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Algorithms, Closest Pair - Tuesday, February 2, 2021

1. What must be the running time of `ClosestSplitPair` if the `ClosestPair` algorithm is to have a running time of $O(n \lg n)$?

```
FUNCTION ClosestPair(px, py)
    n = px.length
    IF n == 2
        RETURN px[0], px[1], dist(px[0], px[1])

    left_px = px[0 ..< n//2]
    left_py = [p FOR p IN py IF p.x < px[n//2].x]
    pl, ql, dl = ClosestPair(left_px, left_py)

    right_px = px[n//2 ..< n]
    right_py = [p FOR p IN py IF p.x ≥ px[n//2].x]
    pr, qr, dr = ClosestPair(right_px, right_py)

    d = min(dl, dr)
    ps, qs, ds = ClosestSplitPair(px, py, d)

    RETURN Closest(pl, ql, dl, pr, qr, dr, ps, qs, ds)
```

2. What is the running time of the nested for-loop (looping over j)?

```
FUNCTION ClosestSplitPair(px, py, d)
    n = px.length
    x_median = px[n//2].x
    middle_py = [p FOR p IN py IF x_median - d < p.x < x_median + d]

    closest_d = INFINITY, closest_p = closest_q = NONE
    FOR i IN [0 ..< middle_py.length - 1]
        FOR j IN [1 ..= min(7, middle_py.length - i)]
            p = middle_py[i], q = middle_py[i + j]
            IF dist(p, q) < closest_d
                closest_d = dist(p, q)
                closest_p = p, closest_q = q

    RETURN closest_p, closest_q, closest_d
```