CS140 - Example NP Complete reduction

Show that Clique is NP-Complete. (Given a graph G does the graph have a clique of size k.)

1. Clique is in NP.

   A solution for click is a set of vertices $V' \subseteq V$. Check the following:

   - Check that $|V'| = k$
   - For each pair of vertices $u, v \in V'$ check that $(u, v) \in E$.

   There are $O(V^2)$ checks and each check is $O(V)$, so the overall run-time is $O(V^3)$.

2. To show that Clique is NP-Hard we show: Independent-Set $\leq_p$ Clique.

   (We're assuming Independent-Set is NP-Complete.)

   - Reduction: given an instance $\langle G, k \rangle$ of Independent-Set, we transform it into an instance of Clique, $\langle G', k' \rangle$, as follows:
     - Let $V' = V$, i.e., copy all of the vertices.
     - For all pairs of vertices $u, v \in V$, if $(u, v) \notin E$, add an edge (u, v) to G', i.e., E' will consist of all the edges *not* in G.
     - Set $k' = k$.
   - Reduction in polynomial time: The reduction takes time $O(V^2)$ to create $\langle G', k' \rangle$, which is polynomial wrt the original problem instance.
   - "yes" $\leftrightarrow$ "yes"
     - "yes" for Independent-Set $\rightarrow$ "yes" for Clique
       A "yes" for Independent-Set means that there are k vertices in G such where no edge exists between these vertices. In G' these vertices will therefore be fully connected forming a clique of size k.
     - "yes" for Clique $\rightarrow$ "yes" for Independent-Set
       A "yes" for Clique means there is a clique of size k in G'. In G these vertices will not have any edges between them, so they will represent an independent set.