

Introductions

Dr. | Prof | Professor Dave | Kauchak

Pronouns: he/him/his

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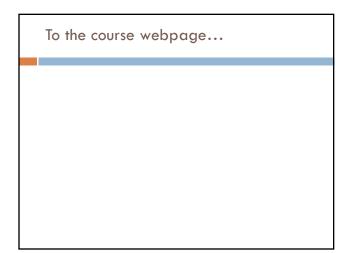
Algorithms

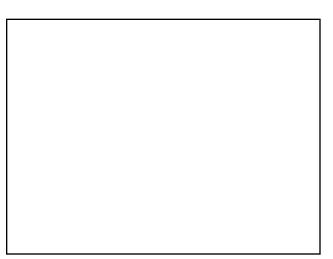
"For me, great algorithms are the poetry of computation. Just like verse, they can be terse, allusive, dense and even mysterious. But once unlocked, they cast a brilliant new light on some aspect of computing." – Francis Sullivan

What is an algorithm?

Example algorithms

sort a list of numbers find a route from one place to another (cars, packet routing, phone routing, ...) find the longest common substring between two strings add two numbers microchip wiring/design (VLSI) solve sudoku cryptography compression (file, audio, video) spell checking pagerank classify a web page





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Pseudocode

A way to discuss how an algorithm works that is language agnostic and without being encumbered with actual implementation details.

Should give enough detail for a person to undersand, analyze and implement the algorithm.

Pseudocode examples

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Pseudocode convections

array indices start at 1 not 0

we may use notation such as $^{\infty}$, which, when translated to code, would be something like Integer.MAX VALUE

use shortcuts for simple function (e.g. swap) to make pseudocode simpler

we'll often use \leftarrow instead of = to avoid ambiguity

indentation specifies scope

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An example

Prove the sum of two odd integers is even

Proof techniques?
example/counterexample
enumeration
by cases
by inference (aka direct proof)
trivially
contrapositive
contradiction
induction (strong and weak)

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Proofs

What is a proof?

A deductive argument showing a statement is true based on previous knowledge (axioms)

Why are they important/useful? Allows us to be sure that something is true In algs: allow us to prove properties of algorithms

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Proving something about a sequence of events by:

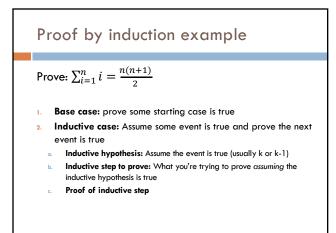
- 1. first: proving that some starting case is true and
- 2. then: proving that if a given event in the sequence were true then the next event would be true

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Proof by induction (weak)

- 1. Base case: prove some starting case is true
- 2. Inductive case: Assume some event is true and prove the next event is true
 - a. Inductive hypothesis: Assume the event is true (usually k or k-1)
 - b. **Inductive step to prove:** What you're trying to prove *assuming* the inductive hypothesis is true
 - Proof of inductive step

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Prove:
$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Show it is true for n = 1

$$\sum_{i=1}^{n} i = 1 = \frac{1 * 2}{2}$$

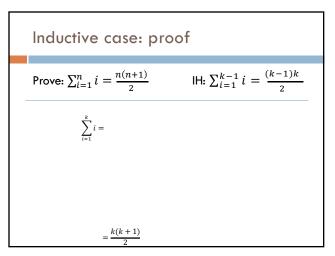
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Inductive case

Prove:
$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Inductive hypothesis: assume n = k - 1 is true $\sum_{i=1}^{k-1} i = \frac{(k-1) * k}{2}$

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Inductive case: proof		
Prove: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$	IH: $\sum_{i=1}^{k-1} i = \frac{(k-1)k}{2}$	
$\sum_{i=1}^{k} i = k + \sum_{i=1}^{k-1} i$	by definition of sum	
$= k + \frac{(k-1)*k}{2}$ $= \frac{2k}{2} + \frac{(k-1)*k}{2}$	by IH	
$=\frac{2k+(k-1)\cdot k}{2}$ $=\frac{k^2+k}{2}$	Why does this work?	
$=\frac{k(k+1)}{2}$		

Inductive case

Prove: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Prove:

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Inductive hypothesis: assume n = k - 1 is true

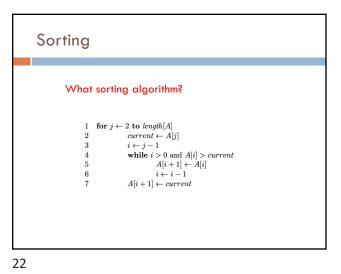
 $\sum_{i=1}^{k-1} i = \frac{(k-1)k}{2}$

 $\sum_{i=1}^k i = \frac{k(k+1)}{2}$

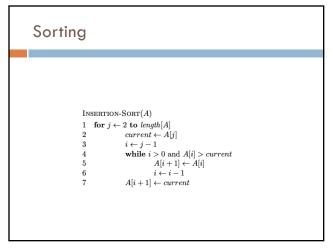
Sorting

Input: An array of numbers A Output: The number in sorted order, i.e.,

 $A[i] \leq A[j] \; \forall i < j$



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	$ \begin{array}{llllllllllllllllllllllllllllllllllll$		
Does it terminate?			
ls it correct?			
How long does it take to run?			
Memory usage?			

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