Quicksort Implementation

https://cs.pomona.edu/classes/cs140/

Outline

Topics and Learning Objectives

- Learn how quicksort works
- Learn how to partition an array

Exercise

- Partitioning

Extra Resources

- https://me.dt.in.th/page/Quicksort/
- https://www.youtube.com/watch?v=ywWBy6J5gz8
- CLRS Chapter 7

Quicksort

- A practical and simple algorithm
- The running time = O(n lg n)
- Superior to other O(n lg n) in some respects
- The hidden constants are small (hidden by Big-O)
- Our first stochastic algorithm

Quicksort

Input : an array of n elements in any order

Output : a reordering of the input array such that the elements are in non-decreasing order

Key idea of Quicksort: **partition** the array around a pivot element

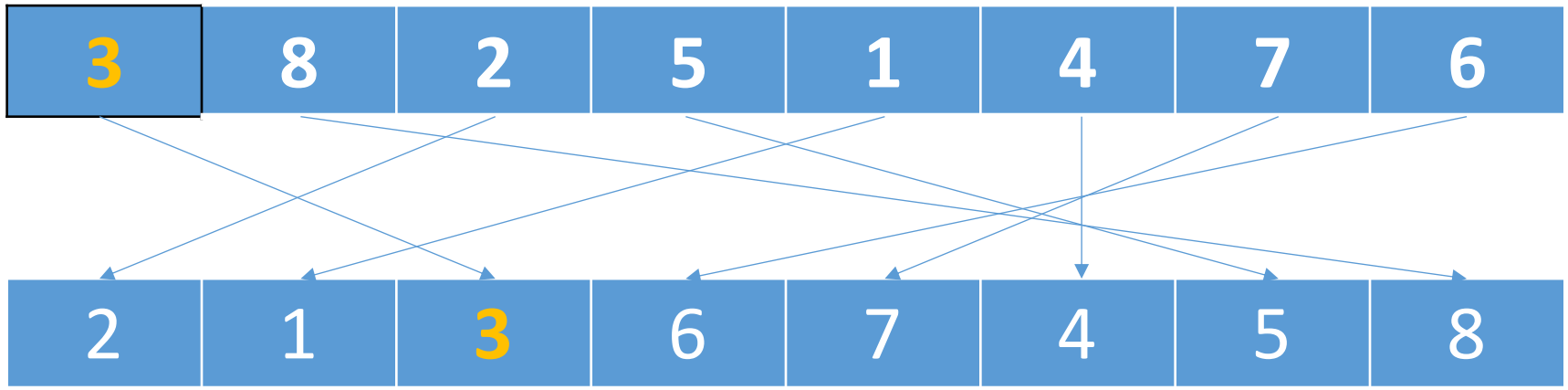
Key concept of Quicksort

- Pick an element and call it the pivot
- Partition (rearrange) the elements so that:
  - Everything to the left of the pivot is less than the pivot
  - Everything to the right of the pivot is greater than the pivot
  - Let's ignore ties for now

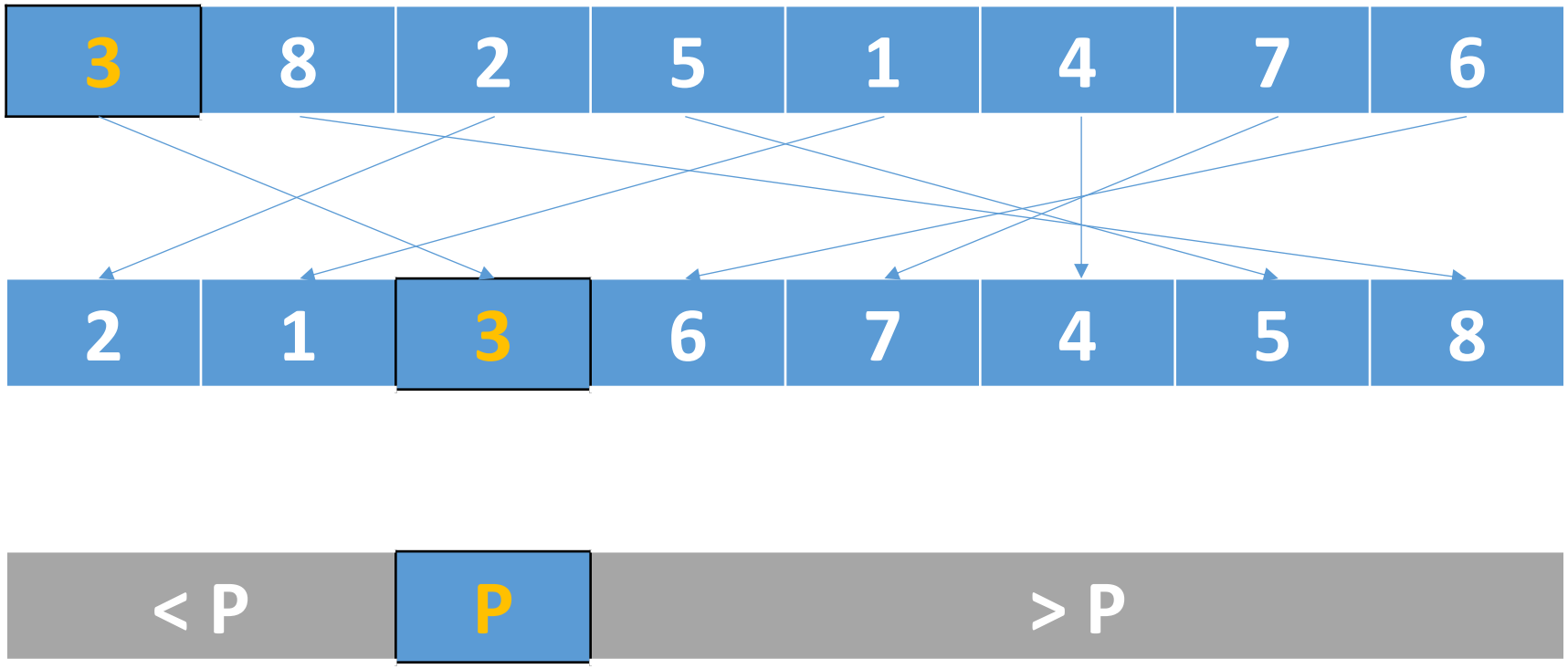
- This is a partial sorting into "buckets"
- What can you tell me about the pivot?
- **Pivot is now in the correct spot (we've made progress!)**

What would be the running time of calling partition on every element?

Partitioning



Partitioning



Pivot around “hello”

[“hello”, “are”, “you”, “how”, “today”, “doing”, “class”]

Quicksort (NOT IN-PLACE PARTITIONING)

```
1.  FUNCTION BadQuicksort(array)
2.      IF array.length ≤ 1
3.          RETURN array
4.
5.      pivot_index = ChoosePivot(array.length)
6.      left_array, right_array = Partition(array, pivot_index)
7.
8.      left_sorted = BadQuicksort(left_array)
9.      right_sorted = BadQuicksort(right_sorted)
10.
11.     RETURN left_sorted ++ array[pivot_index] ++ right_sorted
```

What is the recurrence equation for Quicksort?

Partitioning the Easy Way

- How would you partition? (how did we perform a merge?)
- Copy all elements to a new array

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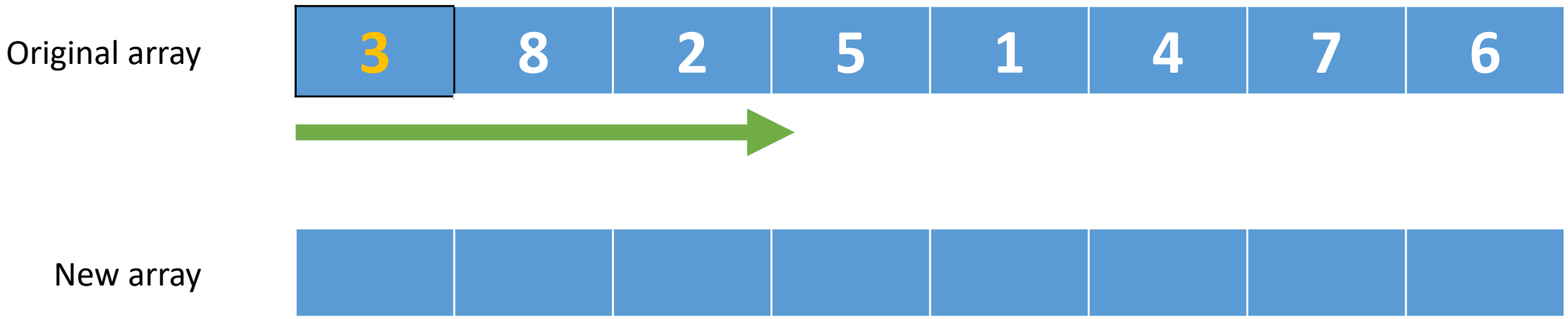

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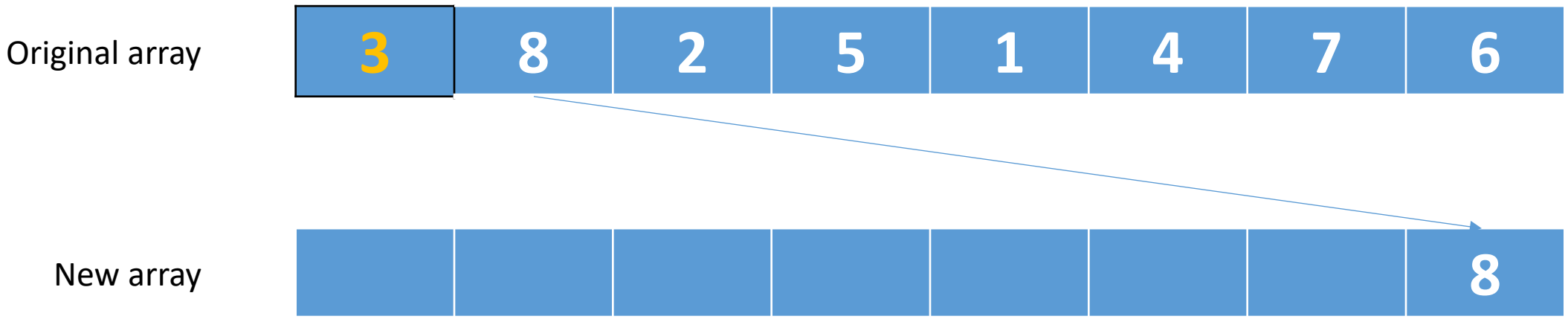

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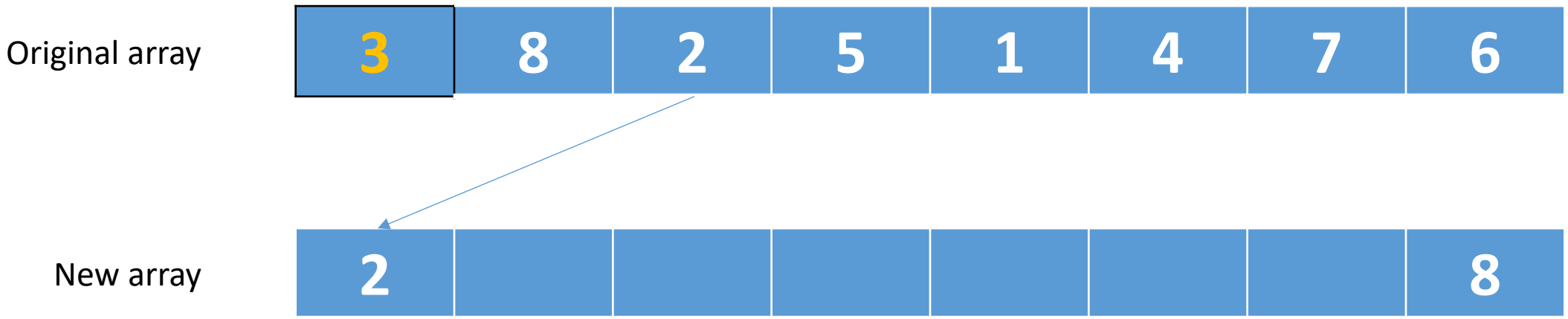

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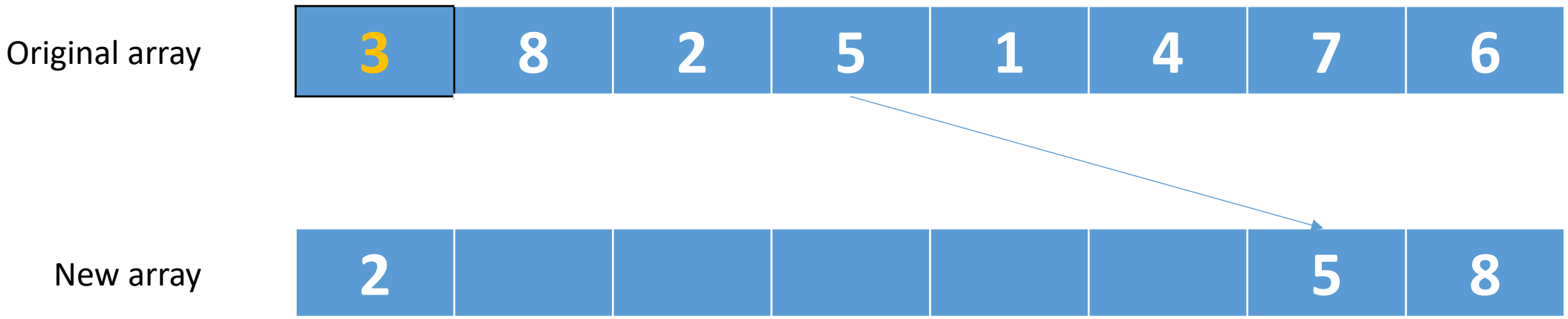

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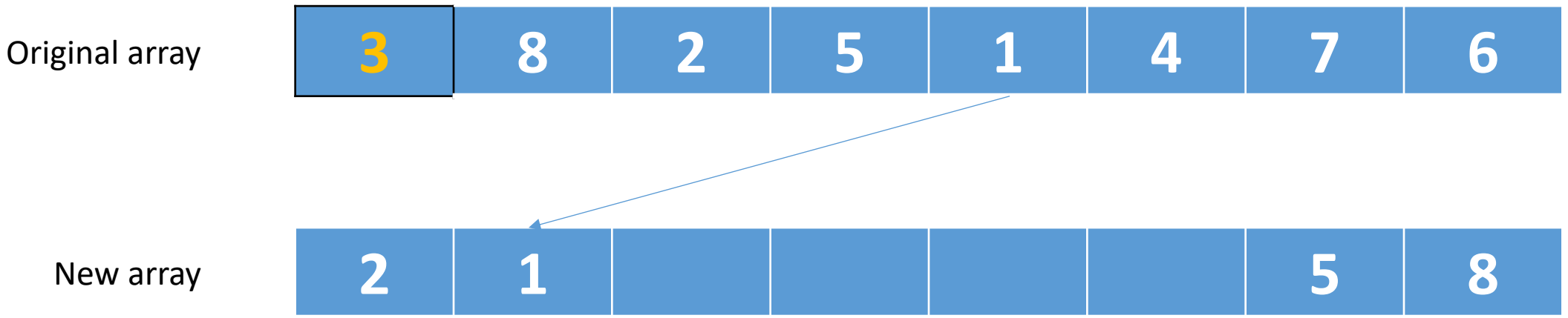

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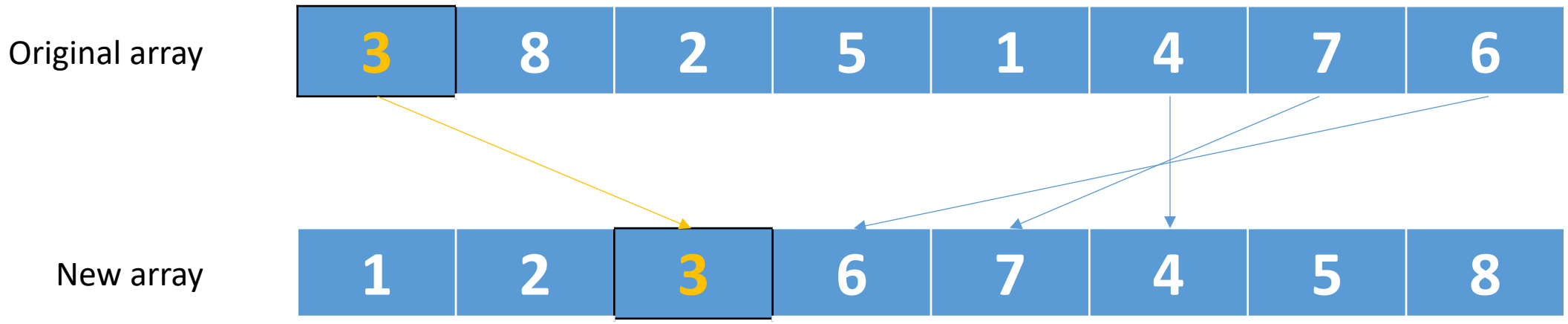

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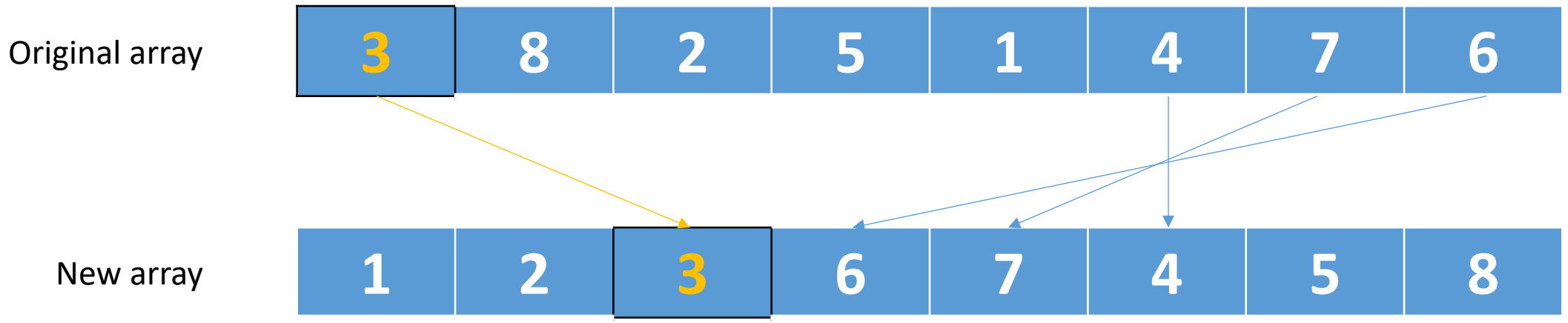


- This would be like merge sort.
- Lots of memory allocations (one for each node in the recursion tree).

Partitioning the Easy Way

- Nothing inherently wrong with this approach **in theory**
- But can we do the same thing without the extra memory?

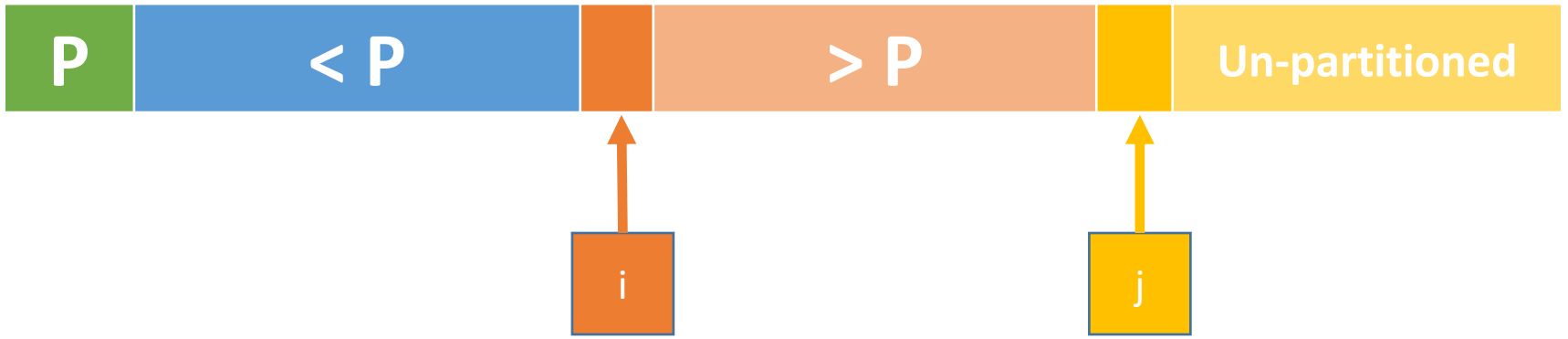
- Note: implementing **merge sort** “in-place” is possible
- You can do so with an iterative (stack based) approach

Partitioning In-Place

- For now, assume that the pivot is in the first spot of a subarray
- (we can swap the pivot with the first spot if needed)

- Idea: gradually build up a subarray that is correctly partitioned by scanning through the array

Partitioning In-Place



Index one to the right of the “smaller-than“ partition

Index one to the right of the “larger-than” partition

Index one to the right of the "smaller-than" partition

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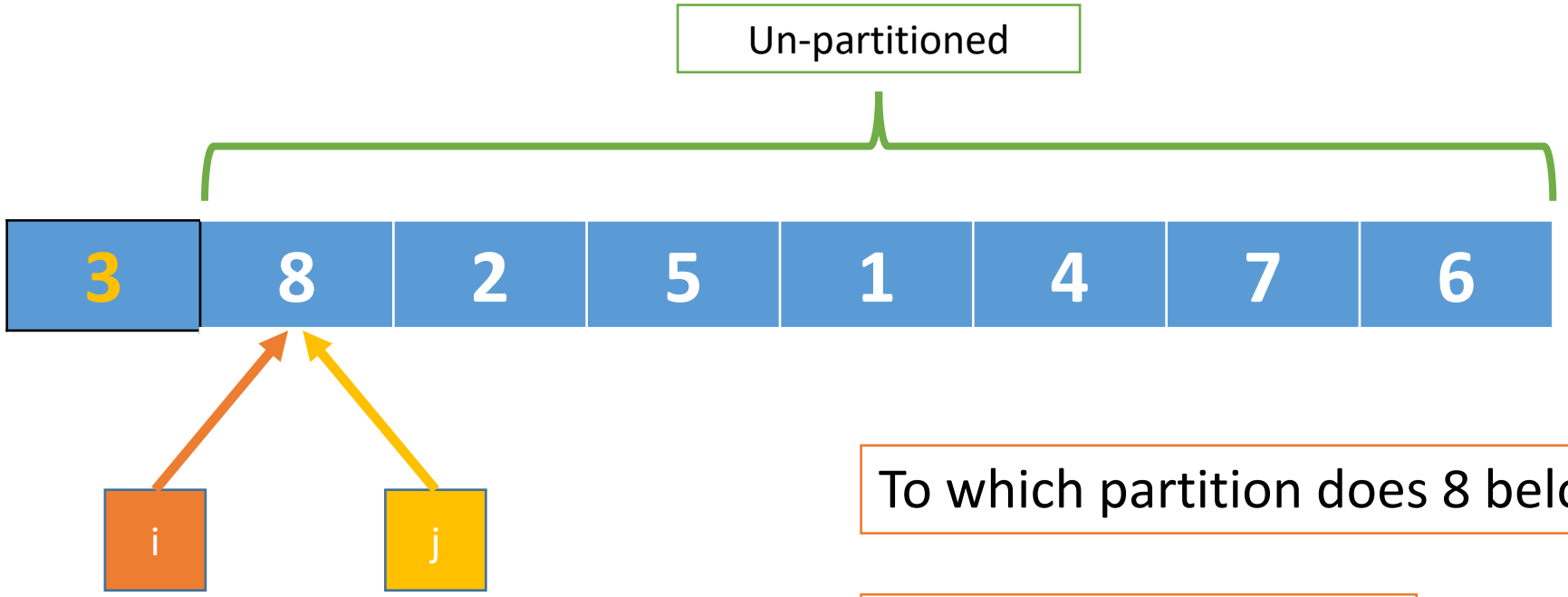


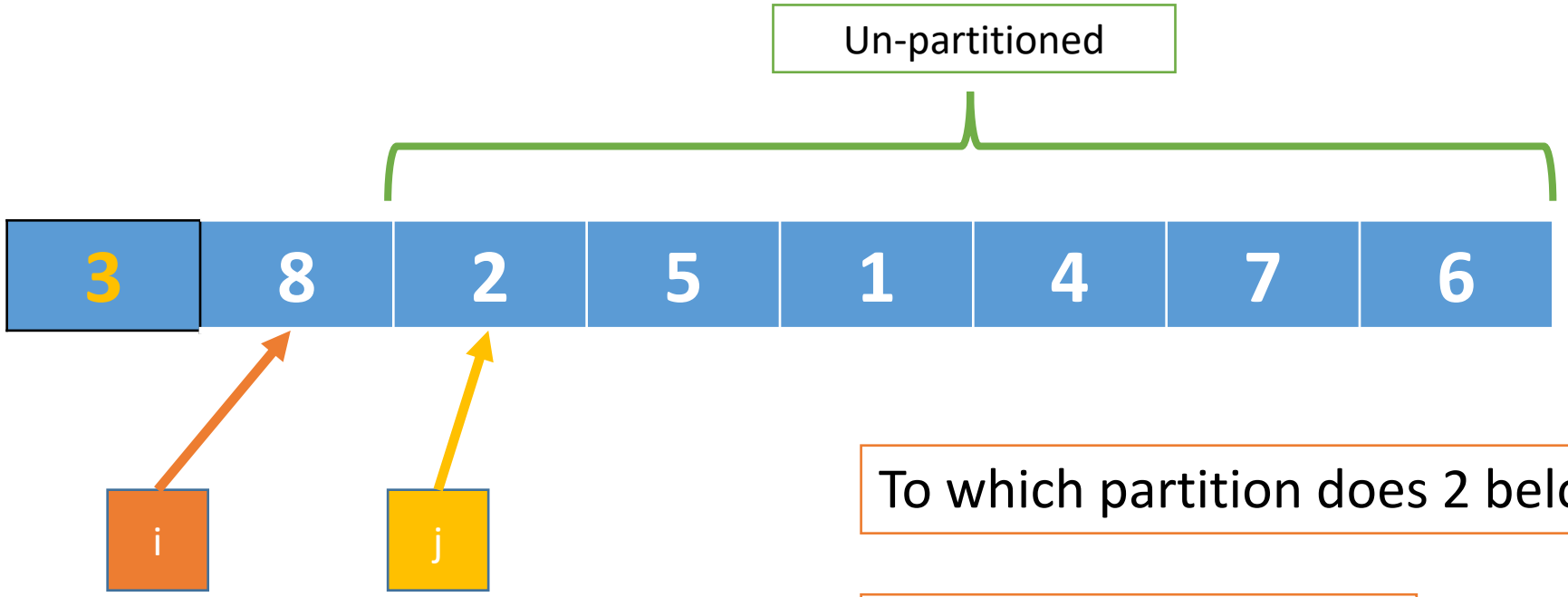
To which partition does 8 belong?

How do I put it there?

How should we initialize i and j?

Index one to the right of the “smaller-than“ partition

Index one to the right of the “larger-than” partition

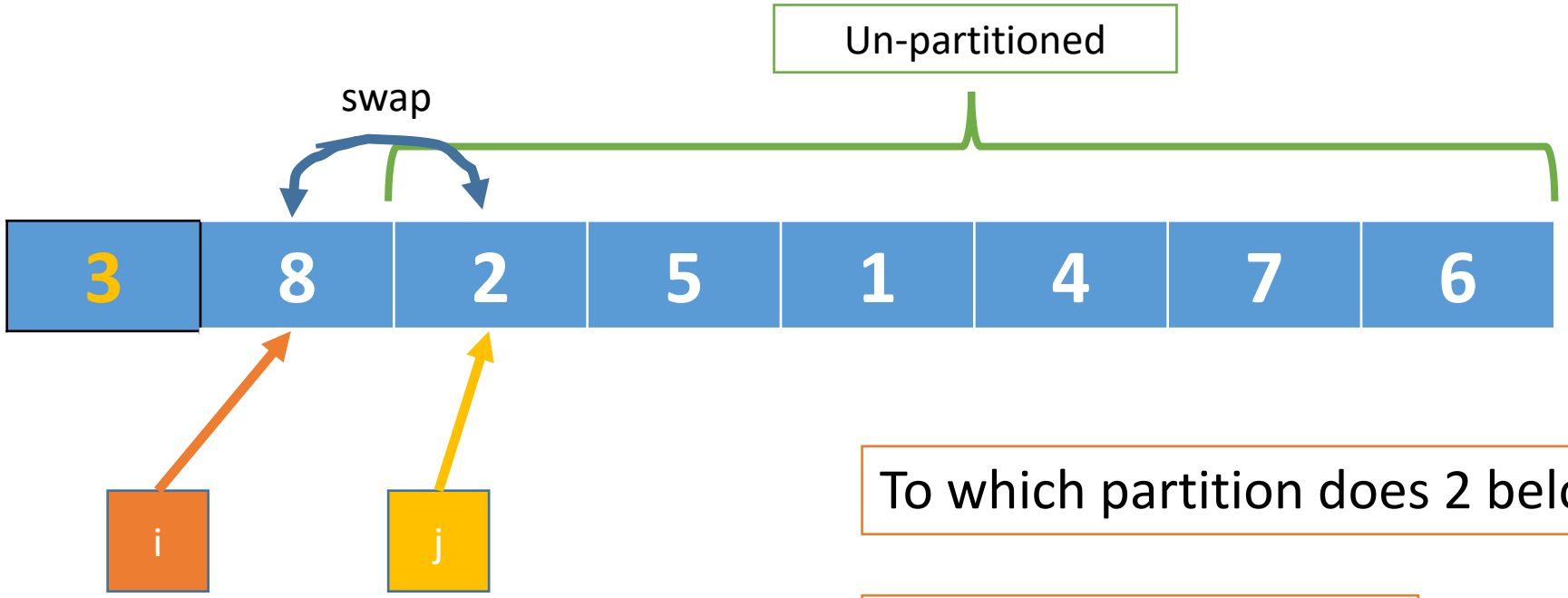


To which partition does 2 belong?

How do I put it there?

Index one to the right of the “smaller-than“ partition

Index one to the right of the “larger-than” partition



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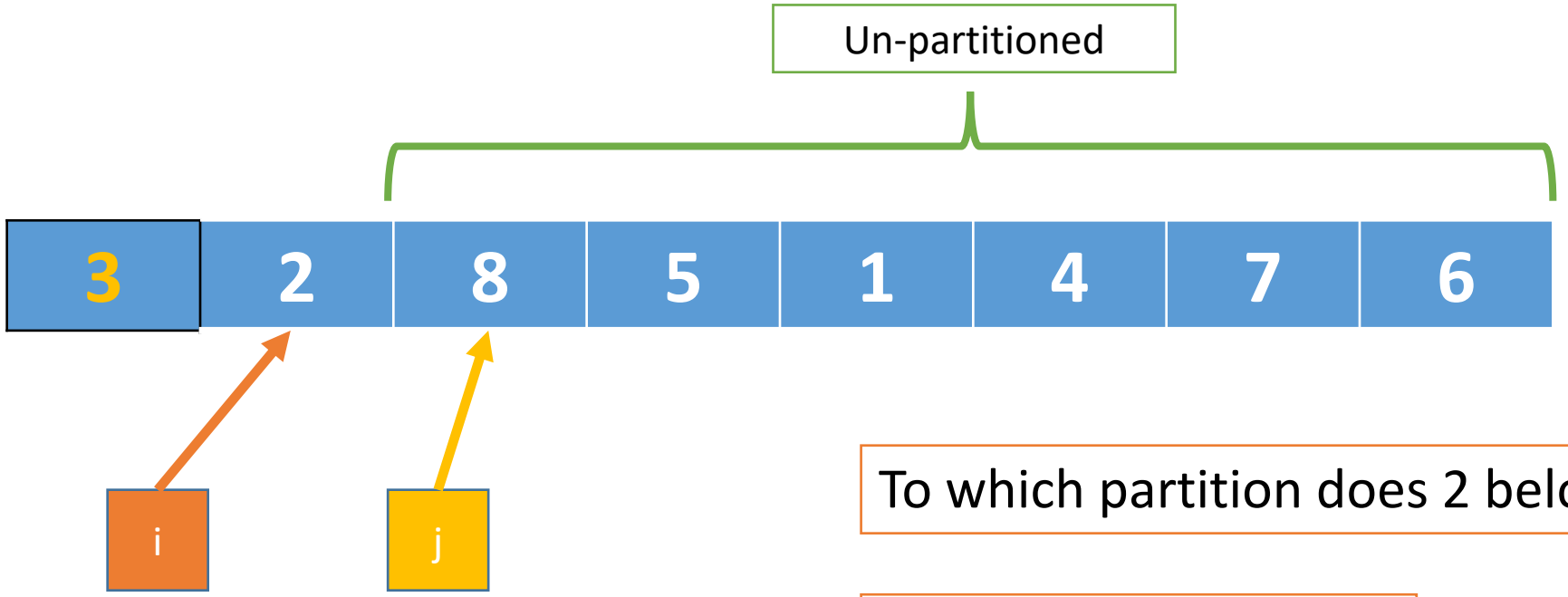


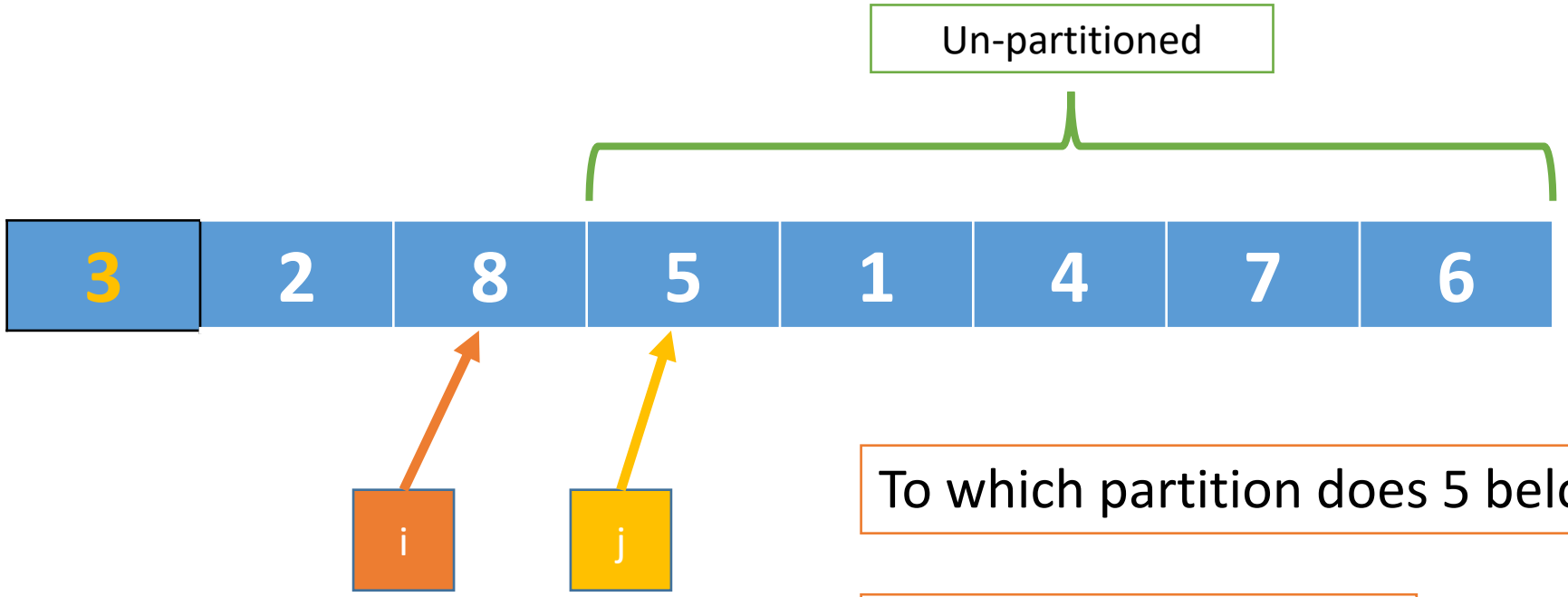
To which partition does 2 belong?

How do I put it there?

Now what?

Index one to the right of the “smaller-than“ partition

Index one to the right of the “larger-than” partition

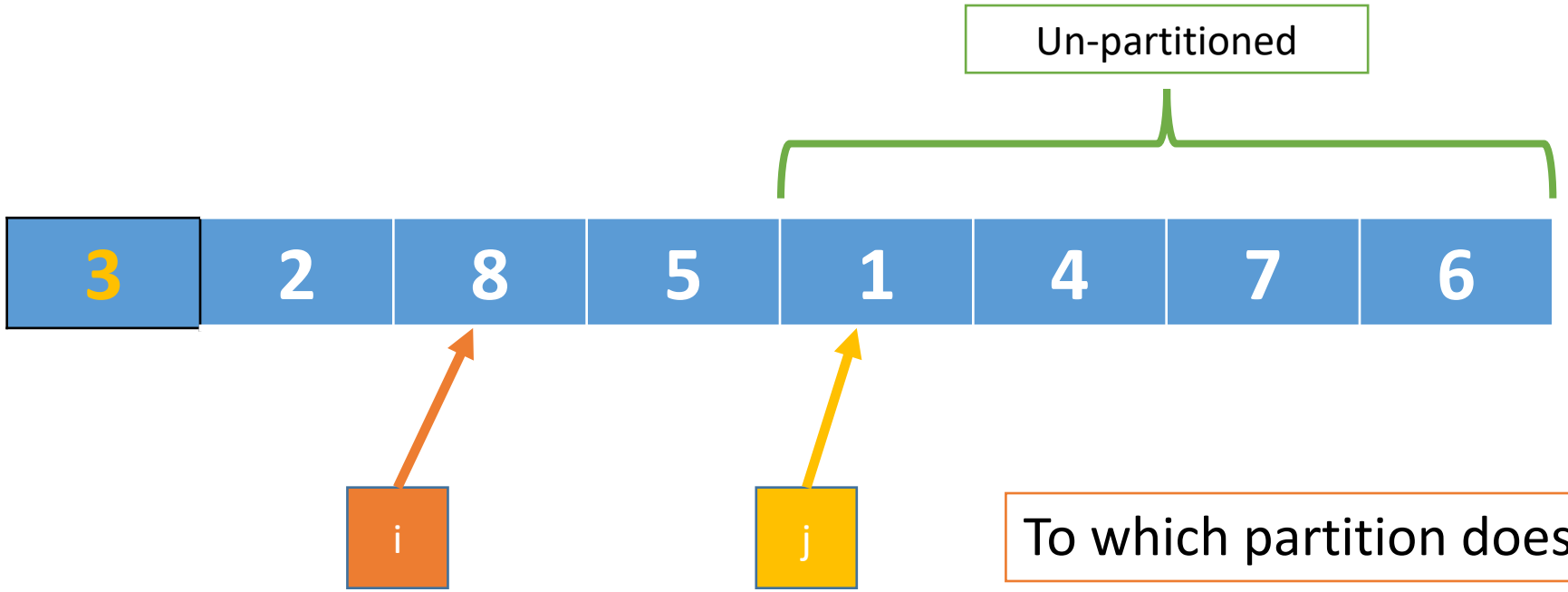


To which partition does 5 belong?

How do I put it there?

Index one to the right of the “smaller-than“ partition

Index one to the right of the “larger-than” partition

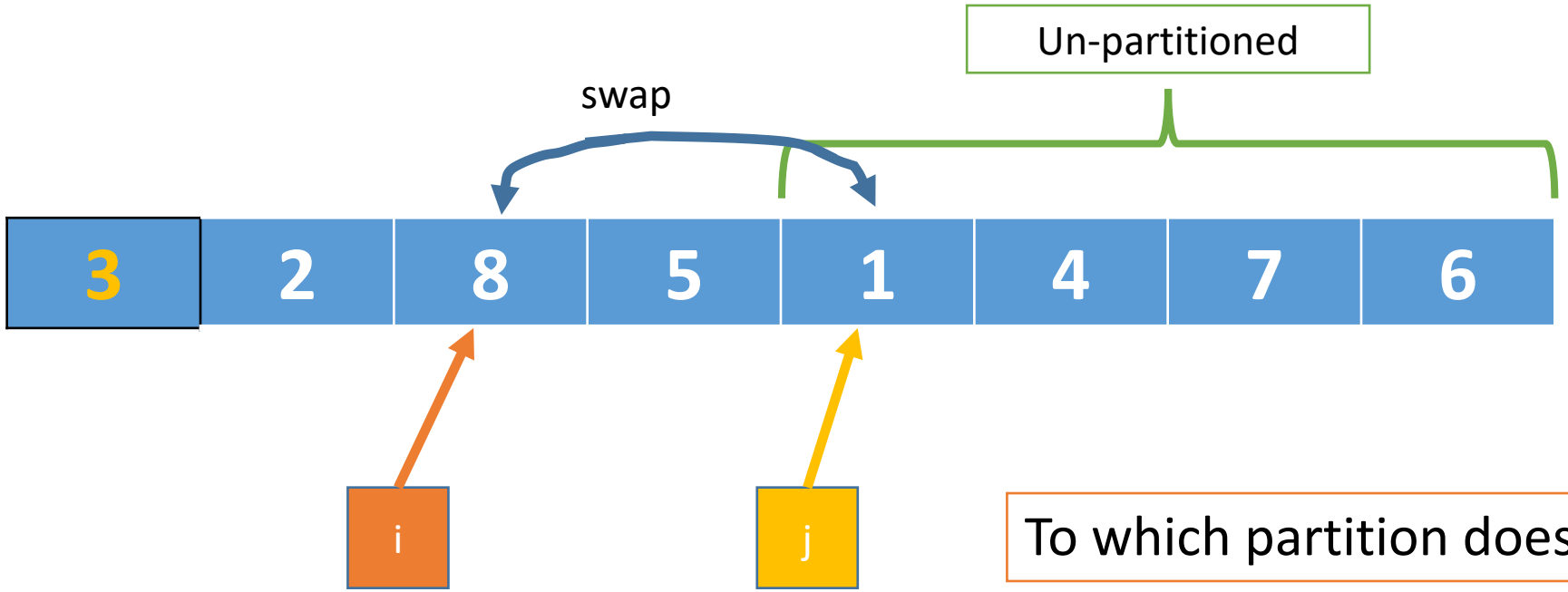


To which partition does 1 belong?

How do I put it there?

Index one to the right of the “smaller-than“ partition

Index one to the right of the “larger-than” partition

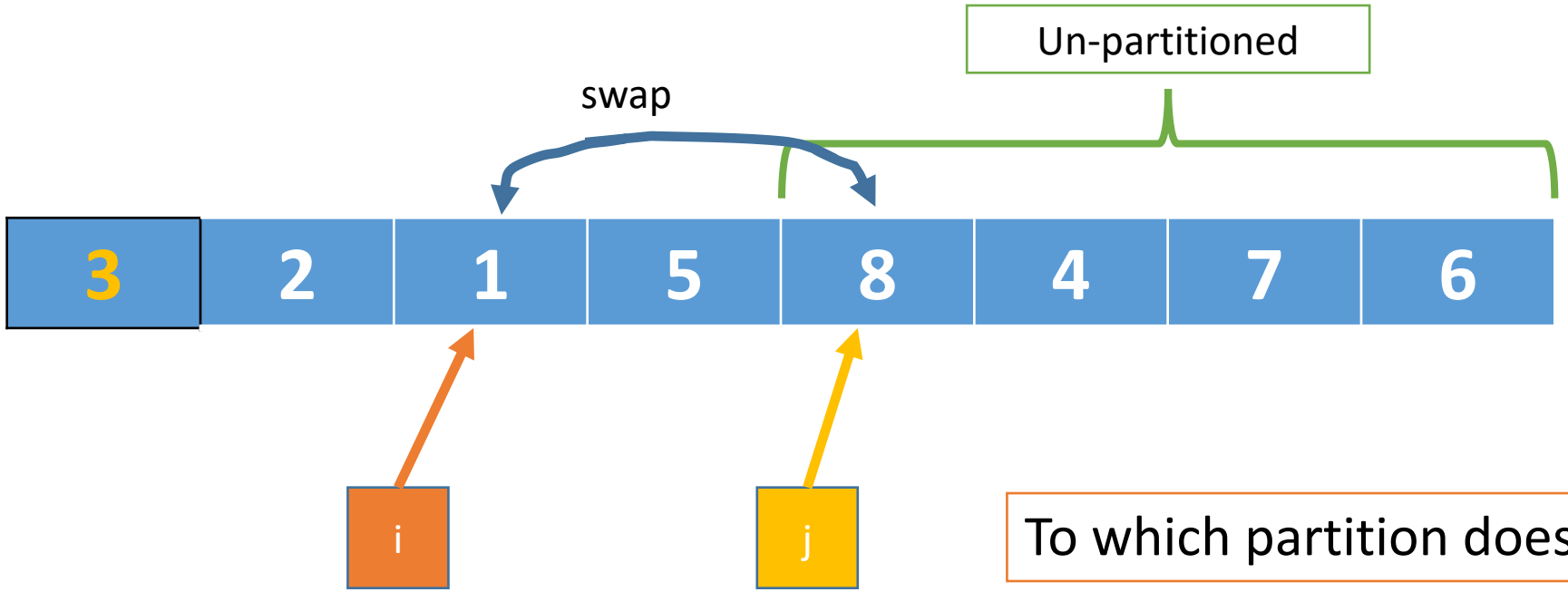


To which partition does 1 belong?

How do I put it there?

Index one to the right of the "smaller-than" partition

Index one to the right of the "larger-than" partition



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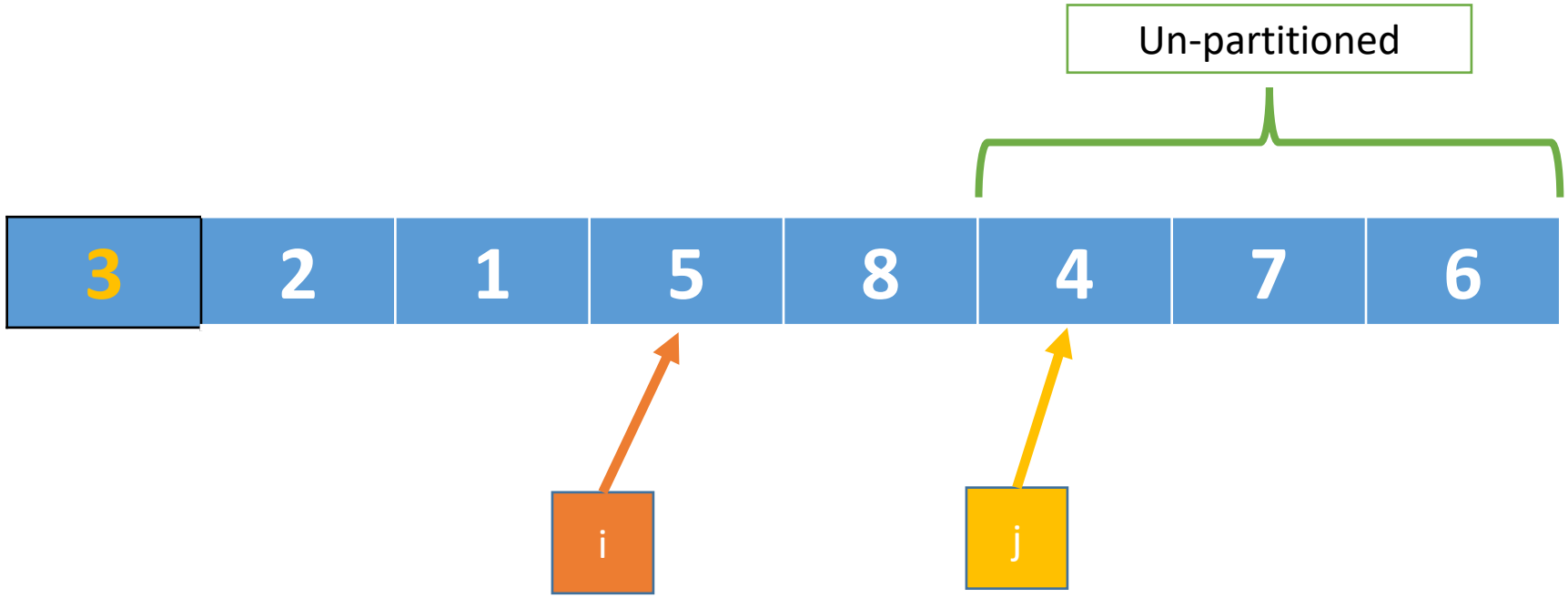

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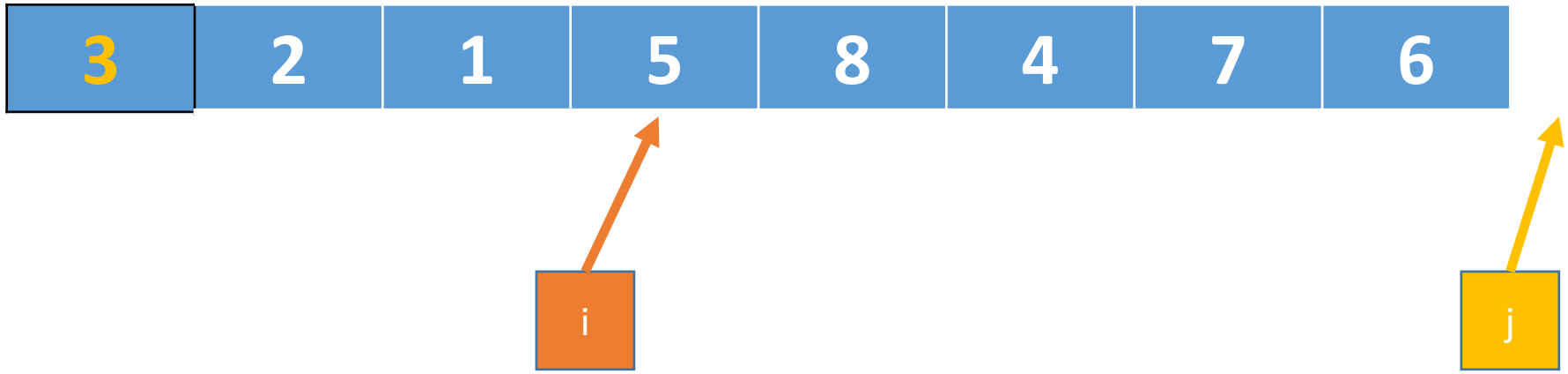

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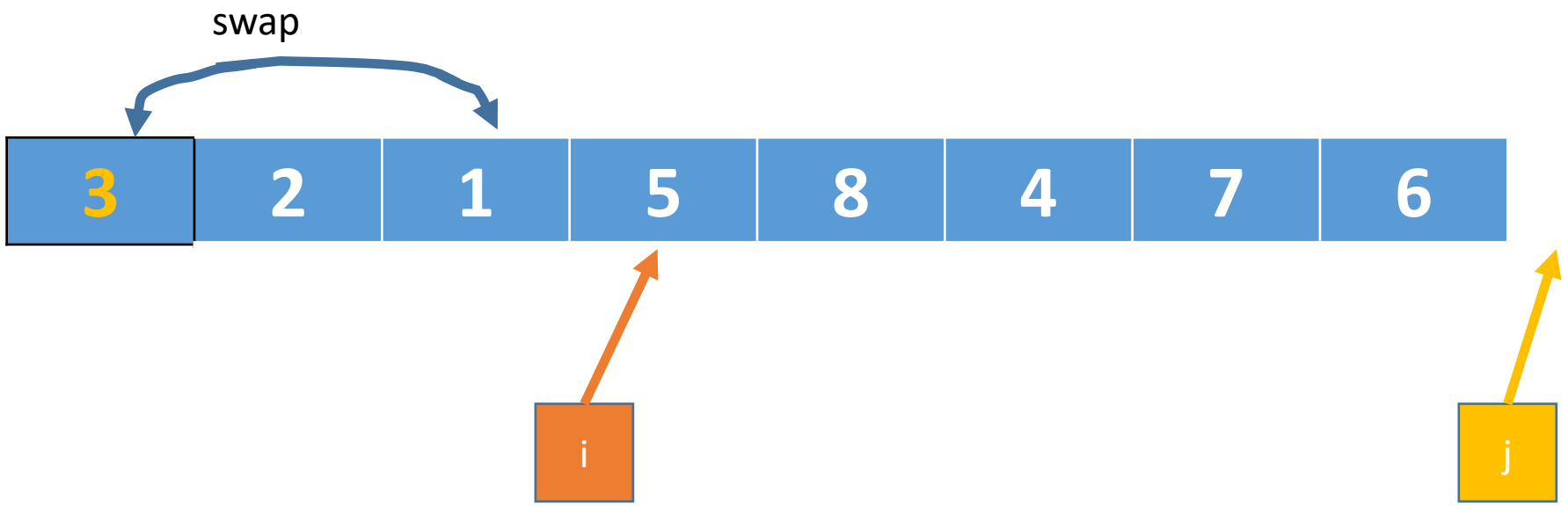


1
2
3
5
8
4
7
6

What is the asymptotic running time?

```
1.  FUNCTION Partition(array, left_index, right_index)
2.      # Partition the subarray array[left_index ..< right_index]
3.      # around the value at left_index
4.
5.      pivot_value = array[left_index]
6.
7.      i = left_index + 1
8.      FOR j IN [left_index + 1 ..< right_index]
9.          IF array[j] < pivot_value
10.             swap(array, i, j)
11.             i = i + 1
12.
13.     swap(array, left_index, i - 1)
14.     RETURN i - 1
```

1. O(n), where n is right_index - left_index

2. In-place
   no extra memory

```
1. FUNCTION QuickSort(array, left_index, right_index)
2.     IF 
3.         RETURN
4. 
5.     MovePivotToLeft(left_index, right_index)
6.     pivot_index = Partition(array, left_index, right_index)
7. 
8.     QuickSort(array, )
9.     QuickSort(array, )
```

Our `Partition` function expects the pivot element to be at `left_index`

How would you call `QuickSort`?

```
1. FUNCTION QuickSort(array, left_index, right_index)
2.     IF left_index ≥ right_index
3.         RETURN
4.
5.     MovePivotToLeft(left_index, right_index)
6.     pivot_index = Partition(array, left_index, right_index)
7.
8.     QuickSort(array, left_index, pivot_index)
9.     QuickSort(array, pivot_index + 1, right_index)
```

Our `Partition` function expects the pivot element to be at `left_index`