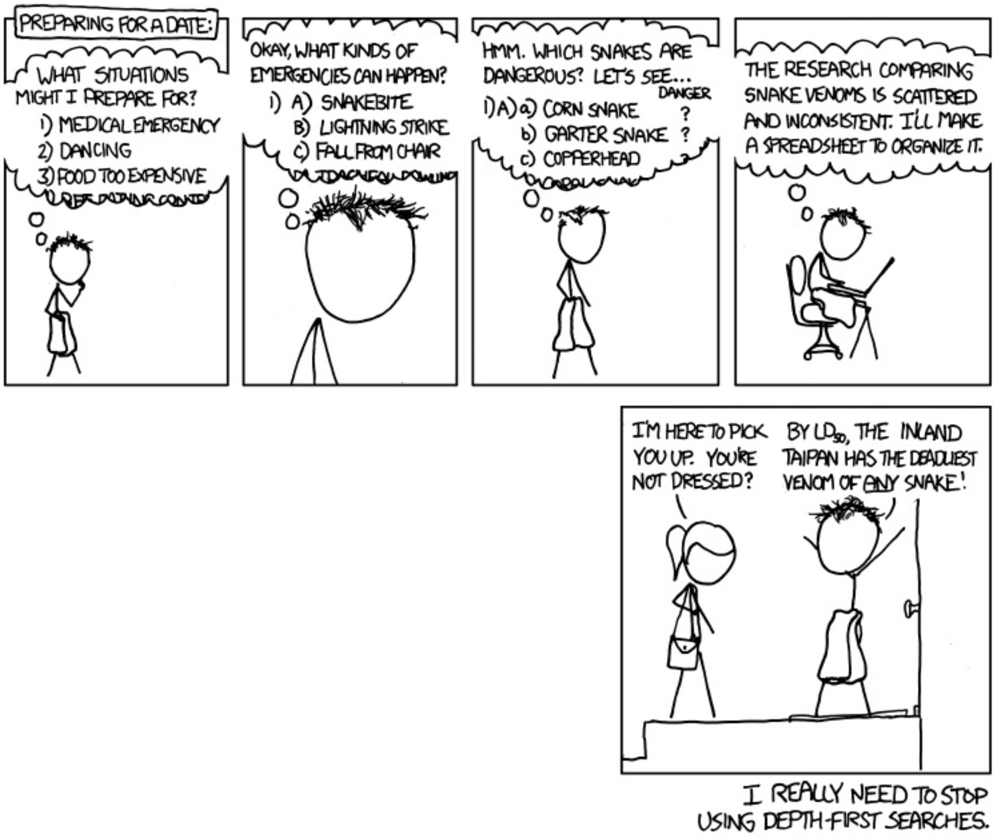
CS140 - Assignment 9

Due: Sunday, Apr. 9th at 8pm


PREPARING FOR A DATE:
WHAT SITUATIONS MIGHT I PREPARE FOR? 1) MEDICAL EMERGENCY 2) DANCING 3) FOOD TOO EXPENSIVE
OKAY, WHAT KINDS OF EMERGENCIES CAN HAPPEN? 1) A) SNAKEBITE B) LIGHTNING STRIKE C) FALL FROM CHAIR
HMM. WHICH SNAKES ARE DANGEROUS? LET'S SEE... 1)A)a) CORN SNAKE b) GARTER SNAKE c) COPPERHEAD
DANGER ? ? ?
THE RESEARCH COMPARING SNAKE VENOMS IS SCATTERED AND INCONSISTENT. I'LL MAKE A SPREADSHEET TO ORGANIZE IT.
I'M HERE TO PICK YOU UP. YOU'RE NOT DRESSED?
BY LD50, THE INLAND TAIPAN HAS THE DEADLIEST VENOM OF ANY SNAKE!
I REALLY NEED TO STOP USING DEPTH-FIRST SEARCHES.

http://xkcd.com/761/

Notes:

- Many of the algorithms below can be accomplished by either modifying the graph and applying a known algorithm or slightly modifying a known algorithm. Try thinking of these *first* as they will save you a lot of work, and writing :) I don't expect long answers, but be precise.
- You will be graded on efficiency!
- If not specified in the problem, you may assume whatever graph representation makes your algorithm more efficient (adjacency list or adjacency matrix). State which one you are using.

1. [**8 points**] State whether the following statements about a graph G that is undirected and connected are true or false and justify your answer.

   (a) Prim's algorithm works correctly if G has negative edge weights.

   (b) The shortest path between two nodes is always part of some MST.

2. [**5 points**] Write pseudocode for an algorithm which, given an undirected graph G and a particular edge e in it, determines whether G has a cycle containing e. What is the runtime of this algorithm?

3. [**8 points**] Often there are multiple shortest paths between nodes of a graph. Write pseudocode for an algorithm that given an undirected, unweighted graph G and nodes $u, v \in V$, outputs the number of distinct shortest paths from u to v. What is the running time?

4. [**5 points**] Given a directed graph $G = (V, E)$ with positive edge weights and a particular node $v_i \in V$, give an efficient algorithm for finding the shortest paths between **all pairs of nodes**, with the one restriction that these paths must all pass through v_i. Give the runtime of your algorithm. Points will be deducted for an inefficient algorithm.

   Hints:

   - Any path in this problem can be seen as two parts, the part to v_i and the part from v_i.
   - Look at how we determined if a graph was strongly connected.

5. [**5 points**] If a graph does not have a negative cycle, when calculating the shortest paths from a given vertex using the Bellman-Ford algorithm, we can stop early and do not need to do all $|V| - 1$ iterations and will still have a correct answer for all the shortest paths from that vertex. Describe how to modify the Bellman-Ford algorithm to stop early when all of the distances are already correct.

6. [**6 points**] Given an undirected graph G with nonnegative edge weights $w_e \geq 0$. Suppose you have calculated the minimum spanning tree of G and also the shortest paths to all nodes from a particular node $s \in V$. Now, suppose that each edge weight is increased by 1, i.e. the new weights are $w'_e = w_e + 1$.

   (a) (3 points) Does the minimum spanning tree change? Give an example where it does or prove that it cannot change.

   (b) (3 points) Do the shortest paths from s change? Given an example where it does or prove that it cannot change.