CS140 - Assignment 1

Due: Sunday, Jan. 29 at 8pm

http://xkcd.com/982/

- This assignment should be done in either pairs or triplets with people from your learning community.
- The purpose of this assignment is to encourage you to get to know your learning community better and to review proof-writing (from cs54) and asymptotics (from cs62).
- This assignment must be typeset using LaTeX. You are encouraged to take the source file for the assignment and modify it by adding your solutions.
- Like the first assignment, you are expected to explain each step of your solution and to present your solutions clearly and precisely. Part of the score on each problem will be for quality of presentation. Note that correct answers without justification are not worth very many points!

1. **[4 points]** Sorting in practice Pick two different sorting functions from a programming language (e.g. `java.util.Arrays.sort`) and state what sorting algorithm is used. They could be two different ways of sorting in the same language or two different languages. Make sure to cite your source. Why do you think these choices were made?

2. **[9 points]** Properties of Logs

   To get you warmed up, here is an example proof showing that $\log_b xy = \log_b x + \log_b y$.

   Let:

   $k = \log_b xy$
   $\ell = \log_b x$
   $m = \log_b y$

   We want to show that $k = \ell + m$.

   - By the definition of logarithms, we know:
   $b^k = xy$
   $b^\ell = x$
   $b^m = y$

   - From these, by properties of exponents
   $b^k = b^\ell b^m = b^{\ell+m}$

   Taking the log of both sides, we obtain $k = \ell + m$, which is what we wanted to show.

   Now give proofs for each of the following properties of logarithms. Write your proofs out carefully. You should assume that a, b, c, n are positive *real numbers* (not necessarily integers).

   (a) $\log_b a^n = n \log_b a$
   (b) $\log_b a = \frac{\log_c a}{\log_c b}$
   (c) $a^{\log_b n} = n^{\log_b a}$

3. **[5 points]** Running Times

   Suppose you have algorithms that execute the following number of operations as a function of the input size n. If you have a computer that can perform 10^{10} operations per second, for each algorithm what is the largest (integer!) input size n for which you would be able to get the result within a minute? Be as precise as possible. But, for once, no explanation necessary!

   (a) $60n^2$
   (b) n^3
   (c) $\sqrt{n}$
   (d) $n \log_2 n$
   (e) 2^n

4. **[52 points]** Asymptotics

   Indicate whether each of the following statements is true or false and then carefully **prove** your answer using the formal definition of Big-O notation and properties of logarithms from problem 1 ((hint, hint). Remember that to show that one of these statements is false, you must obtain a formal contradiction; it does not suffice to just say "false".

   (a) 2^{n+1} is $O(2^n)$.

   (b) 2^{2n} is $O(2^n)$.

   (c) $3n^2 \log_2 n + 16n$ is $O(n^3)$.

   (d) $25 \log_2 8n^{10}$ is $O(\log_{10} n)$.

   (e) $8^{\log_2 n}$ is $O(n^3)$.

5. **[20 points]** Writing proofs

   The objective of these two problems are to reinforce clear and precise writing on mathematical material.

   (a) There are two buses, A and B, about to take students on a field trip. Bus A contains 50 1st grade students. Bus B contains 50 2nd grade students. Before the buses leave, 8 students run out of Bus A and onto Bus B. The teachers then randomly choose 8 of the now 58 students on Bus B and force them to move to Bus A. The buses then (finally!) drive off.

   Are there more 2nd grade students in Bus A or more 1st grade students in Bus B?

   (b) Prove by induction that $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$ for all integers $n \geq 1$. Do you need strong induction? Why or why not?