}$$

Which is bigger (for $x > 1$)?

1) $2^{x+1} = 2^x \cdot 2$

2) $2^x + 2$

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Exponent properties

$$(ab)^c =$$

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Exponent properties

$$(ab)^c = a^c b^c$$

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Exponent properties

$$(ab)^c = a^c b^c$$

Which is bigger?

1) 12^3

2) 4^6

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Exponent properties

$$(ab)^c = a^c b^c$$

Which is bigger?

1) $12^3 = (4*3)^3 = 4^3 3^3$

2) $4^6 = 4^3 4^3$

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Exponent properties

$$(a^b)^c =$$

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Exponent properties

$$(a^b)^c = a^{bc}$$

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Exponent properties

$$(a^b)^c = a^{bc}$$

Which is bigger ($x > 1$)?

1) 2^{2x}

2) 4^x

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Exponent properties

$$(a^b)^c = a^{bc}$$

Which is bigger ($x > 1$)?

1) $2^{2x} = (2^2)^x = 4^x$

2) 4^x

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Pseudocode

A way to discuss how an algorithm works that is language agnostic and without being encumbered with actual implementation details.

Should give enough detail for a person to understand, analyze and implement the algorithm.

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Pseudocode examples

```

MYSTERY1(A)
1   $x \leftarrow -\infty$ 
2  for  $i \leftarrow 1$  to  $\text{length}[A]$ 
3      if  $A[i] > x$ 
4           $x \leftarrow A[i]$ 
5  return  $x$ 

MYSTERY2(A)
1  for  $i \leftarrow 1$  to  $\lfloor \text{length}(A)/2 \rfloor$ 
2      swap  $A[i]$  and  $A[\text{length}(A) - (i - 1)]$ 

```

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Pseudocode conventions

array indices start at 1 not 0

we may use notation such as ∞ , which, when translated to code, would be something like Integer.MAX VALUE

use shortcuts for simple function (e.g. swap) to make pseudocode simpler

we'll often use \leftarrow instead of $=$ to avoid ambiguity

indentation specifies scope

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Proofs

What is a proof?

A deductive argument showing a statement is true based on previous knowledge (axioms)

Why are they important/useful?

Allows us to be sure that something is true

In algs: allow us to prove properties of algorithms

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Proof techniques?

example/counterexample

enumeration

by cases

by inference (aka direct proof)

trivially

contrapositive

contradiction

induction (strong and weak)