

In-Class Worksheet

Discrete Math & Functional Programming— CSCI 054— Spring 2024

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Given an implication $p \Rightarrow q$, we can define the following related concepts:

- converse: $q \Rightarrow p$
- inverse: $\neg p \Rightarrow \neg q$
- contrapositive: $\neg q \Rightarrow \neg p$

Which, if any, are logically equivalent to the original implication?

If 2 is an even number, then 3 is an odd number

If x is an even number, then $x + 1$ is an odd number

Define the predicates:

- $\text{rested}(n) = \text{“}n \text{ got at least 8 hours of sleep in the past 24 hours”}$
- $\text{bornMA}(n) = \text{“}n \text{ was born in Massachusetts”}$

Which, if any, of the following propositions is True? Justify your answer.

- $\exists n \text{ in this room} : \text{rested}(n)$
- $\forall n \text{ in this room} : (\text{rested}(n) \Rightarrow \text{bornMA}(n))$
- $\exists n \text{ currently enrolled at Pomona College} : (\text{rested}(n) \wedge \text{bornMA}(n))$
- $\forall \text{ currently enrolled at Pomona College} : (\text{rested}(n) \vee \text{bornMA}(n))$

Is the following a theorem? Explain.

$$[\forall x \in S : P(x)] \vee [\forall x \in S : \neg P(x)]$$