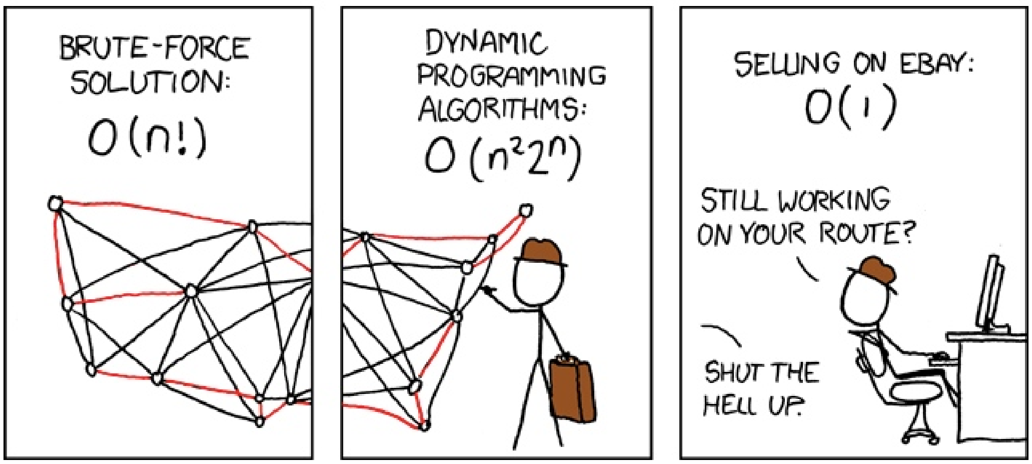
CS140 - Assignment 2

Due: Sunday, Sept. 11 at 11:59pm



http://xkcd.com/399/

This problem set should be done in the pairs within your learning group. If your group has five members, you may do a group of three.

You must use LaTeX to format your solution; one person should upload the pdf to gradescope.

1. **[10 points]** The Geometric Series

   (a) Use induction on n to show that for all integers $n \geq 0$

   $$1 + a + a^2 + a^3 + \ldots + a^n = \frac{a^{n+1} - 1}{a - 1}$$

   where a is some arbitrary real number other than 1. Please make sure to write your proof carefully, with a base case, induction hypothesis, induction step, and conclusion.

   (b) Explain where your induction proof relied on the fact that $a \neq 1$.

   (c) What does the sum evaluate to when $a = 1$?

2. **[12 points]** Solving Recurrences

   Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for $n \leq 2$. Make your bounds as tight as possible (in other words, give Θ bounds where possible) and make sure to justify your answers. (Note: you may want to read appendix A.1 in the textbook, which has some summation properties.)

(a) $T(n) = 4T(n/2) + cn$

(b) $T(n) = T(n-1) + n$

(c) $T(n) = T(n-1) + 1/n$

(d) $T(n) = T(9n/10) + n$

3. **[20 points]** 3-part sort

Consider the following sorting algorithm: First sort the first two-thirds of the elements in the array. Next sort the last two thirds of the array. Finally, sort the first two thirds again. Notice that this algorithm does not allocate any extra memory; all the sorting is done inside array A. Here's the code:

ThreeSort (A,i,j)
  if $A[i] > A[j]$
    swap $A[i]$ and $A[j]$
  if $i + 1 \geq j$
    return
  $k = \lfloor (j - i + 1)/3 \rfloor$.
  ThreeSort$(A, i, j - k)$ **Comment:** Sort first two-thirds.
  ThreeSort$(A, i + k, j)$ **Comment:** Sort last two thirds.
  ThreeSort$(A, i, j - k)$ **Comment:** Sort first two-thirds again!

(a) Give an informal but convincing explanation (not a rigorous proof by induction) of why the approach of sorting the first two-thirds of the array, then sorting the last two-thirds of the array, and then sorting again the first two-thirds of the array yields a sorted array. A few well-chosen sentences should suffice here.

(b) Find a recurrence relation for the worst-case running time of ThreeSort.

(c) Next, solve the recurrence relation using a recursion tree.

(d) Double check that you got the right answer in the previous part by solving the recurrence using the master method.

(e) How does the worst-case running time of ThreeSort compare with the worst-case running times of Insertion Sort, Selection Sort, and Mergesort?