Graphs and Connectivity

https://cs.pomona.edu/classes/cs140/

Outline

Topics and Learning Objectives

- Discuss the basics of graphs
- Introduce graph searching

Exercise

- Graph search

Extra Resources

- Introduction to Algorithms, 3rd, Chapter 22
- Algorithms Illuminated Part 2: Chapter 7

Graphs

Represent pairwise relationships

Tons of uses

- Physical connections : roads (driving directions), network routing (phone), …
- Relationship groups : social networks, similar purchases, …
- Problem solving : each vertex may represent a partial part of the problem, and each edge is a step/move (e.g., Sudoku)

Tons of algorithms

- Cuts, clustering, searching, partitioning, contracting, …

Graphs

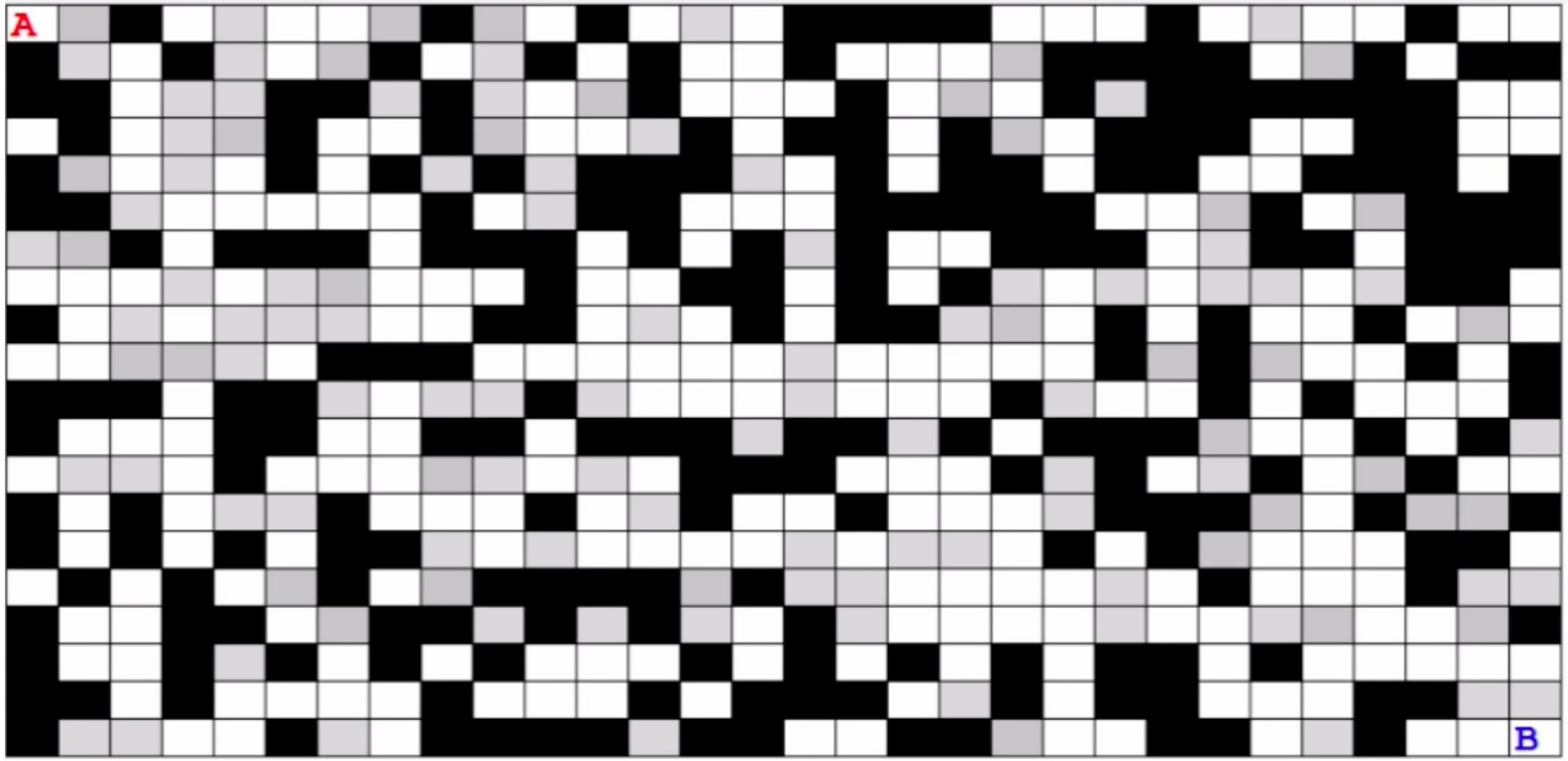
For many reasons, graph algorithms are extremely important.

They are a ubiquitous tool for solving many engineering problems

- Signal traces on a PCB
- Balancing the load on a server
- Balancing the load across cores on a computer
- Scheduling the delivery of packages via drone
- Scheduling the path of an automated robot that is grabbing your Amazon purchase from shelves in a warehouse
- Topological networks
- Data mirroring across a network
- Modeling an ecology
- Modeling the nervous system
- The list goes on and on

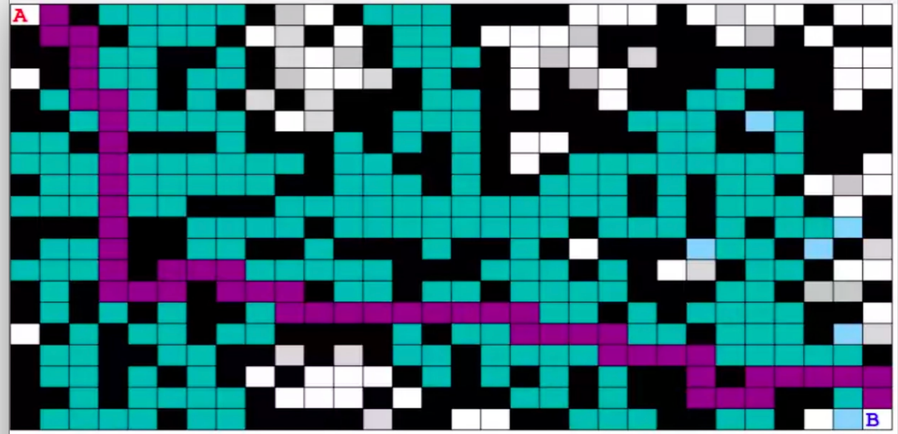
For this reason, you will often be asked graph-related questions during interviews

Algorithm



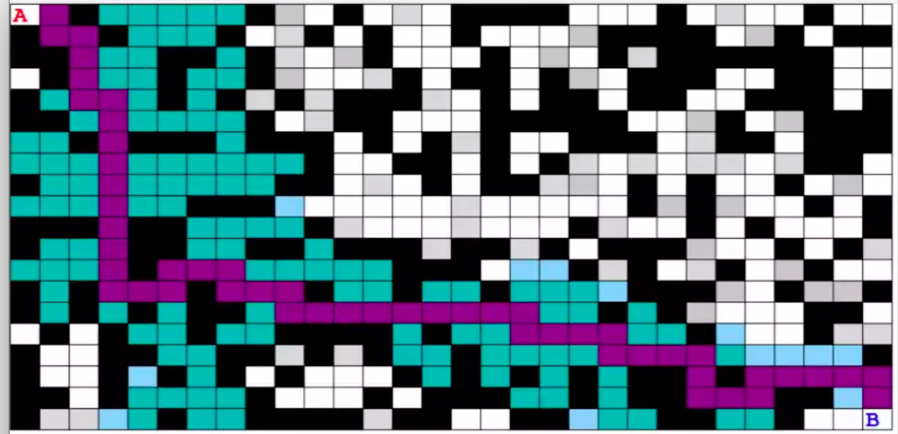
	Operations	Unexplored	Open	Closed	Path Length/Cost
Dijkstra:	0	0	0	0	?/?
A*:	0	0	0	0	?/?
Breadth-First:	0	0	0	0	?/?
Depth-First:	0	0	0	0	?/?

Algorithm: Dijkstra's



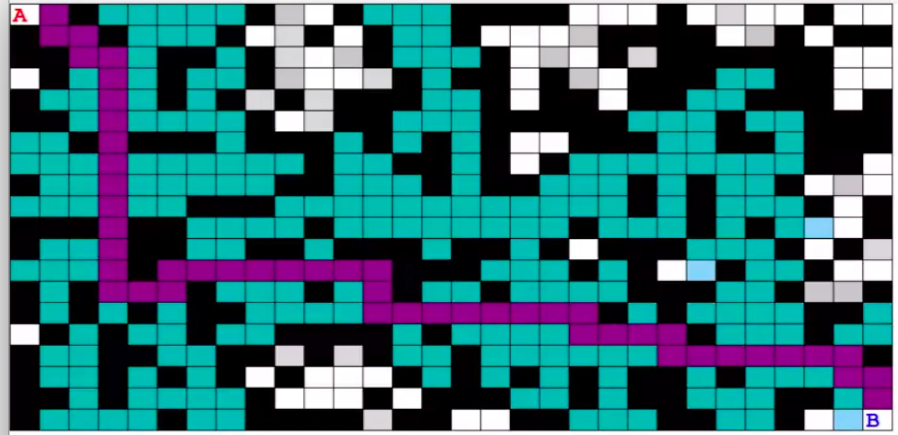
	Operations	Unexplored	Open	Closed	Path Length/Cost
Dijkstra:	732	80	6	283	53/65
A*:	0	0	0	0	?/?
Breadth-First:	0	0	0	0	?/?
Depth-First:	0	0	0	0	?/?

Algorithm: A*



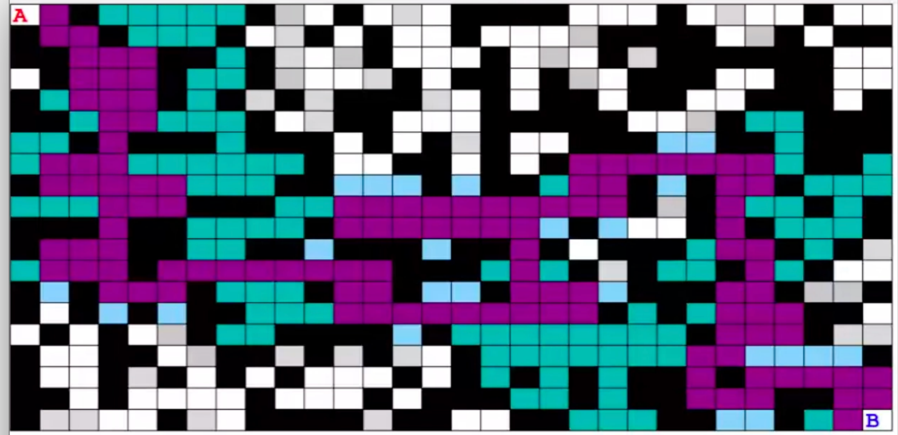
	Operations	Unexplored	Open	Closed	Path Length/Cost
Dijkstra:	732	80	6	283	53/65
A*:	441	188	13	168	53/65
Breadth-First:	0	0	0	0	?/?
Depth-First:	0	0	0	0	?/?

Algorithm: Breadth-First

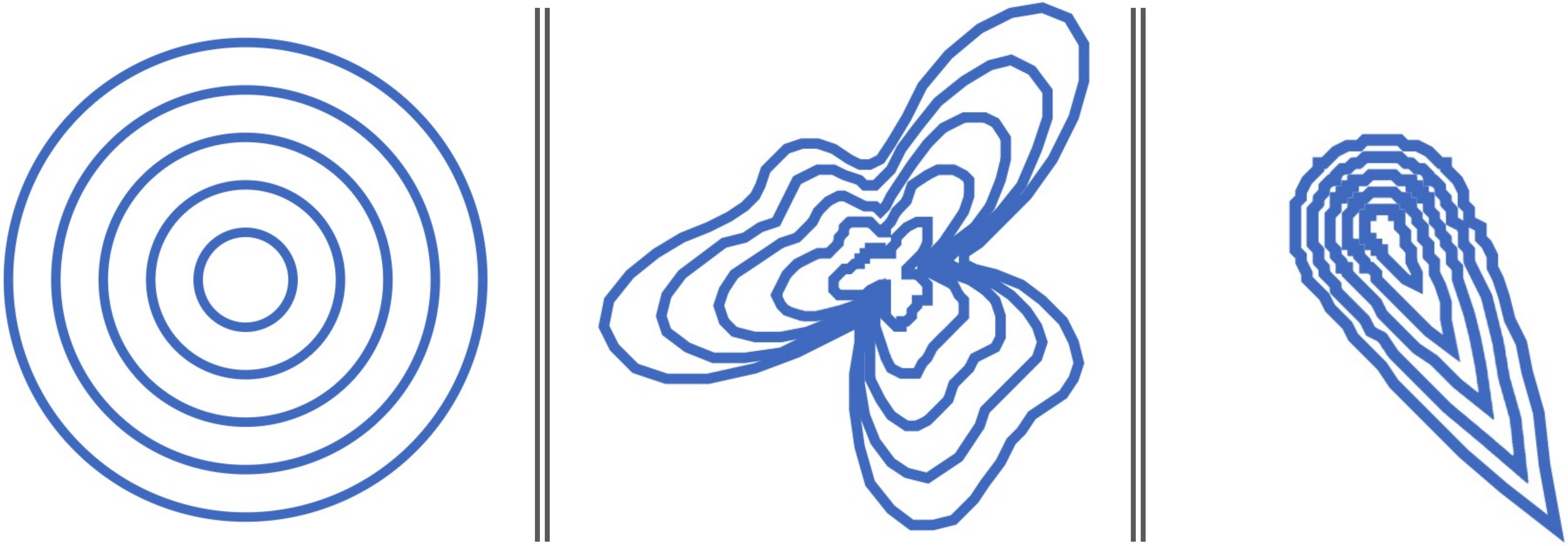


	Operations	Unexplored	Open	Closed	Path Length/Cost
Dijkstra:	732	80	6	283	53/65
A*:	441	188	13	168	53/65
Breadth-First:	367	79	5	288	51/75
Depth-First:	0	0	0	0	?/?

Algorithm: Depth-First



	Operations	Unexplored	Open	Closed	Path Length/Cost
Dijkstra:	732	80	6	283	53/65
A*:	441	188	13	168	53/65
Breadth-First:	367	79	5	288	51/75
Depth-First:	314	130	117	99	117/160



BFS vs Dijkstra's vs A*

https://www.redblobgames.com/pathfinding/a-star/introduction.html

G = (V, E)

G is the standard symbol representing a graph

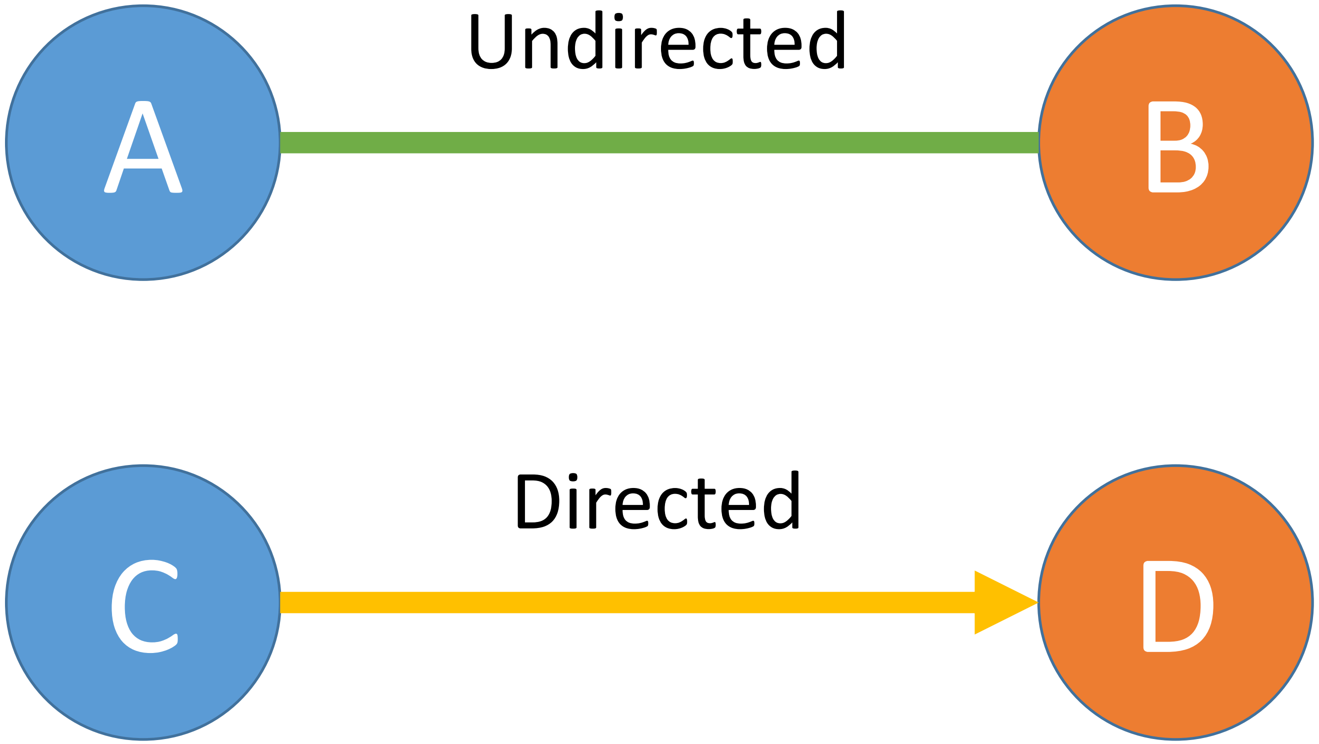
V is the standard symbol representing a set of graph vertices ($|V| = n$)

- Vertices are also sometimes referred to as nodes

E is the standard symbol representing a set of graph edges ($|E| = m$)

- Each edge contains pointers to two vertices, for example: (v1, v2)
- The order of the vertices may or may not matter

Directed and Undirected



Notation for Edges

(A, B) or (B, A)

(C, D)

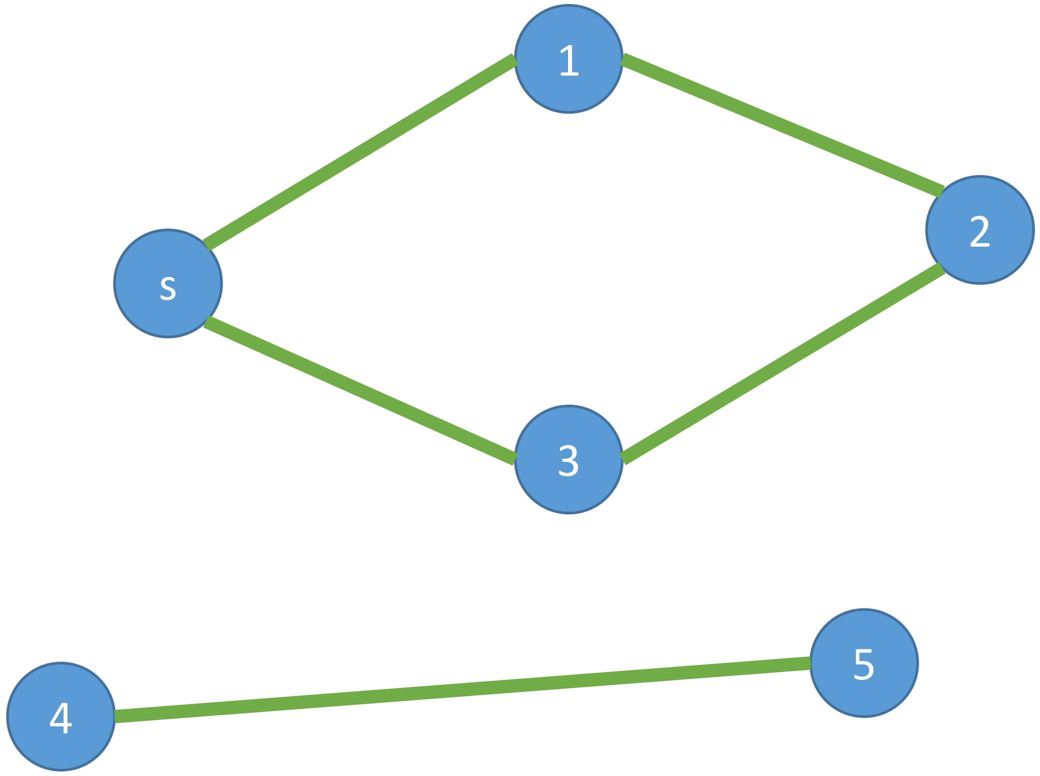
Graph Search and Connectivity

Goals:

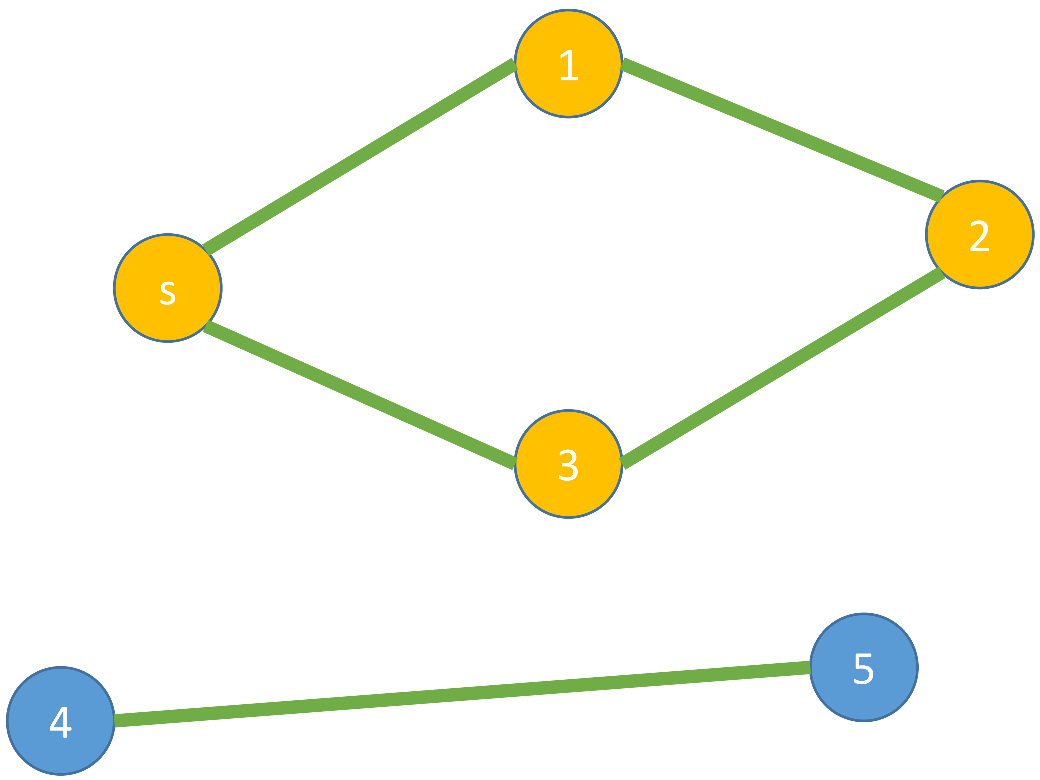
- Find everything that is findable (a “path” from the start node exists)
- Don’t explore anything twice (don’t waste time)

- These operations are done in linear time,
- Note: it is often useful to consider O(n) algorithms as being “free”
  - (when compared to more complex tasks)

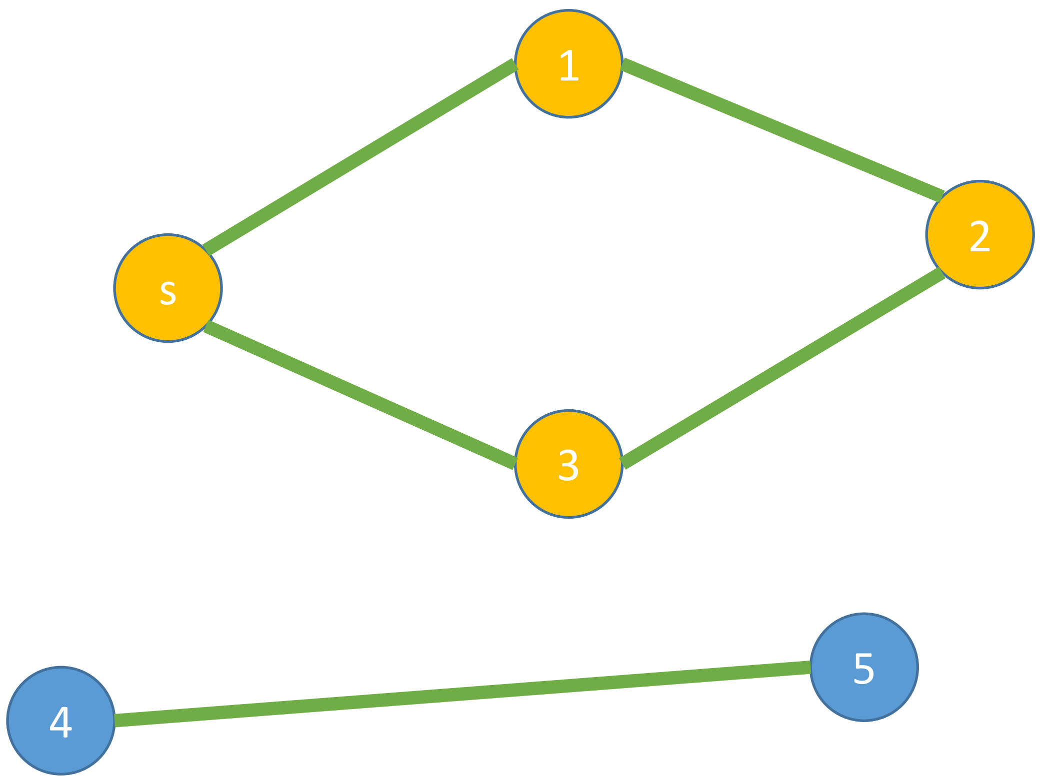
Findable

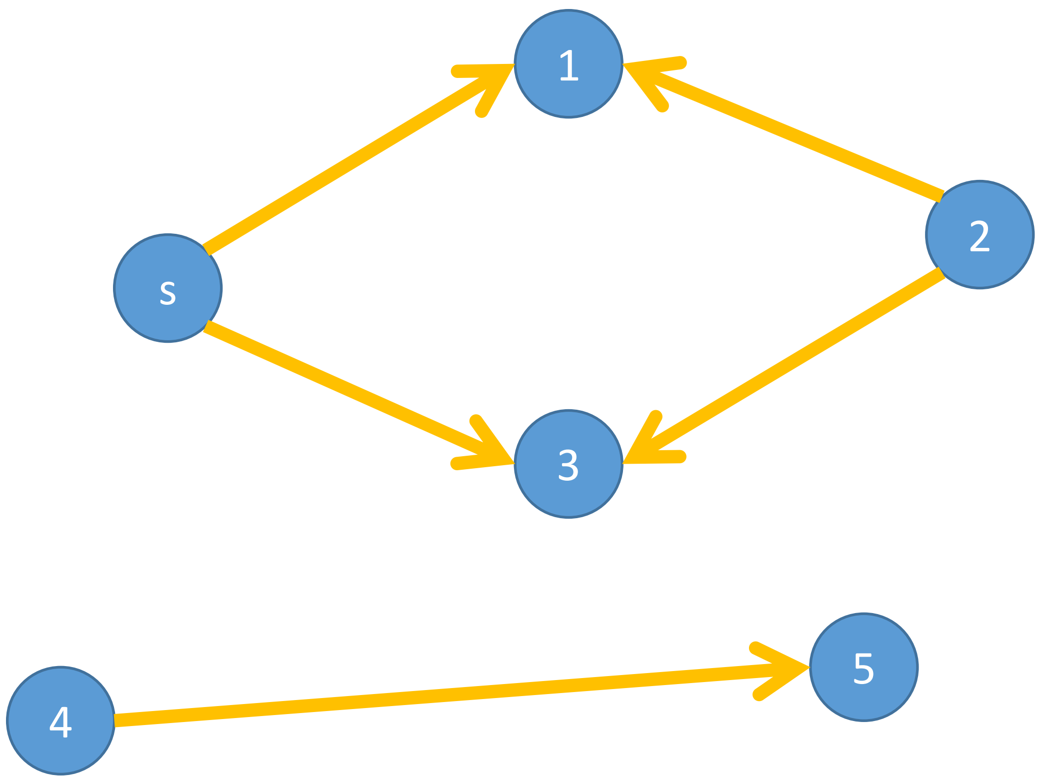


Findable

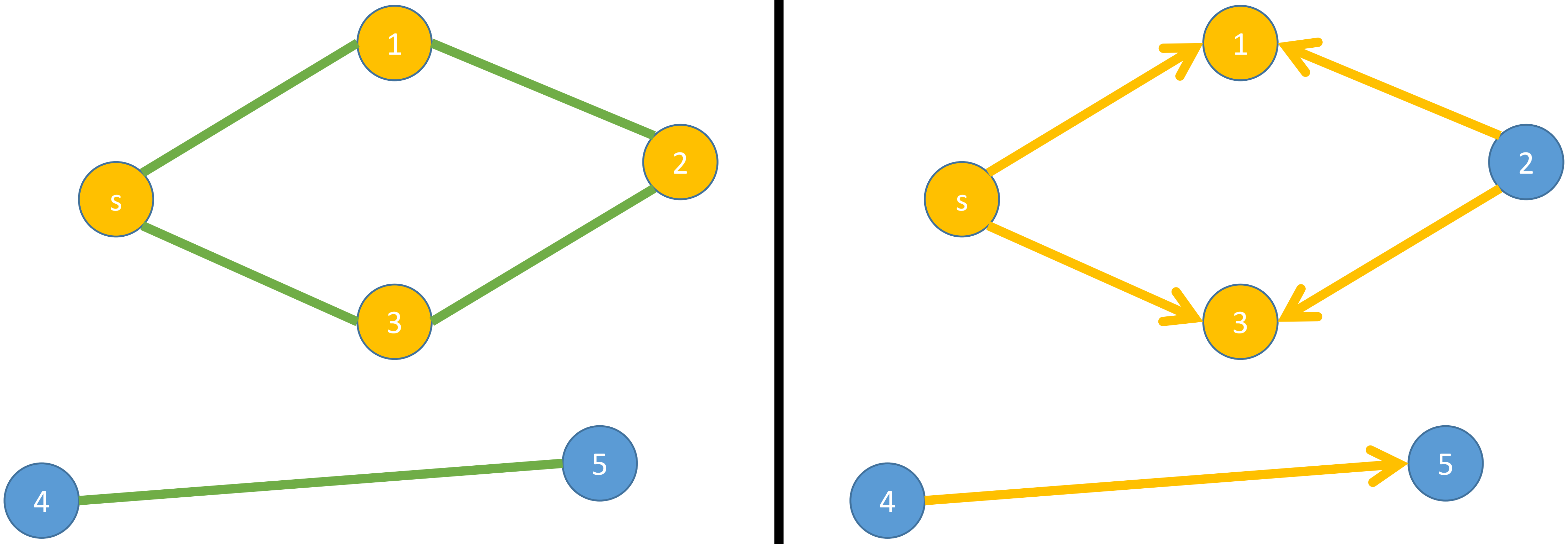


Findable



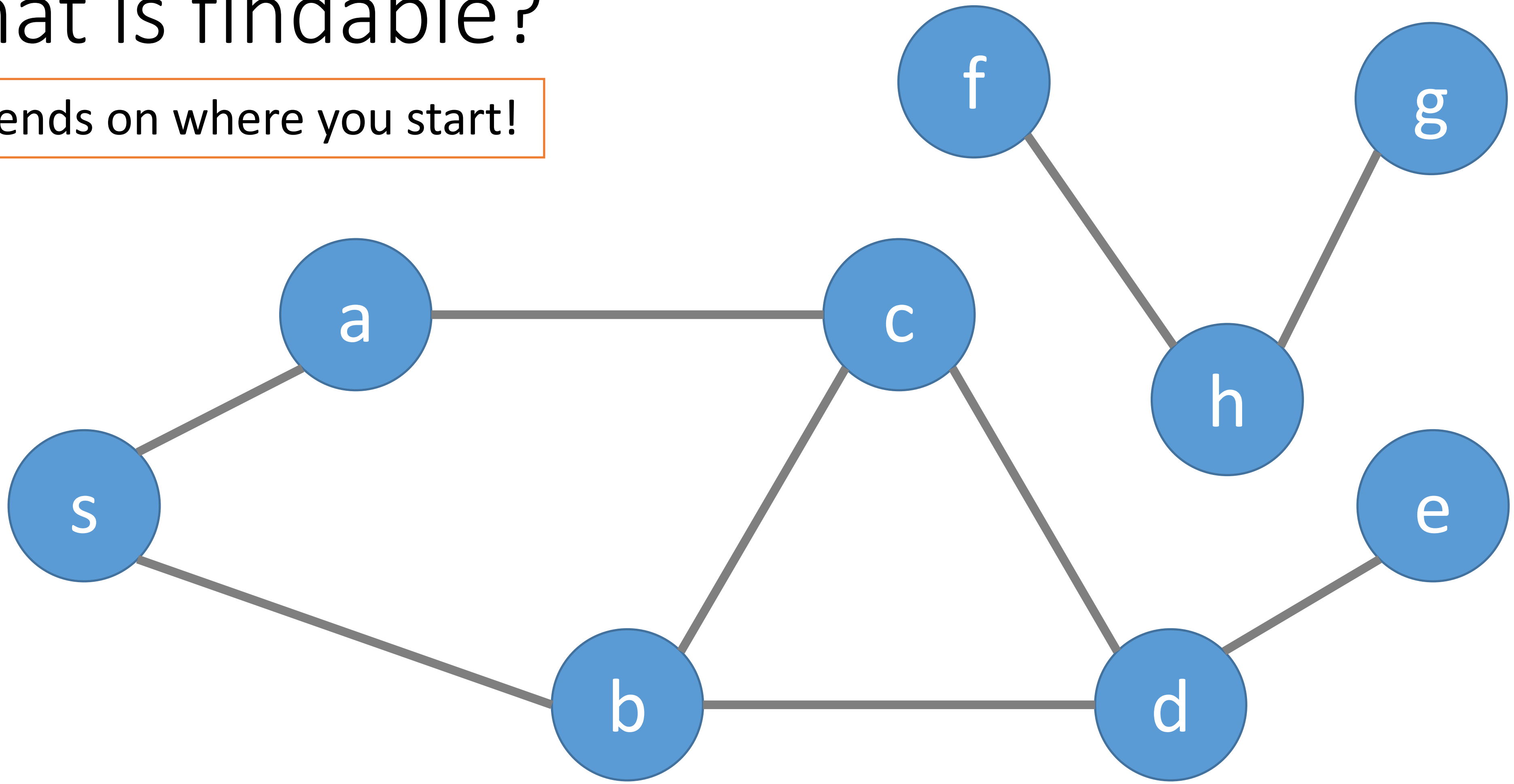


Findable

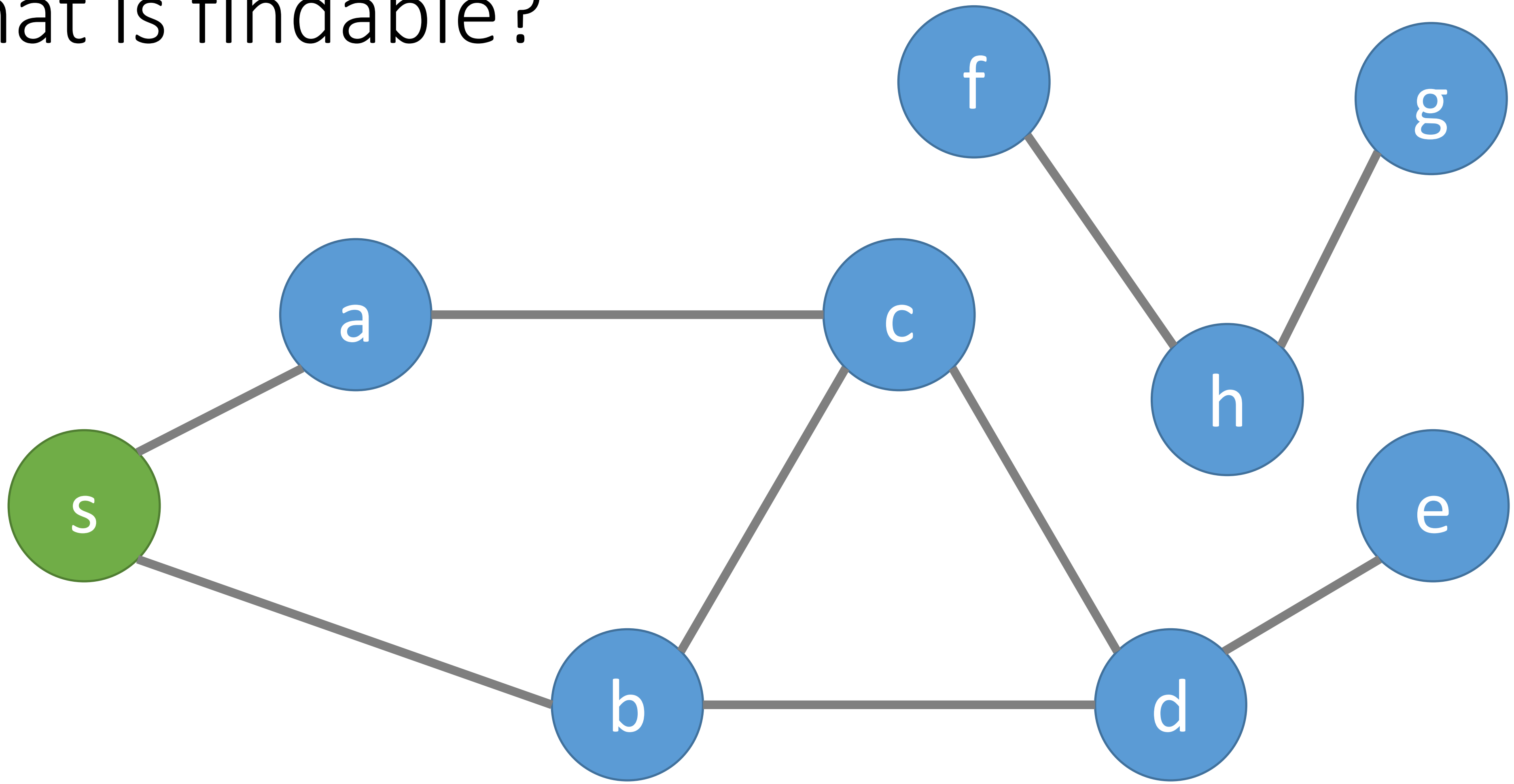


What is findable?

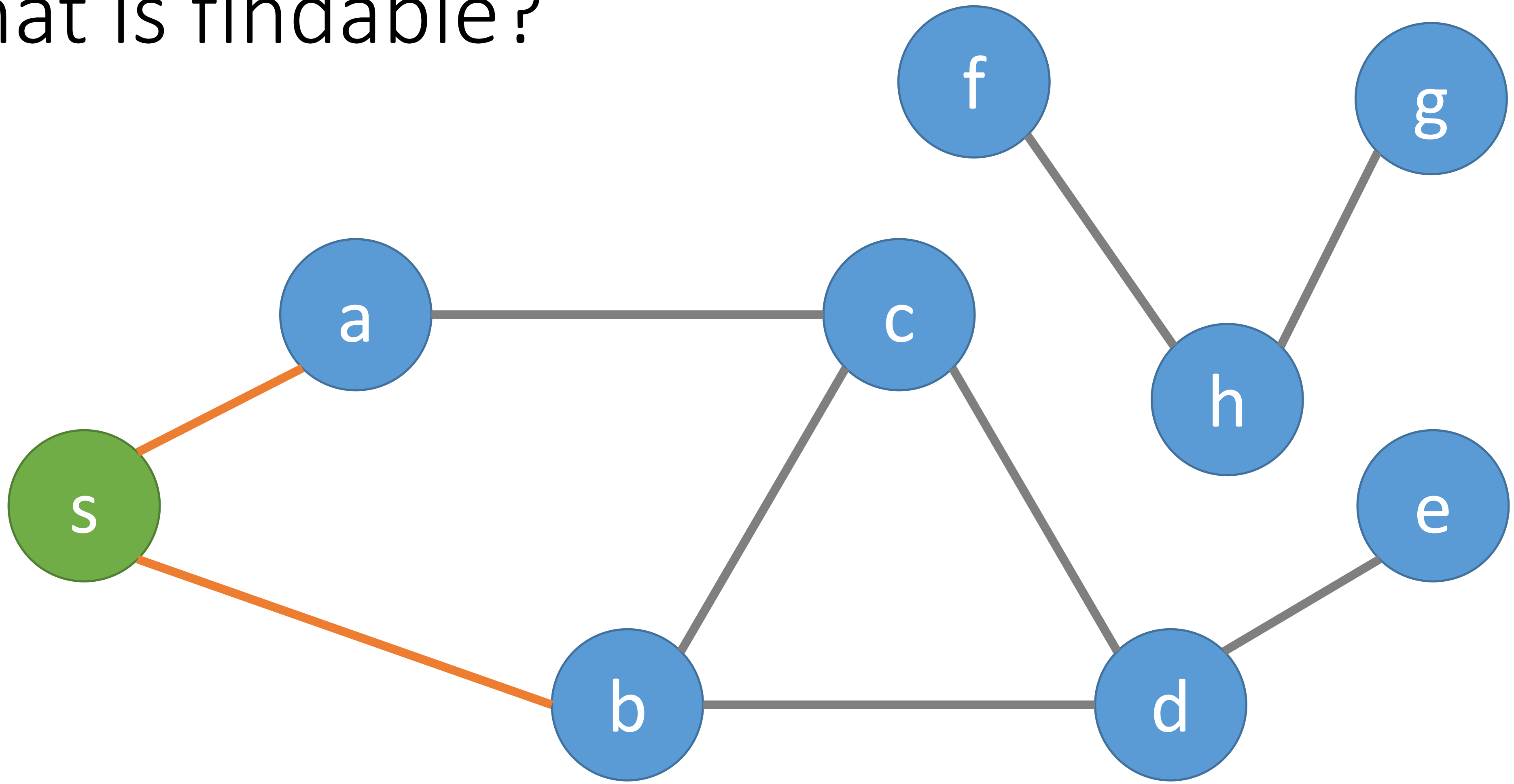
Depends on where you start!



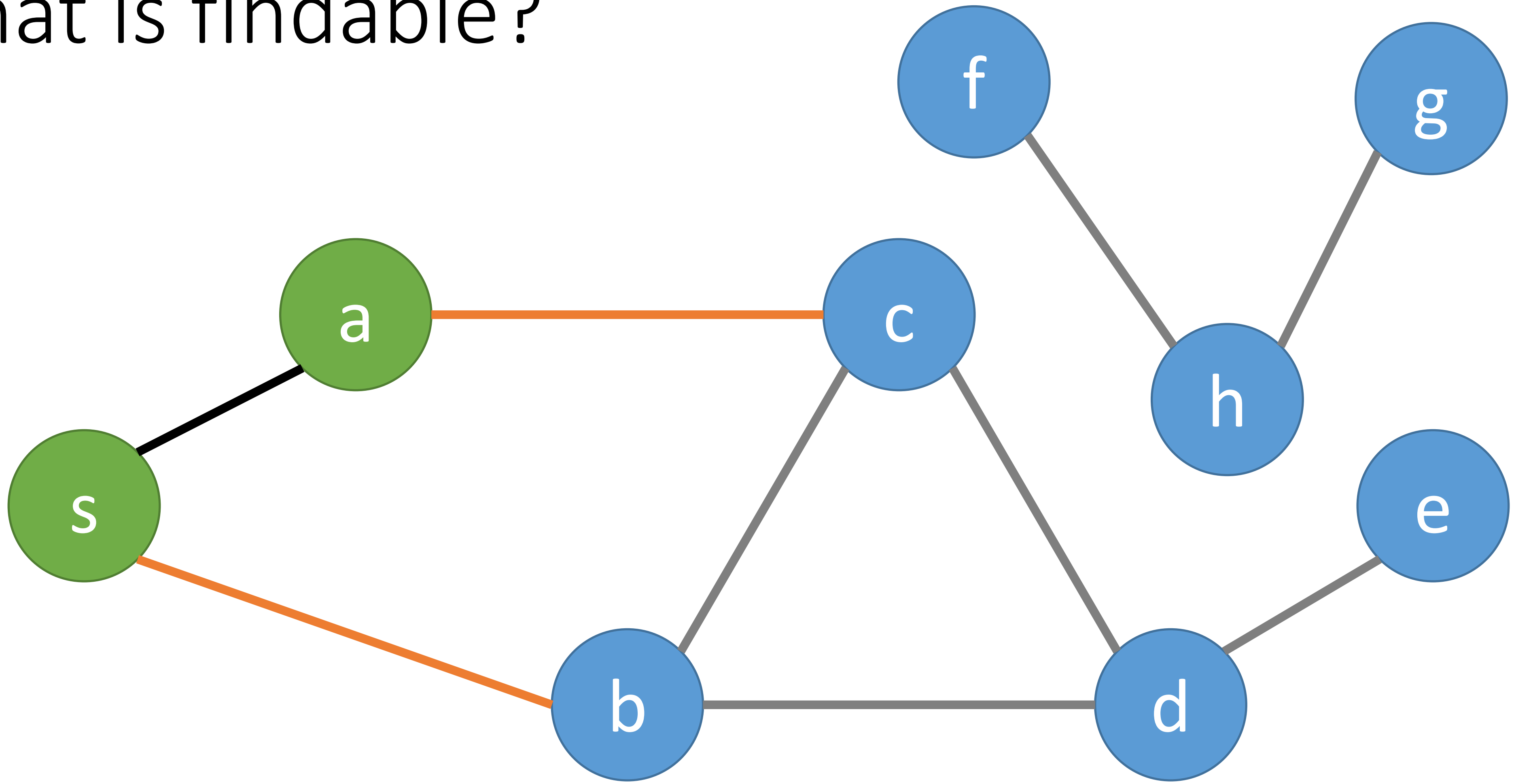
What is findable?



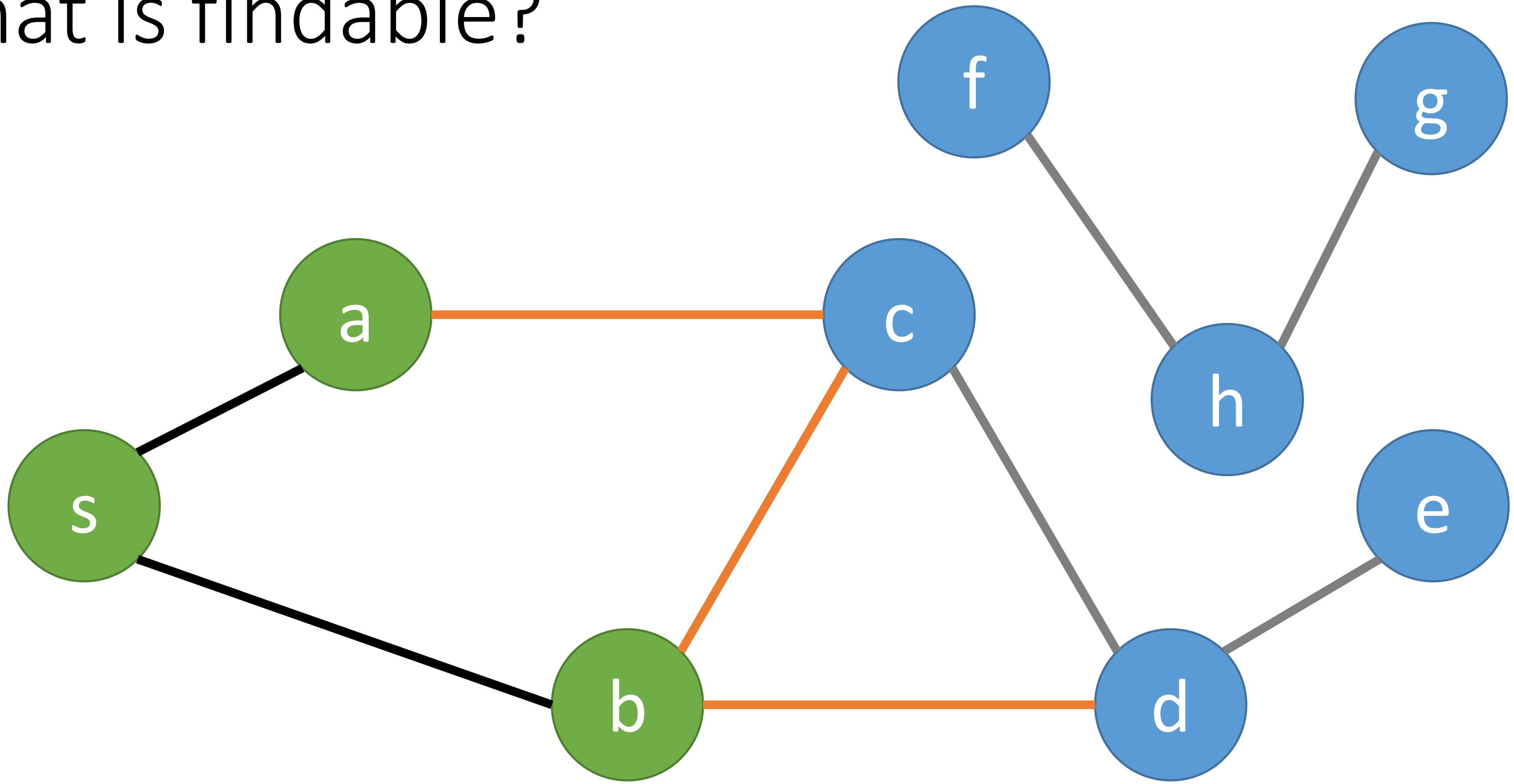
What is findable?



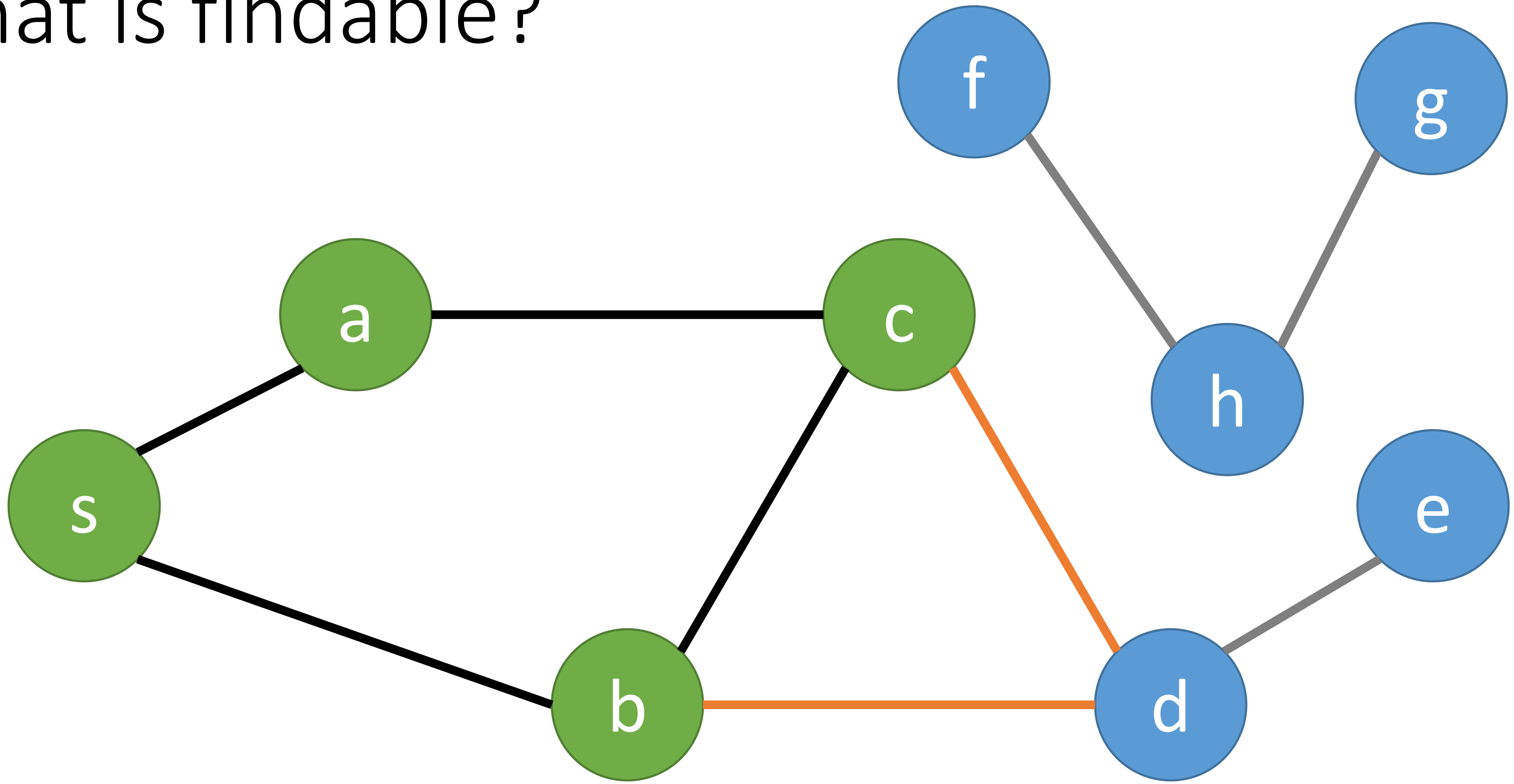
What is findable?



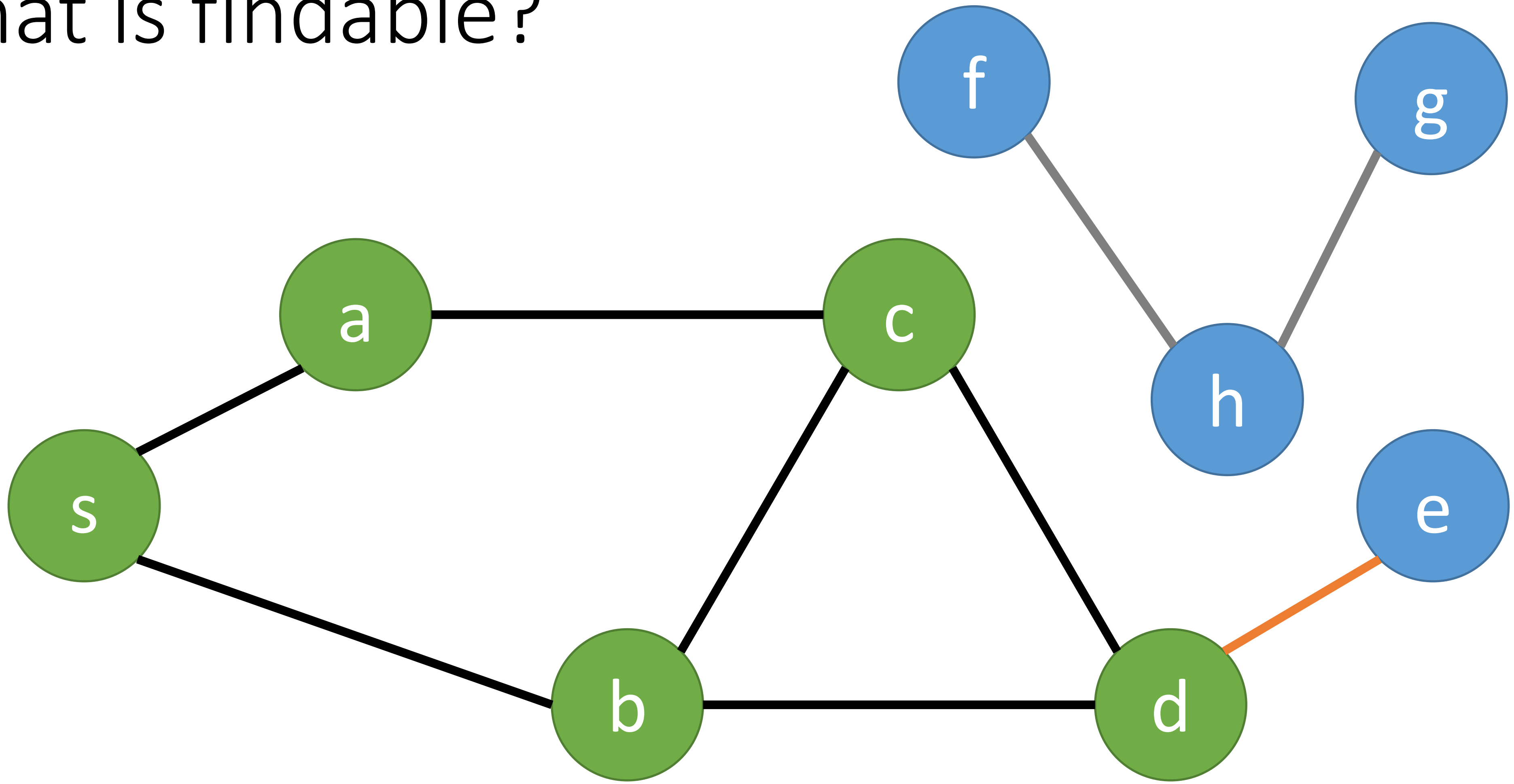
What is findable?



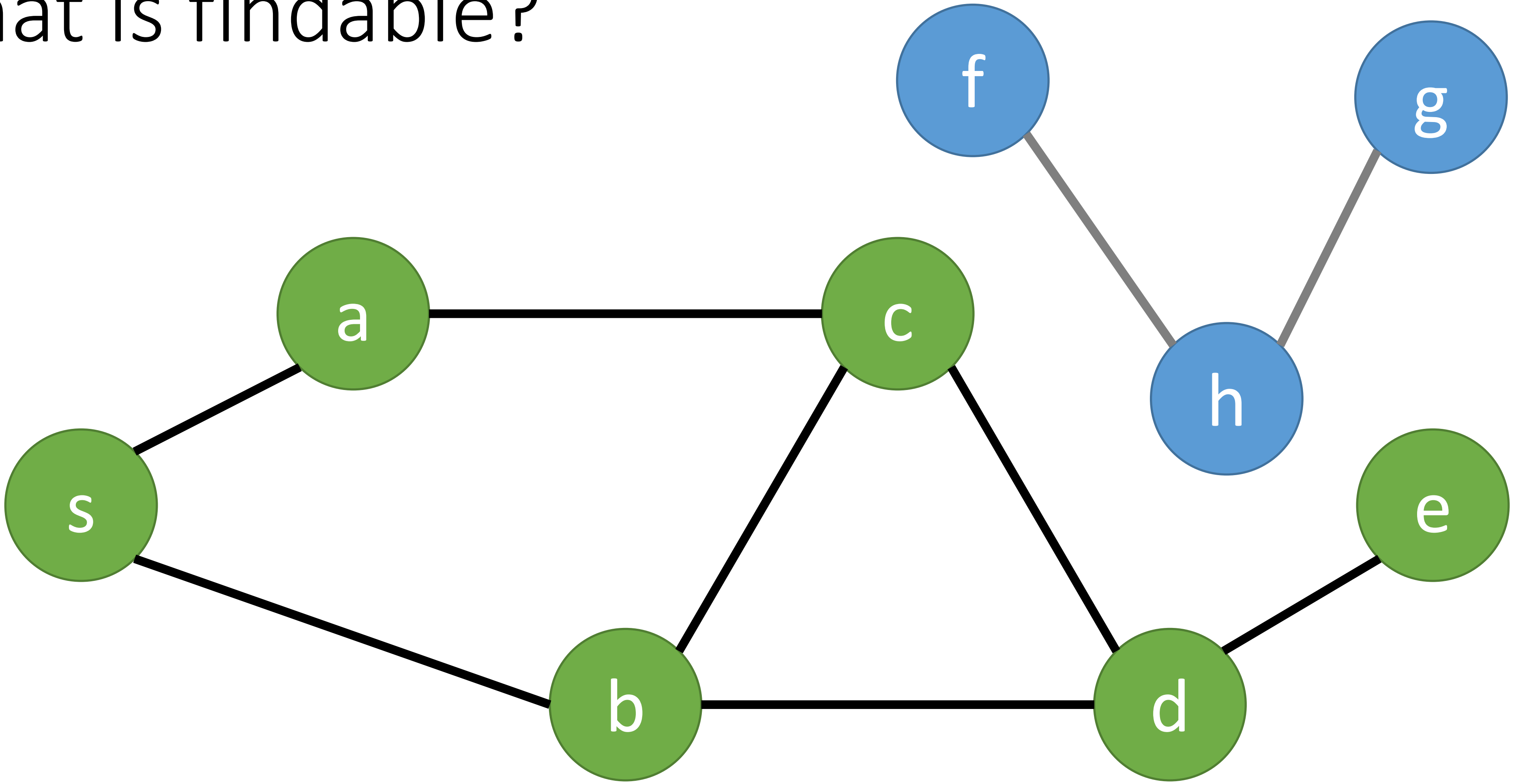
What is findable?



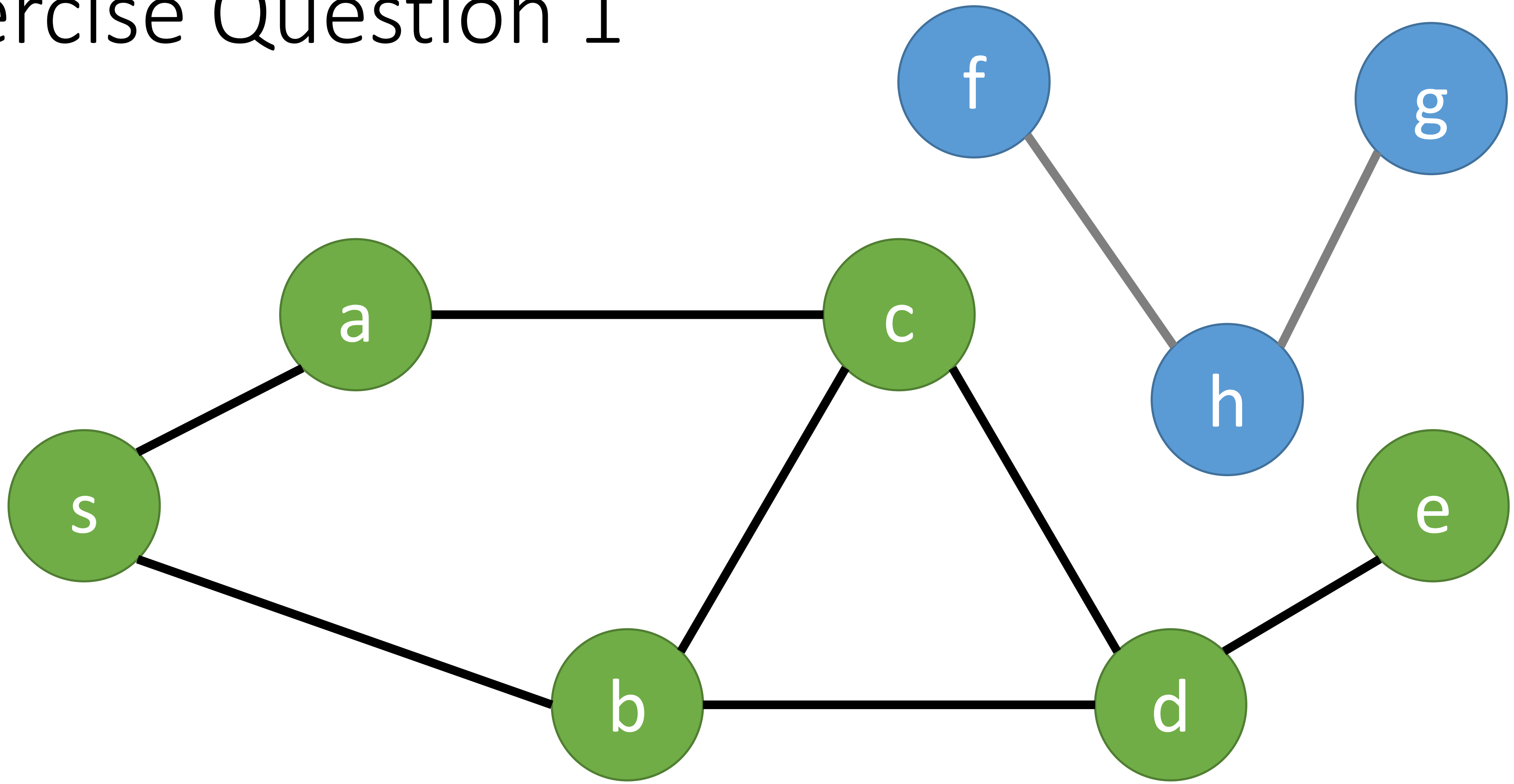
What is findable?



What is findable?



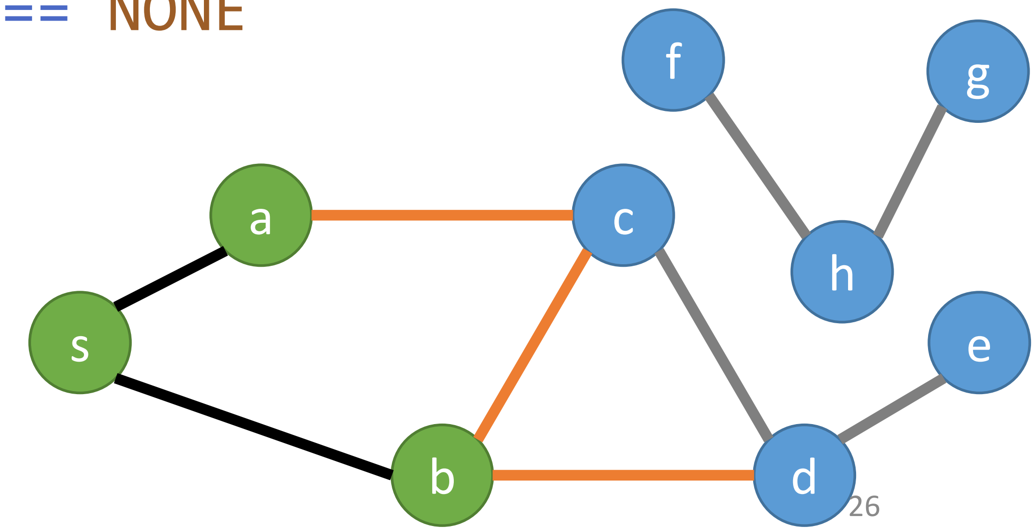
Exercise Question 1



General Algorithm

```
FUNCTION Connectivity(G, start_vertex)
    found = {v: FALSE FOR v IN G.vertices}
    found[start_vertex] = TRUE
    LOOP
        (vFound, vNotFound) = get_valid_edge(G.edges, found)
        IF vFound == NONE || vNotFound == NONE
            BREAK
        ELSE
            found[vNotFound] = TRUE
    RETURN found
```

Find an edge where one vertex has been found and the other vertex has not been found.



General Algorithm Outline

Claim: at the end of this algorithm

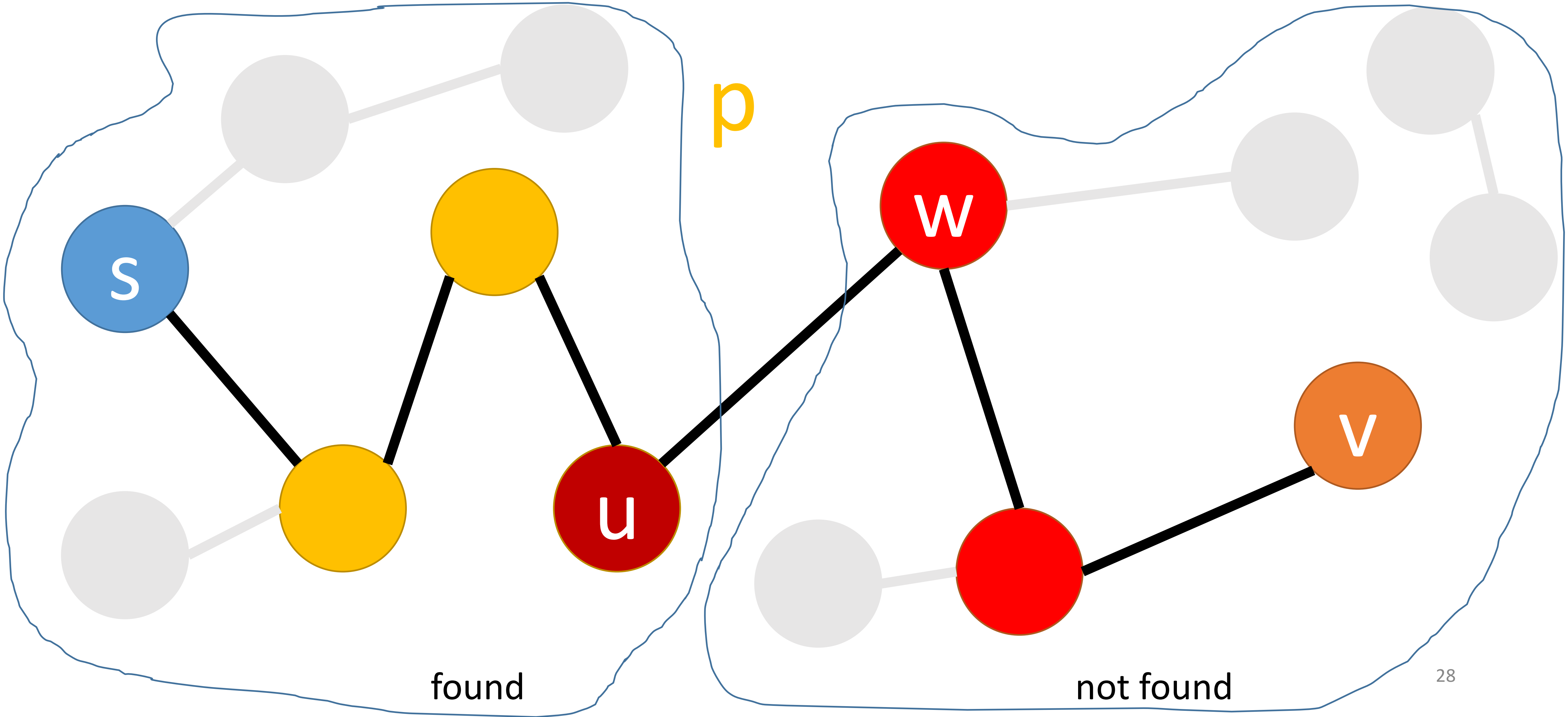
- if v is found
- Then there exists a path from s to v

Proof by contradiction

- Suppose the graph G has a path p from the vertex s to the vertexs v
- Also suppose that upon completion of the algorithm v was not found
- Thus, we have an edge (u, w) such that u is found, and w is not found
- This is **contradictory** to the termination condition of the algorithm

Contradiction

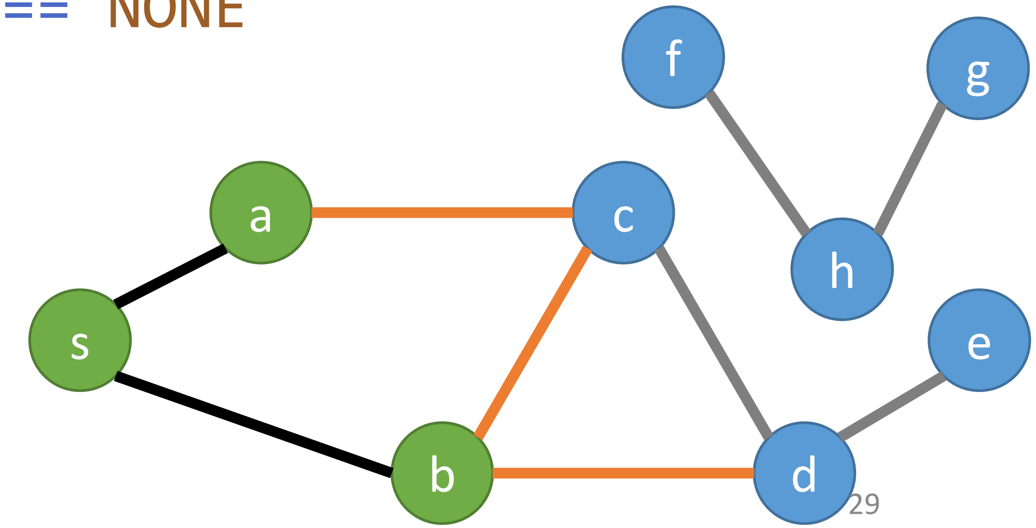
Suppose G has a path p from s to v
Also suppose that upon completion of the algorithm v was not found
Thus we have an edge (u, w) such that u is found and w is not found
This is **contradictory** to the termination condition of the algorithm



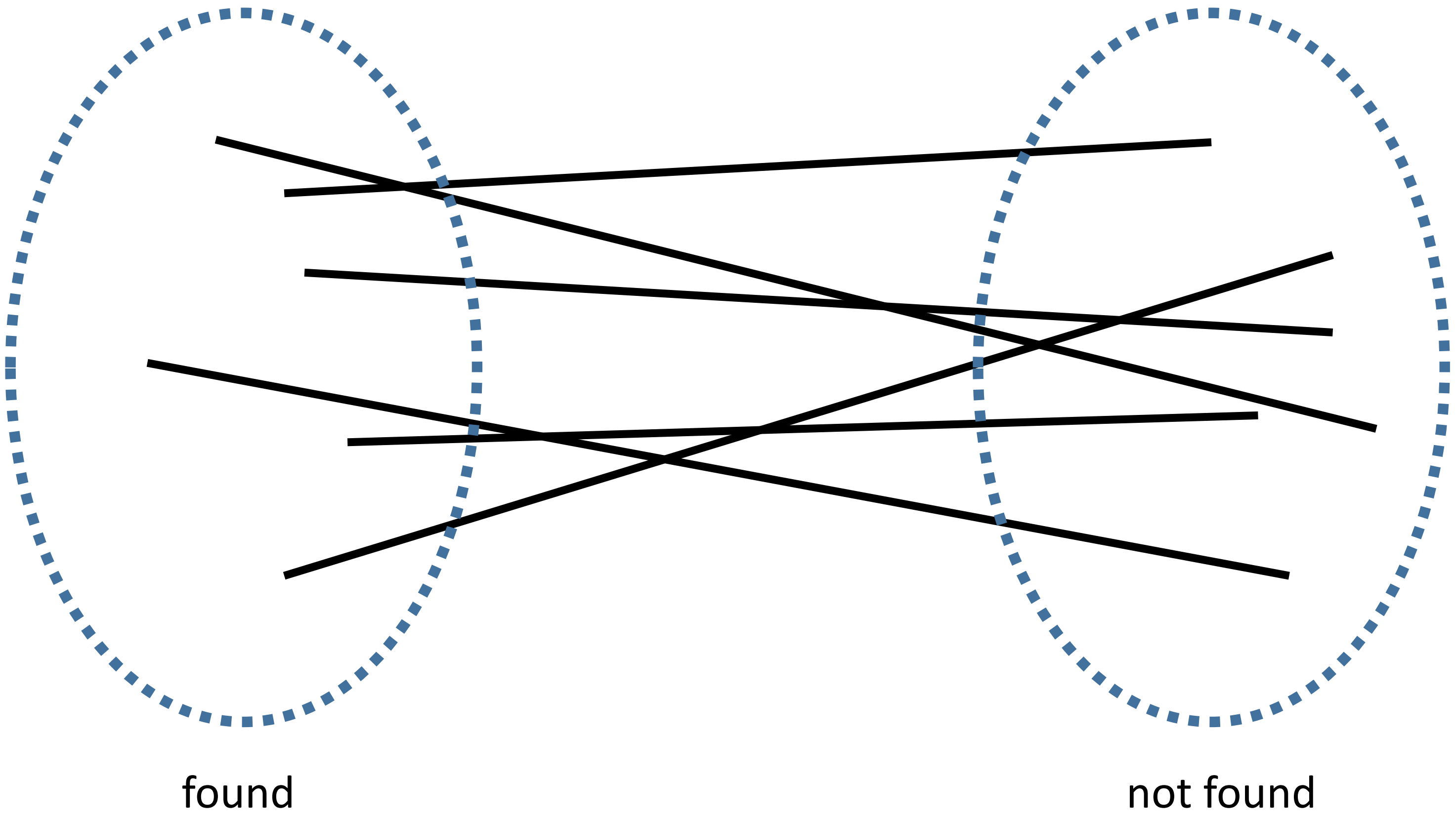
General Algorithm

```
FUNCTION Connectivity(G, start_vertex)
    found = {v: FALSE FOR v IN G.vertices}
    found[start_vertex] = TRUE
    LOOP
        (vFound, vNotFound) = get_valid_edge(G.edges, found)
        IF vFound == NONE || vNotFound == NONE
            BREAK
        ELSE
            found[vNotFound] = TRUE
    RETURN found
```

Find an edge where one vertex has been found and the other vertex has not been found.



How do we choose the <u>next</u> edge?



Two common (and well studied) options

Breadth-First Search

- Explore the graph in layers
- *"Cautious"* exploration
- Use a FIFO data structure (can you think of an example?)

Depth-First Search

- Explore recursively
- A more *"aggressive"* exploration (we backtrack if necessary)
- Use a LIFO data structure (or recursion)