Quicksort Implementation

https://cs.pomona.edu/classes/cs140/

Outline

Topics and Learning Objectives

- Learn how quicksort works
- Learn how to partition an array

Exercise

- Partitioning

Extra Resources

- https://me.dt.in.th/page/Quicksort/
- https://www.youtube.com/watch?v=ywWBy6J5gz8
- CLRS Chapter 7

Quicksort

- A practical and simple algorithm
- The running time = O(n lg n)
- Superior to other O(n lg n) in some respects
- The hidden constants are small (hidden by Big-O)
- Our first stochastic algorithm

Quicksort

Input : an array of n elements in any order

Output : a reordering of the input array such that the elements are in non-decreasing order

Key idea of Quicksort: **partition** the array around a pivot element

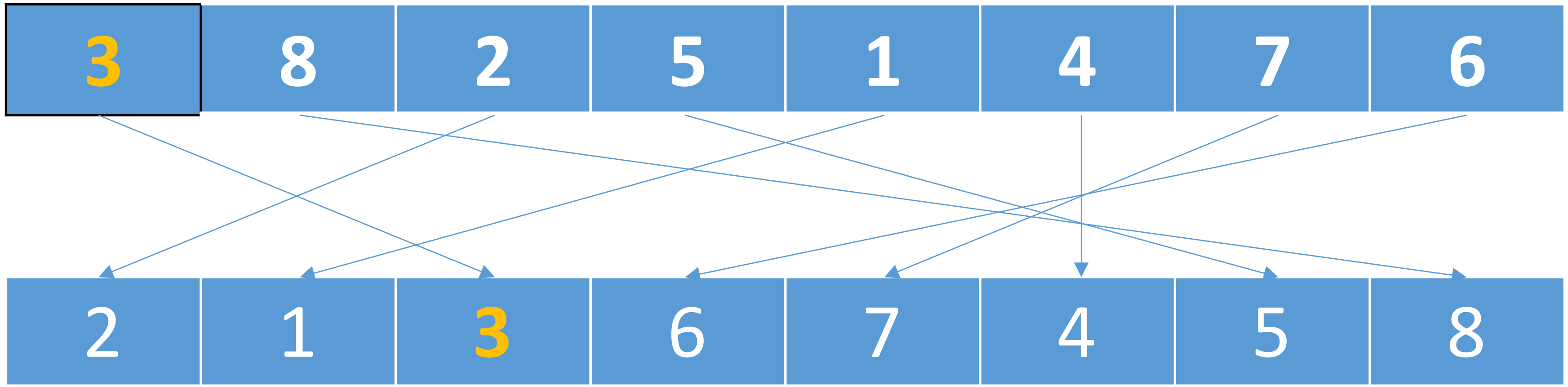
Key concept of Quicksort

- Pick an element and call it the pivot
- Partition (rearrange) the elements so that:
  - Everything to the left of the pivot is less than the pivot
  - Everything to the right of the pivot is greater than the pivot
  - Let’s ignore ties for now

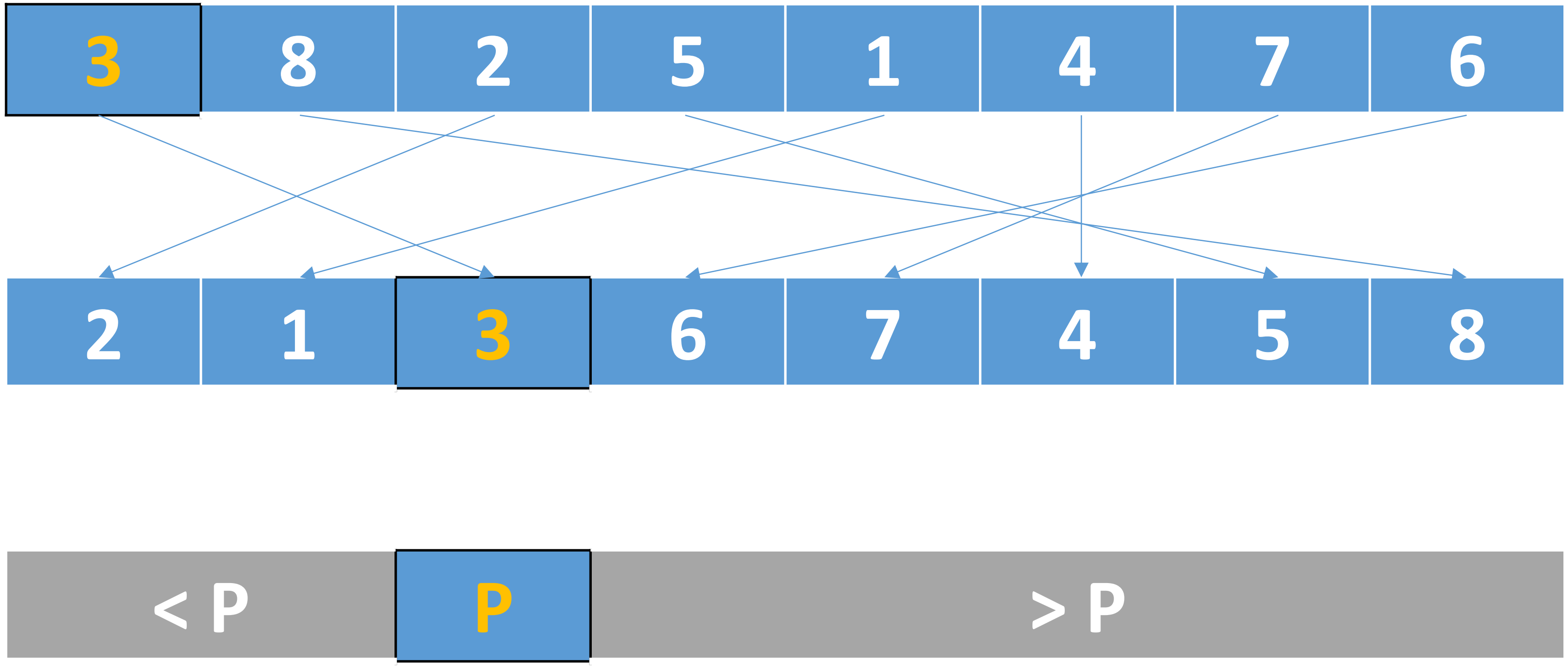
- This is a partial sorting into “buckets”
- What can you tell me about the pivot?
- **Pivot is now in the correct spot (we’ve made progress!)**

What would be the running time of calling partition on every element?

Partitioning



Partitioning



3	8	2	5	1	4	7	6

2	1	3	6	7	4	5	8

< P

P

> P

Pivot around “hello”

[“hello”, “are”, “you”, “how”, “today”, “doing”, “class”]

Quicksort (NOT IN-PLACE PARTITIONING)

```
FUNCTION BadQuicksort(array)
    IF array.length ≤ 1
        RETURN array

    pivot_index = ChoosePivot(array.length)
    left_array, right_array = Partition(array, pivot_index)

    left_sorted = BadQuicksort(left_array)
    right_sorted = BadQuicksort(right_sorted)

    RETURN left_sorted ++ array[pivot_index] ++ right_sorted
```

What is the recurrence equation for Quicksort?

Partitioning the Easy Way

- How would you partition? (how did we perform a merge?)
- Copy all elements to a new array

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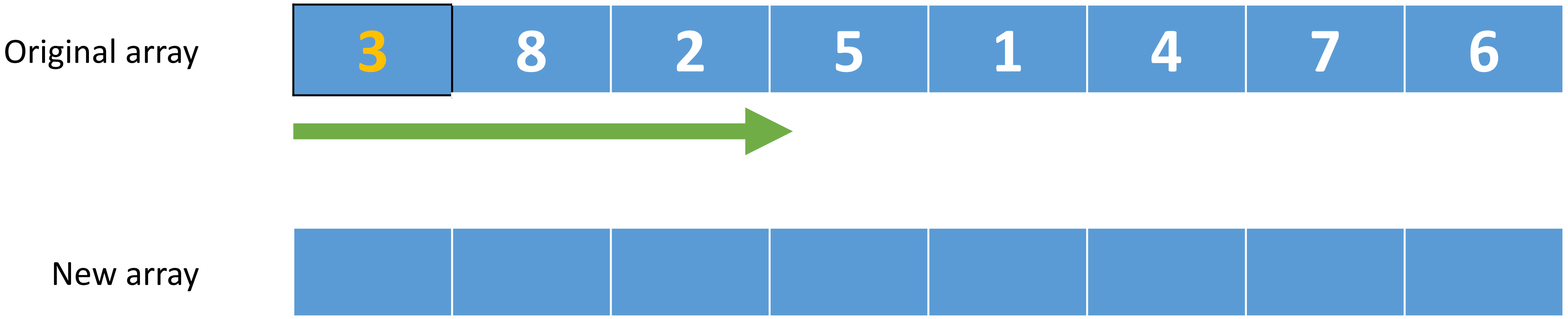

Original array	3	8	2	5	1	4	7	6
New array								

Partitioning the Easy Way

- How would you partition? (how did we perform a merge?)
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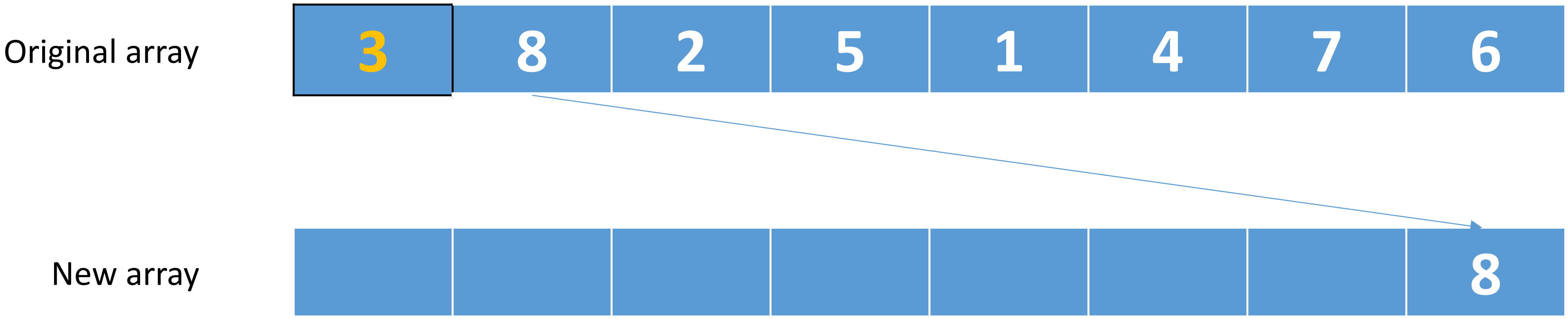
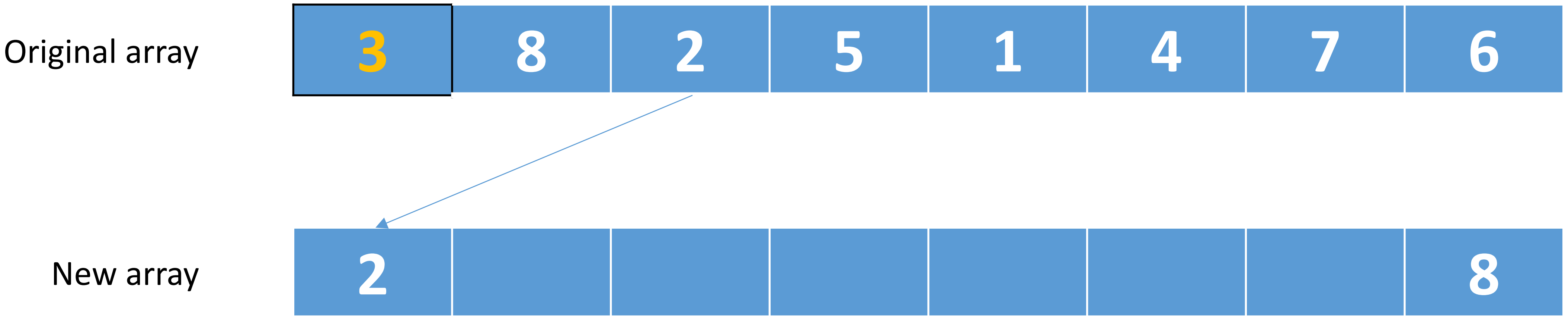


Original array	3	8	2	5	1	4	7	6
New array								8

Partitioning the Easy Way

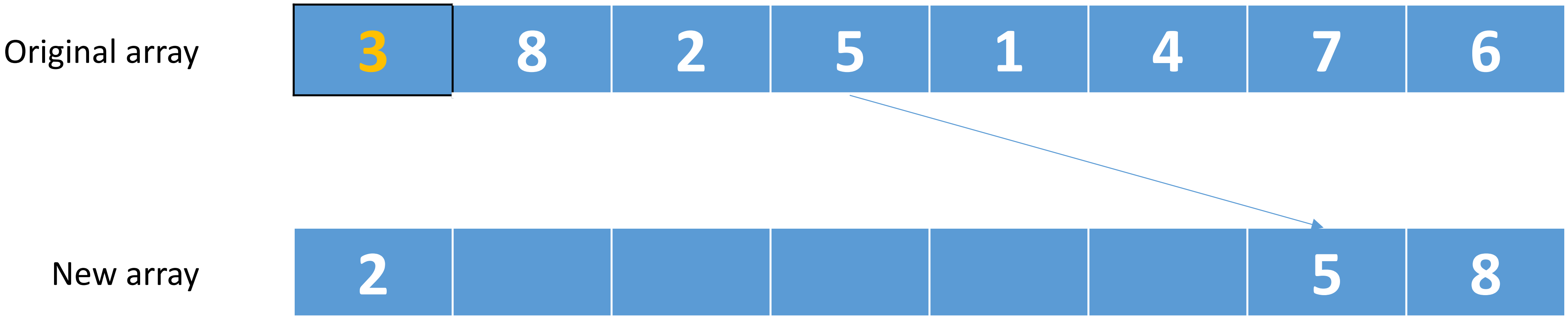
- How would you partition? (how did we perform a merge?)
- Copy all elements to a new array



Original array	3	8	2	5	1	4	7	6
New array	2							8

Partitioning the Easy Way

- How would you partition? (how did we perform a merge?)
- Copy all elements to a new array



Original array	3	8	2	5	1	4	7	6
New array	2						5	8

Partitioning the Easy Way

- How would you partition? (how did we perform a merge?)
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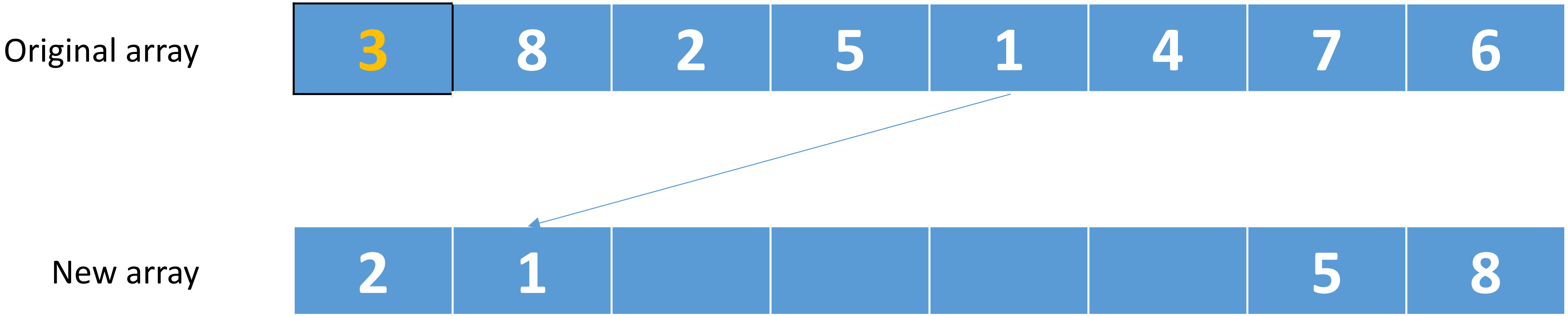
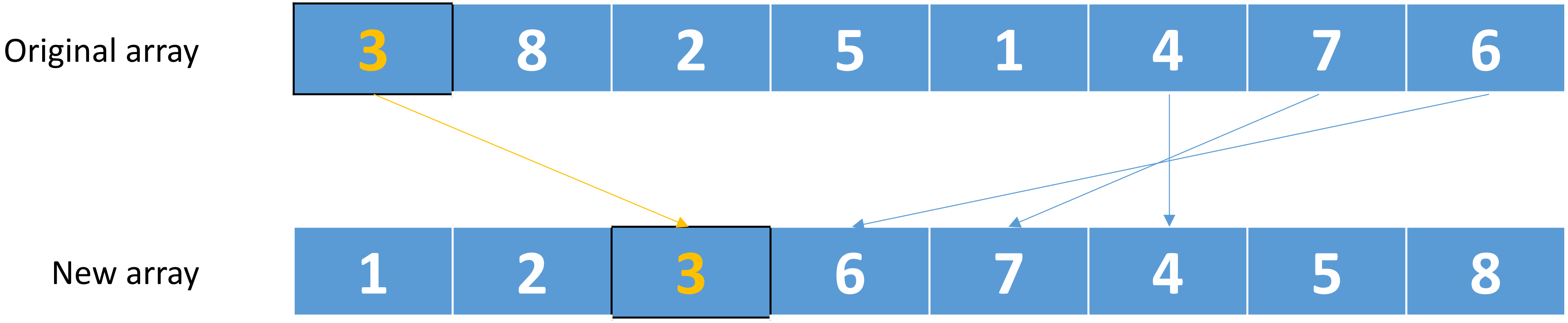


Original array	3	8	2	5	1	4	7	6
New array	2	1					5	8

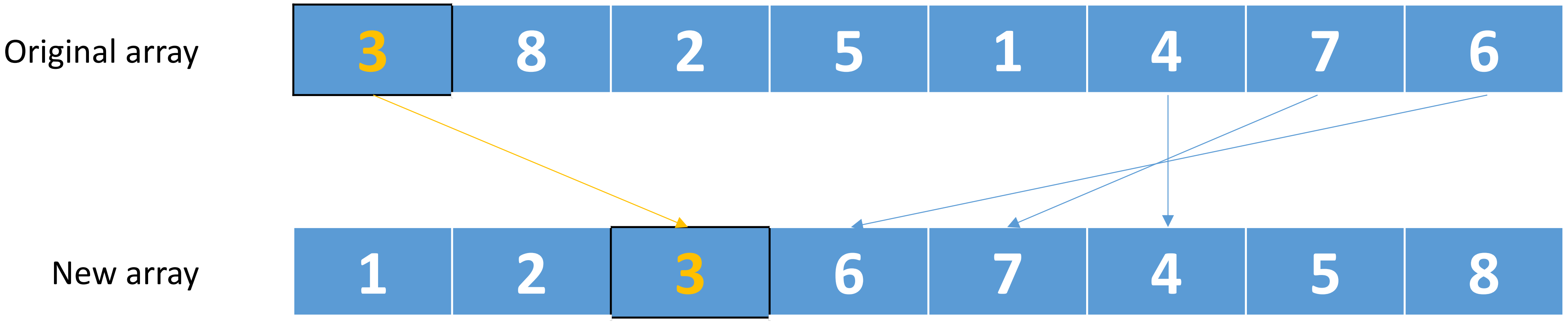
Partitioning the Easy Way

- How would you partition? (how did we perform a merge?)
- Copy all elements to a new array



Partitioning the Easy Way

- How would you partition? (how did we perform a merge?)
- Copy all elements to a new array



Original array 3 8 2 5 1 4 7 6

New array 1 2 3 6 7 4 5 8

- This would be like merge sort.
- Lots of memory allocations (one for each node in the recursion tree).

Partitioning the Easy Way

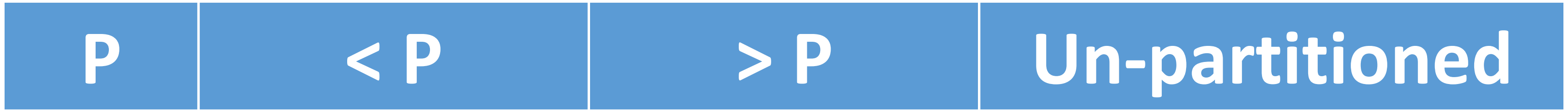
- Nothing inherently wrong with this approach **in theory**
- But can we do the same thing without the extra memory?

- Note: implementing **merge sort** “in-place” is possible
- You can do so with an iterative (stack based) approach

Partitioning In-Place

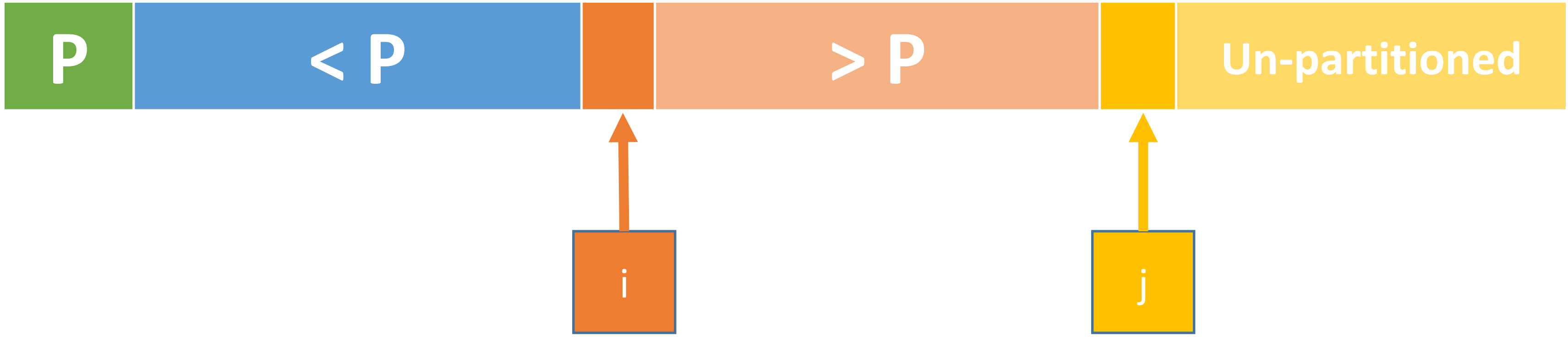
- For now, assume that the pivot is in the first spot of a subarray
- (we can swap the pivot with the first spot if needed)

- Idea: gradually build up a subarray that is correctly partitioned by scanning through the array



P	< P	> P	Un-partitioned

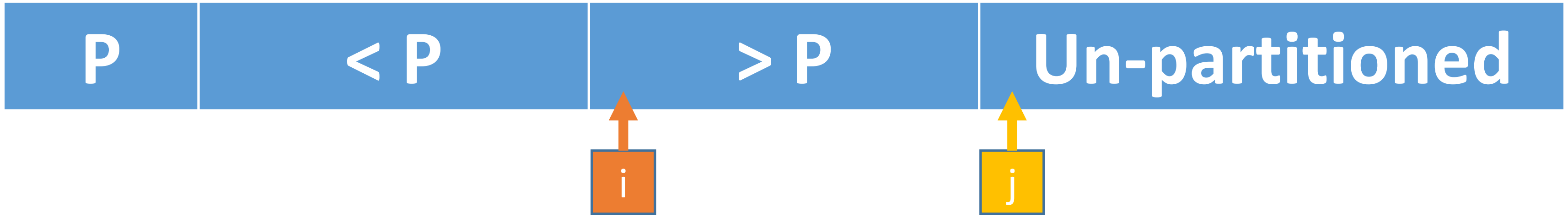
Partitioning In-Place



Index one to the right of the “smaller-than“ partition

Index one to the right of the “larger-than” partition

3	8	2	5	1	4	7	6



P

< P

> P

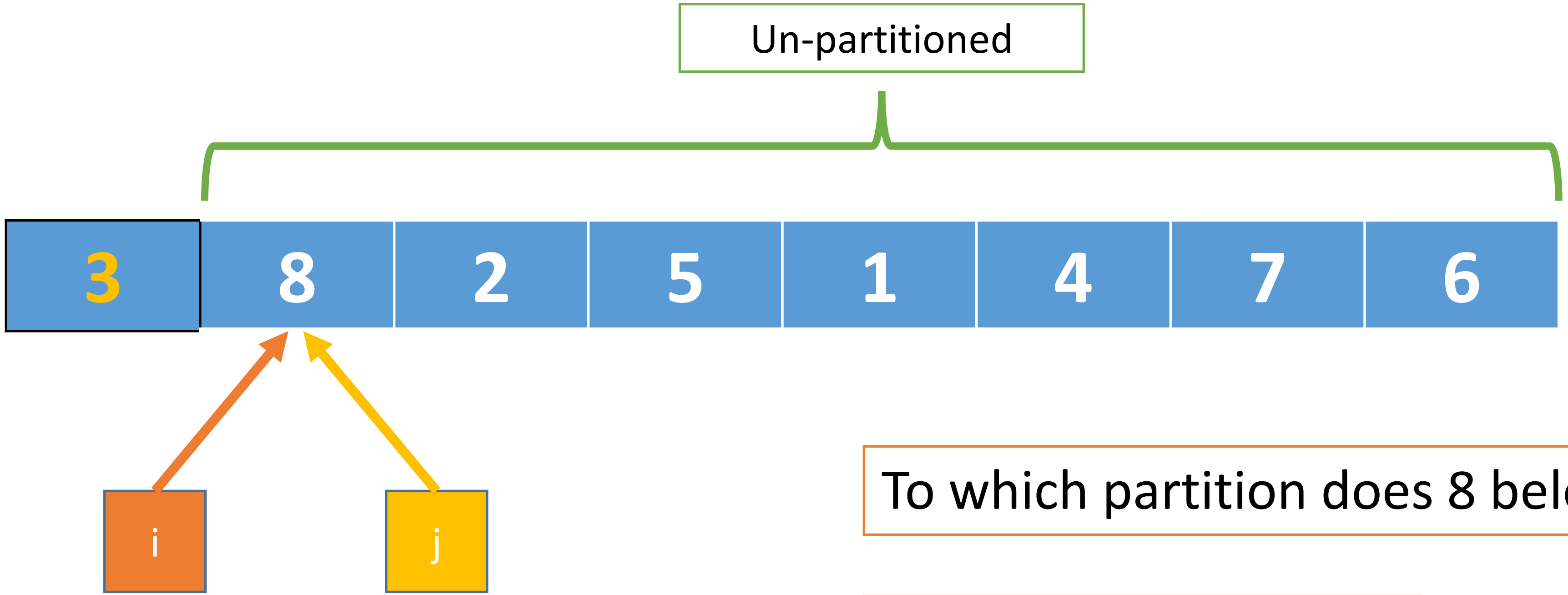
Un-partitioned

i

j

Index one to the right of the “smaller-than“ partition

Index one to the right of the “larger-than” partition



Un-partitioned

3	8	2	5	1	4	7	6

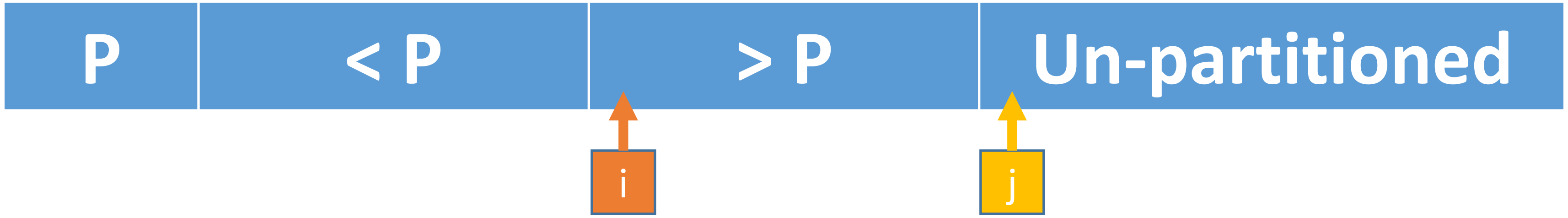
i

j

To which partition does 8 belong?

How do I put it there?

How should we initialize i and j?



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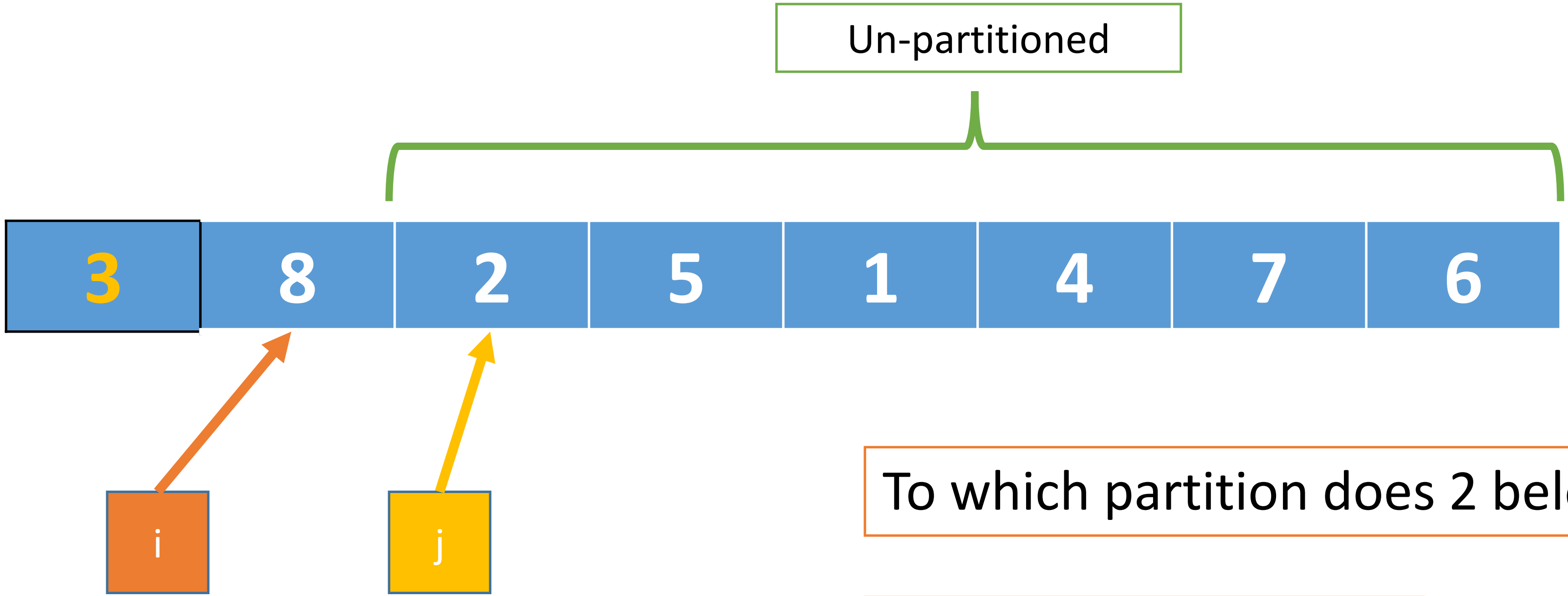
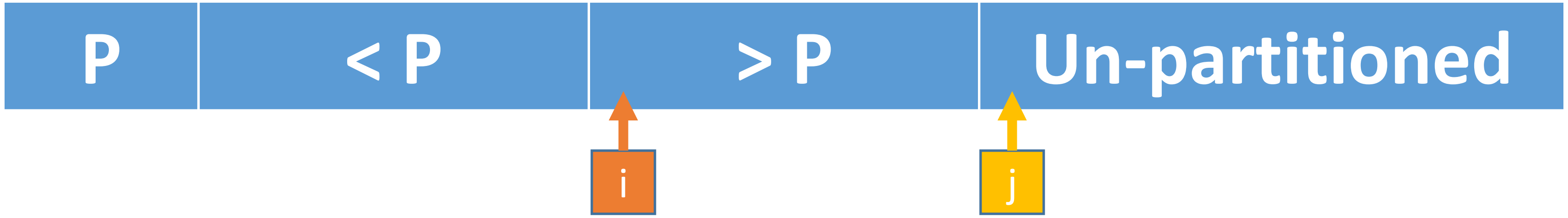


To which partition does 2 belong?

How do I put it there?



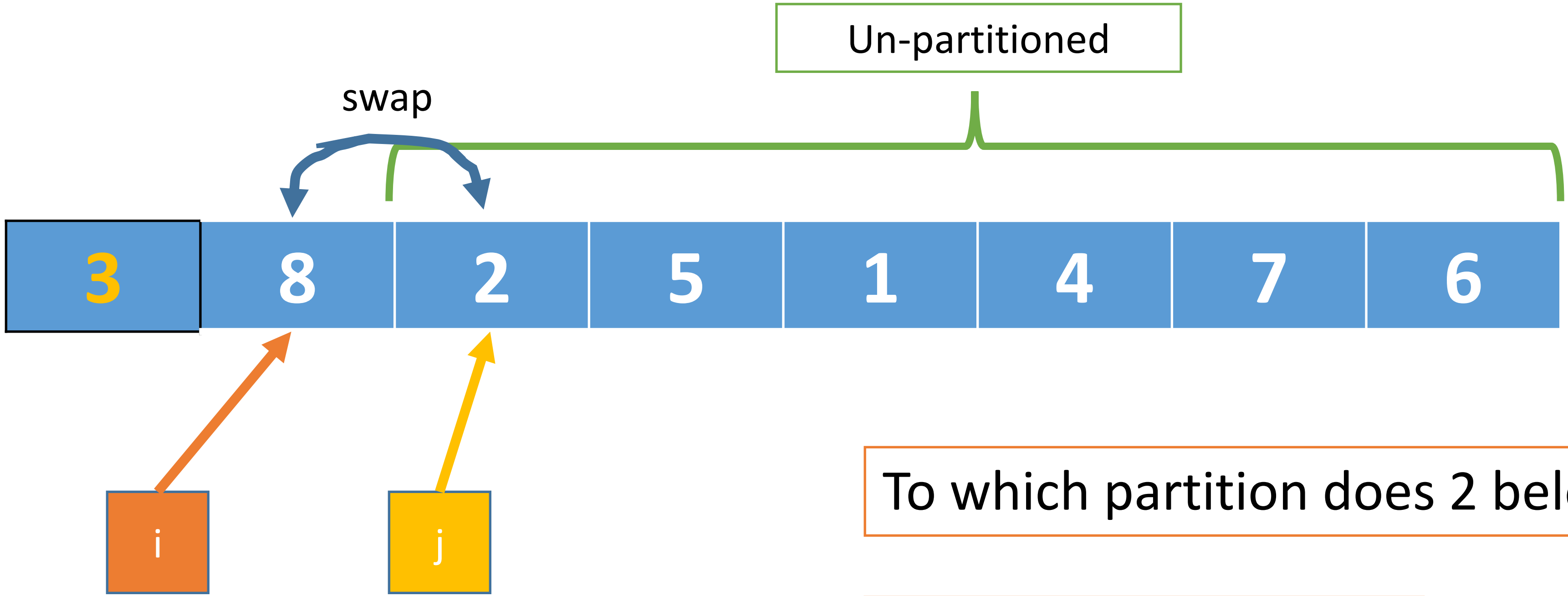
P	< P	> P	Un-partitioned

i

j

Index one to the right of the “smaller-than“ partition

Index one to the right of the “larger-than” partition



Un-partitioned

swap

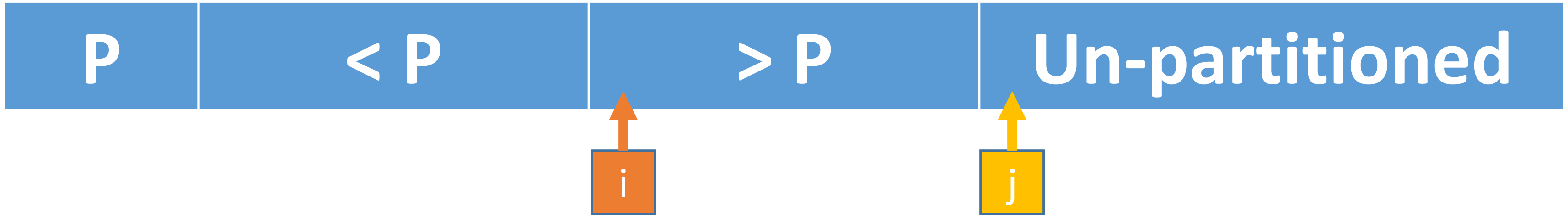
3	8	2	5	1	4	7	6

i

j

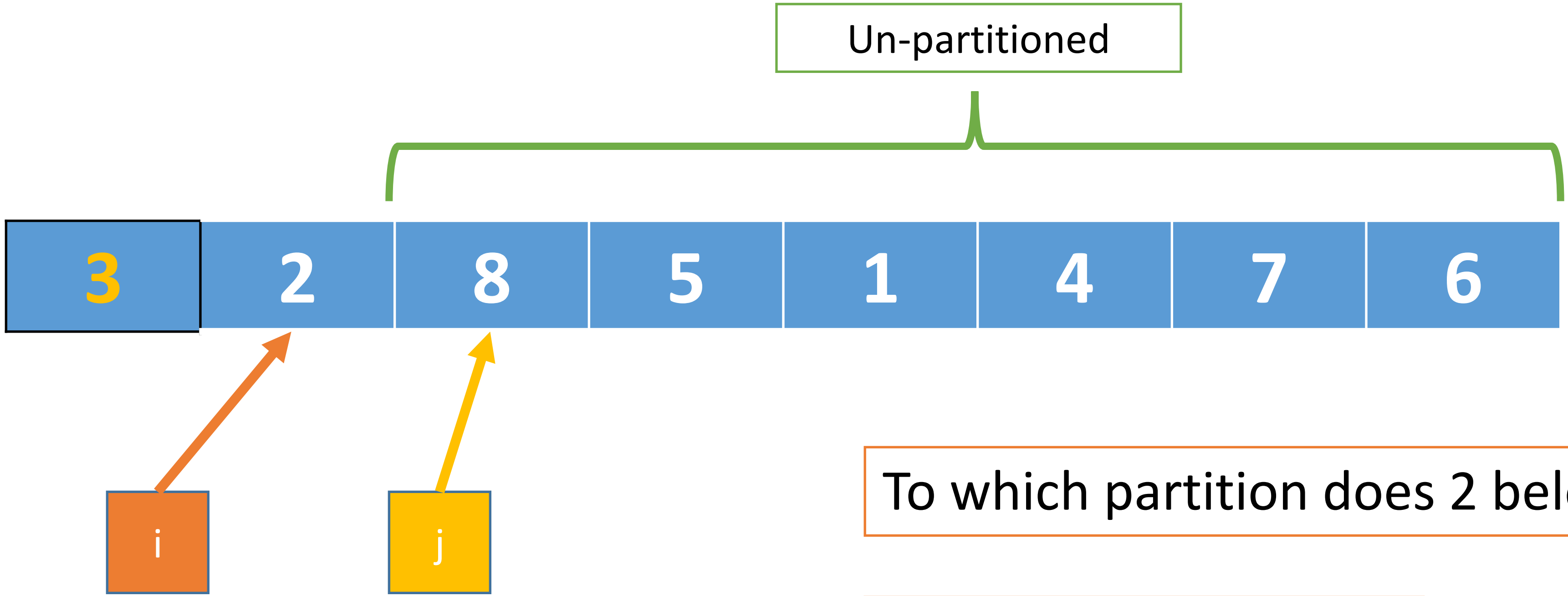
To which partition does 2 belong?

How do I put it there?



Index one to the right of the "smaller-than" partition

Index one to the right of the "larger-than" partition

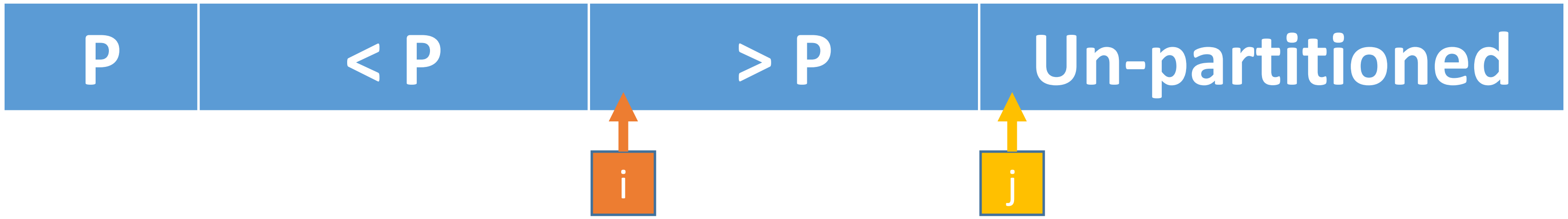


3	2	8	5	1	4	7	6

To which partition does 2 belong?

How do I put it there?

Now what?



P

< P

> P

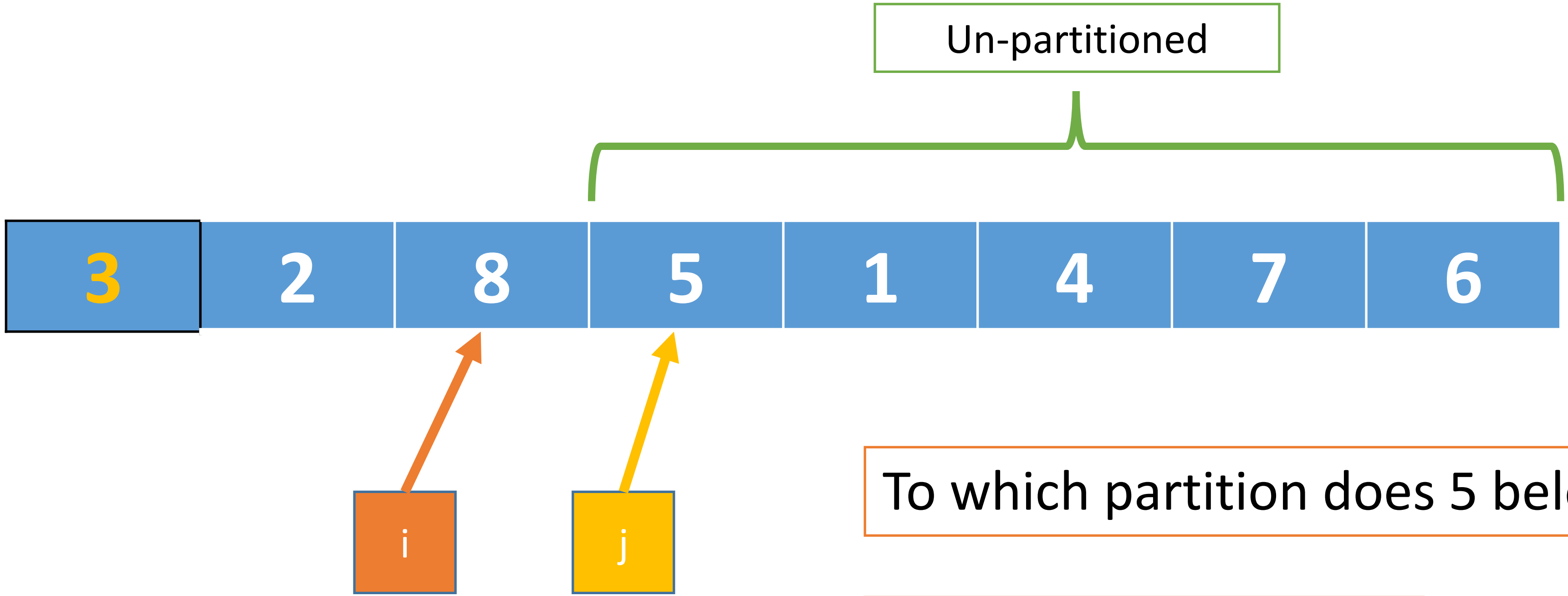
Un-partitioned

i

j

Index one to the right of the "smaller-than" partition

Index one to the right of the "larger-than" partition



Un-partitioned

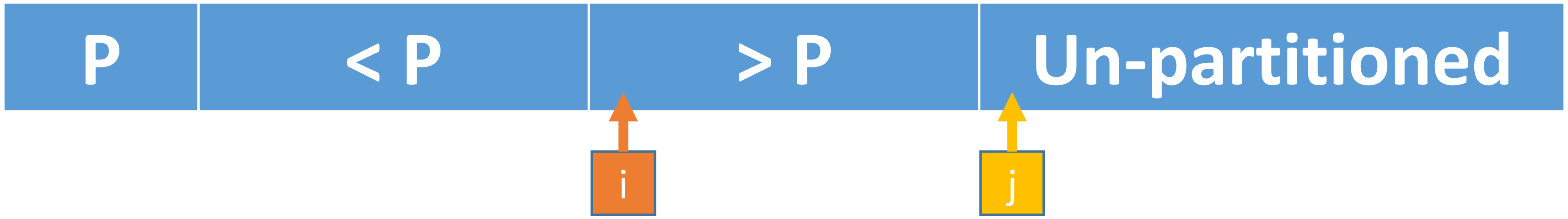
3	2	8	5	1	4	7	6

i

j

To which partition does 5 belong?

How do I put it there?



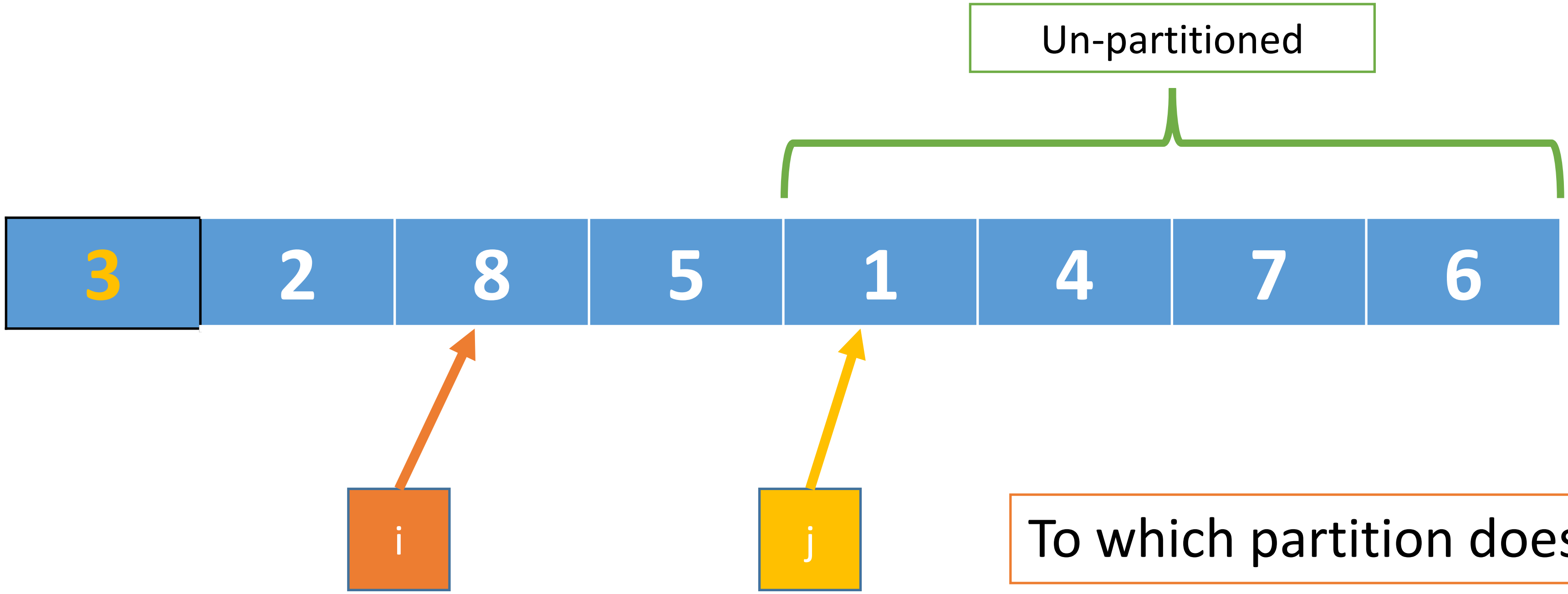
P	< P	> P	Un-partitioned

i

j

Index one to the right of the “smaller-than“ partition

Index one to the right of the “larger-than” partition



Un-partitioned

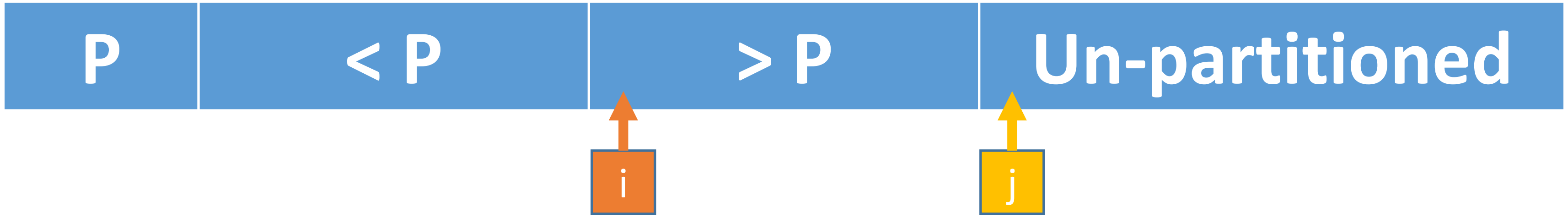
3	2	8	5	1	4	7	6

i

j

To which partition does 1 belong?

How do I put it there?



P

< P

> P

Un-partitioned

i

j

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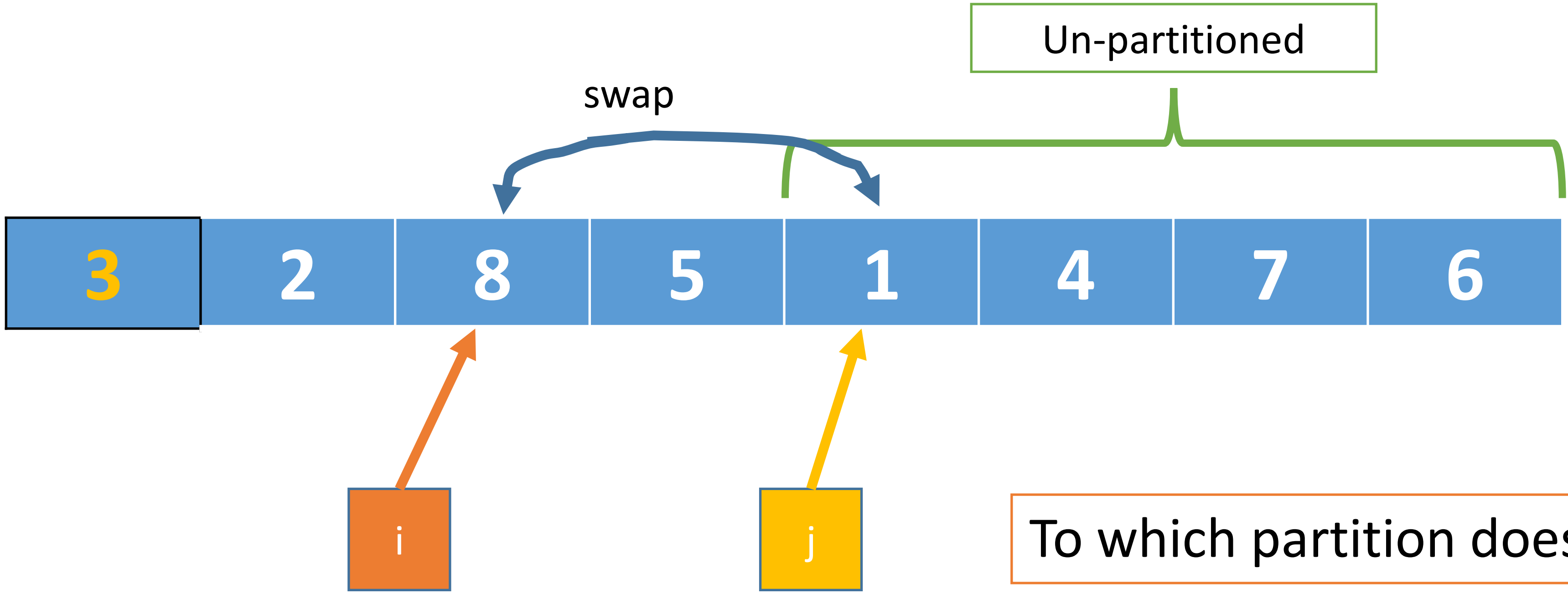
Index one to the right of the “larger-than” partition



Un-partitioned

swap

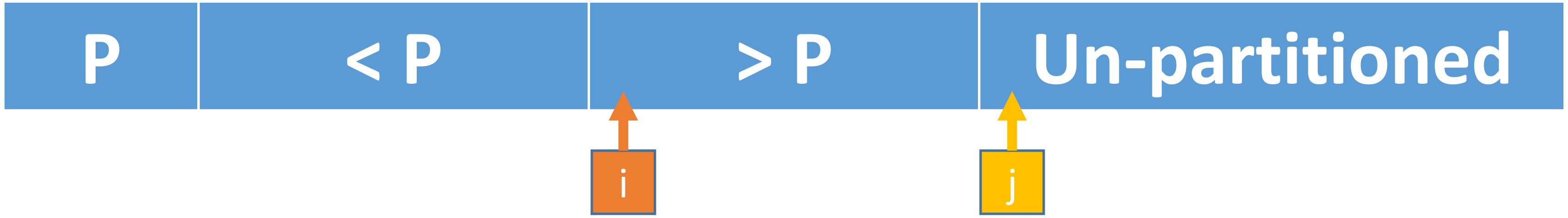
3 2 8 5 1 4 7 6

i

j

To which partition does 1 belong?

How do I put it there?



P	< P	> P	Un-partitioned

i

j

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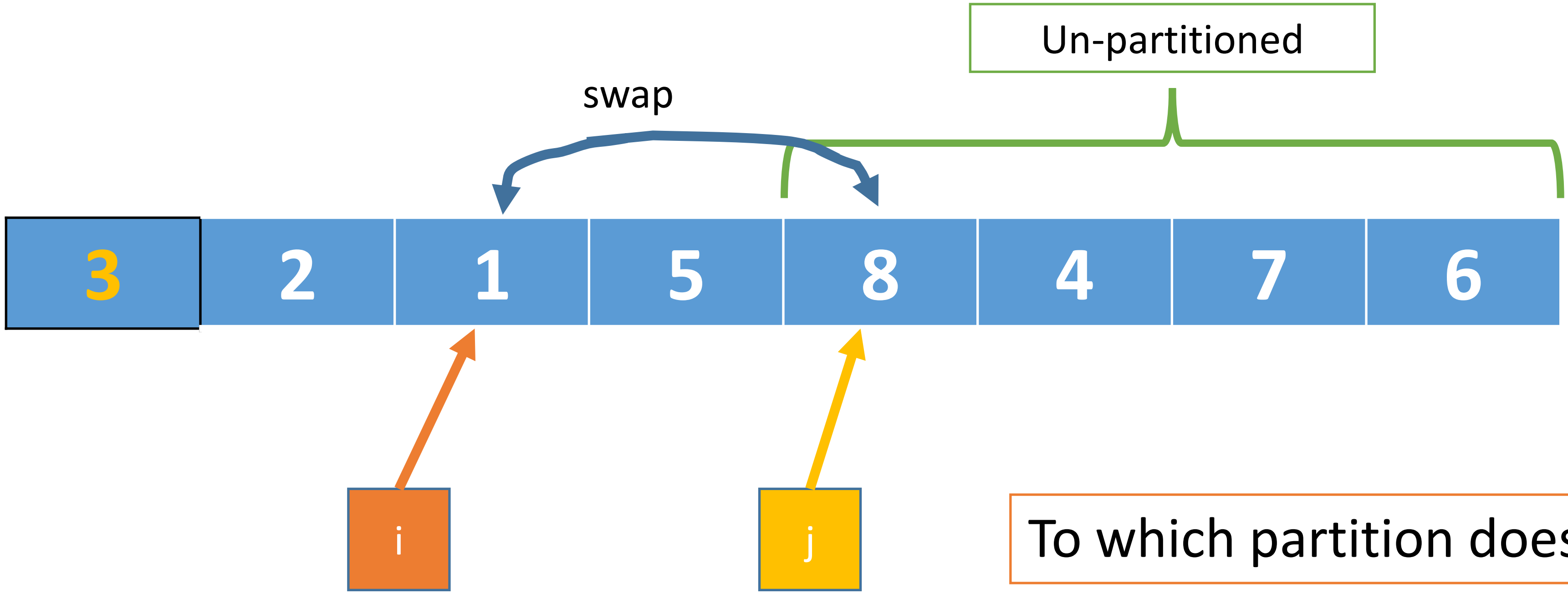
Index one to the right of the "larger-than" partition



Un-partitioned

swap

3	2	1	5	8	4	7	6

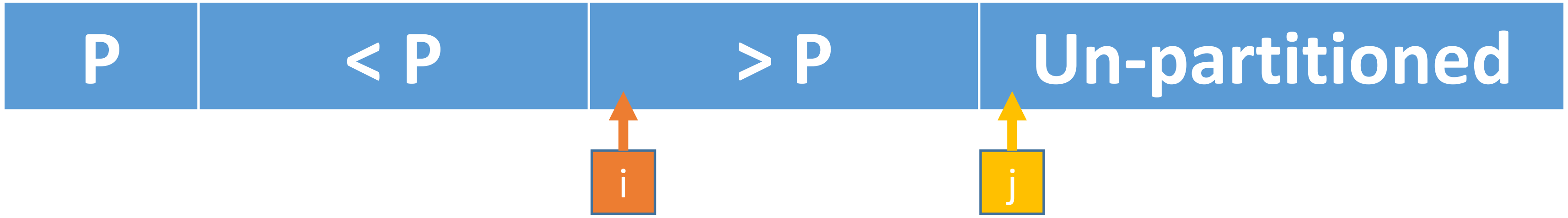
i

j

To which partition does 1 belong?

How do I put it there?

Now what?



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i

j

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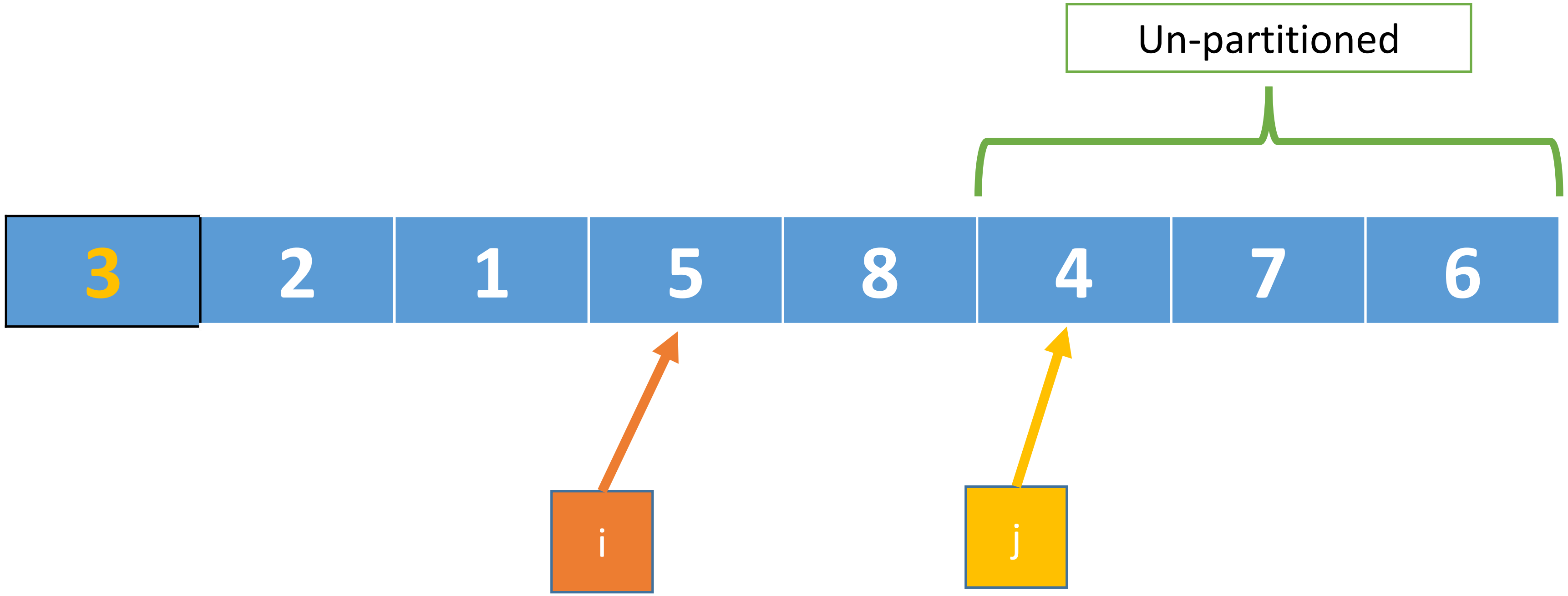
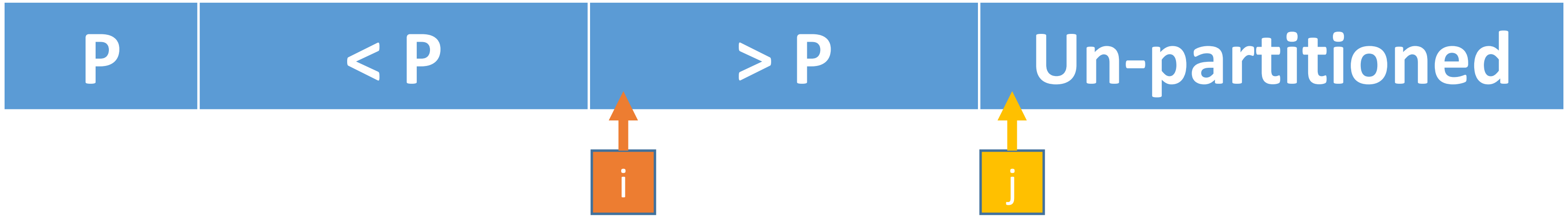


Un-partitioned

3	2	1	5	8	4	7	6

i

j



P

< P

> P

Un-partitioned

i

j

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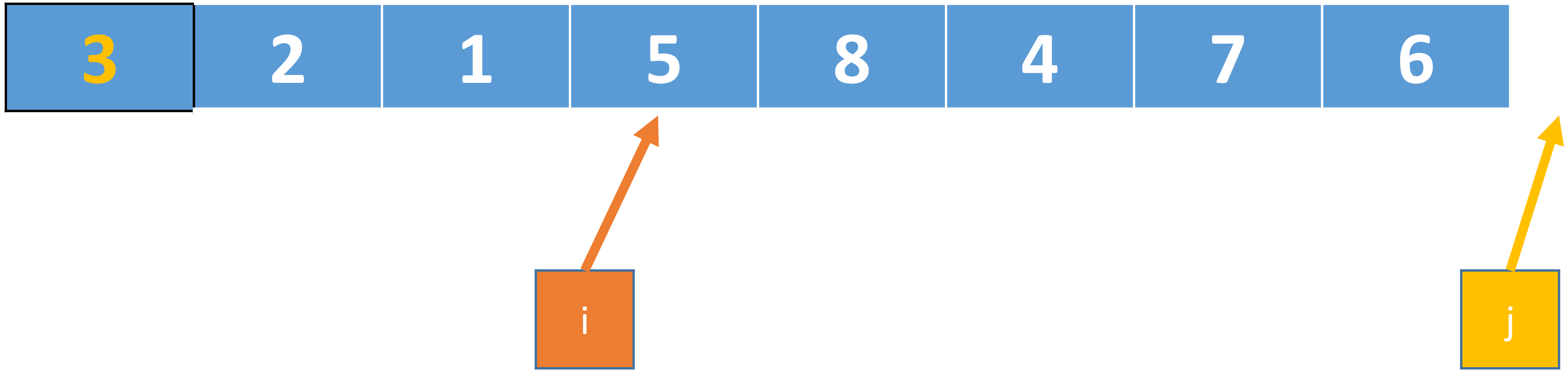
Index one to the right of the “larger-than” partition



3

2

1

5

8

4

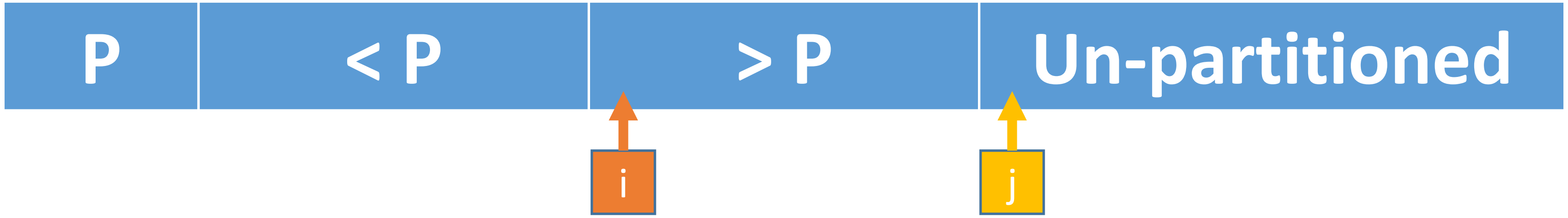
7

6

i

j

Now what?



P | < P | > P | Un-partitioned

i

j

Index one to the right of the "smaller-than" partition

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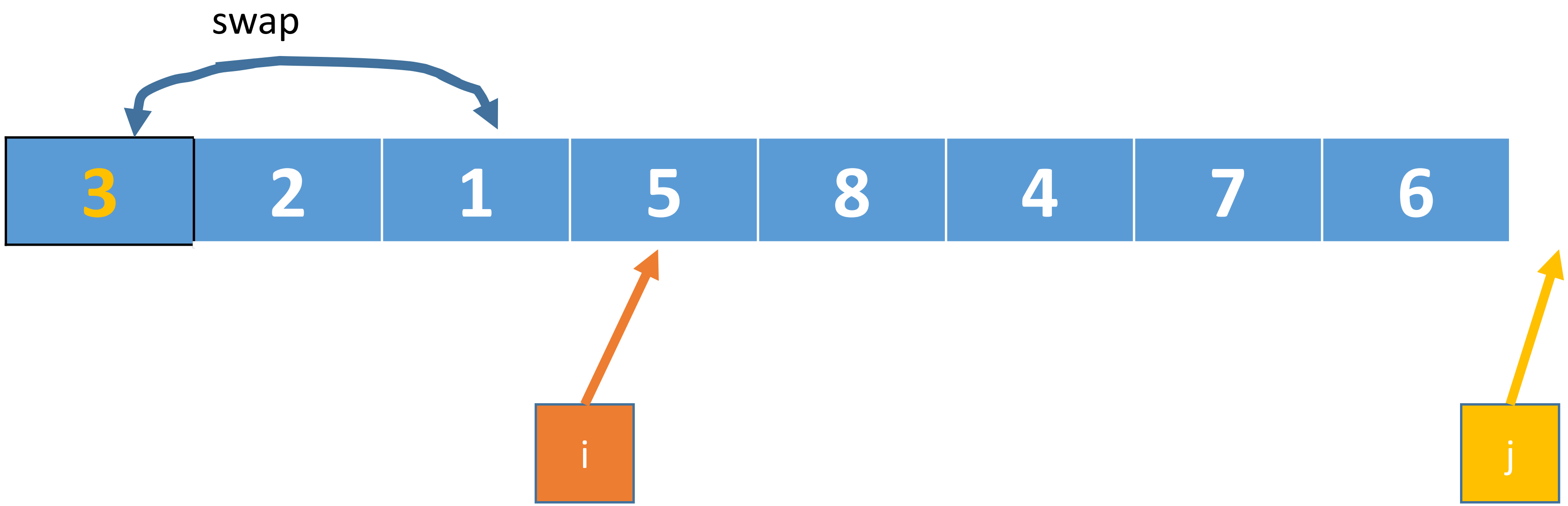

swap

3 | 2 | 1 | 5 | 8 | 4 | 7 | 6

i

j

1	2	3	5	8	4	7	6

What is the asymptotic running time?

```
FUNCTION Partition(array, left_index, right_index)
    # Partition the subarray array[left_index ..< right_index]
    # around the value at left_index

    pivot_value = array[left_index]

    i = left_index + 1
    FOR j IN [left_index + 1 ..< right_index]
        IF array[j] < pivot_value
            swap(array, i, j)
            i = i + 1

    swap(array, left_index, i - 1)
    RETURN i - 1
```

1. O(n), where n is right_index - left_index

2. In-place
   no extra memory

```
FUNCTION QuickSort(array, left_index, right_index)
    IF 
        RETURN

    MovePivotToLeft(left_index, right_index)
    pivot_index = Partition(array, left_index, right_index)

    QuickSort(array, )
    QuickSort(array, )
```

Our Partition function expects the pivot element to be at left_index

How would you call QuickSort?

```
FUNCTION QuickSort(array, left_index, right_index)
    IF left_index ≥ right_index
        RETURN

    MovePivotToLeft(left_index, right_index)
    pivot_index = Partition(array, left_index, right_index)

    QuickSort(array, left_index, pivot_index)
    QuickSort(array, pivot_index + 1, right_index)
```

Our `Partition` function expects the pivot element to be at `left_index`