

# Contracts Made Manifest

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# First-order contracts

`assert( $n > 0$ )`

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`sqrt : { $x:\text{Float} \mid x \geq 0$ }  $\mapsto$  Float`

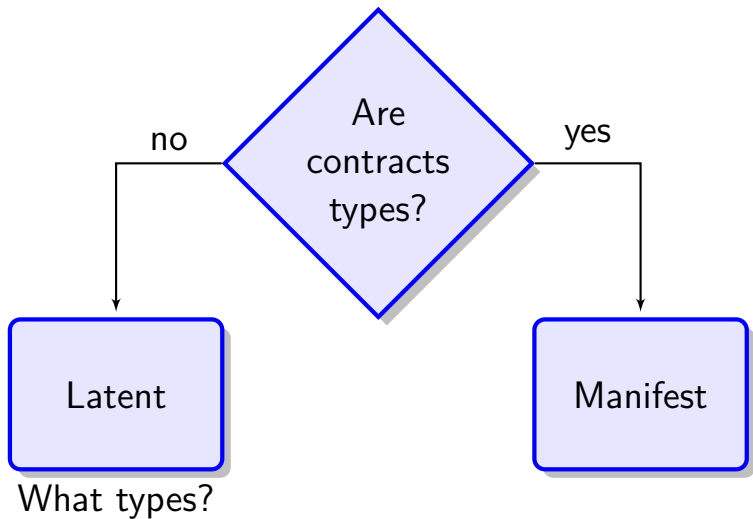
# First-order contracts

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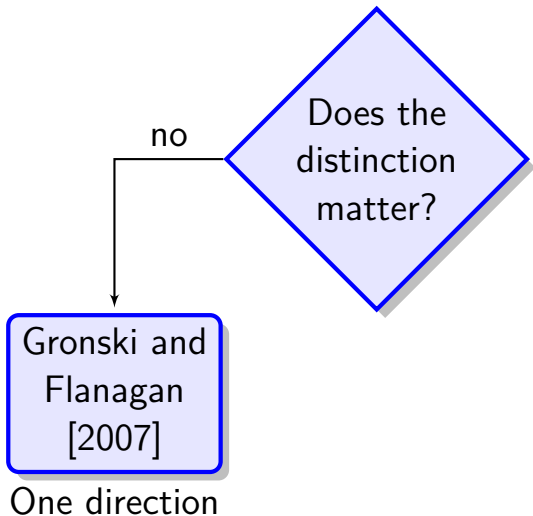
`sqrt : {x:Float |  $x \geq 0$ }  $\mapsto$  Float`

`sqrt : x:{x:Float |  $x \geq 0$ }  $\mapsto$  {y:Float |  $|y^2 - x| < \epsilon$ }`

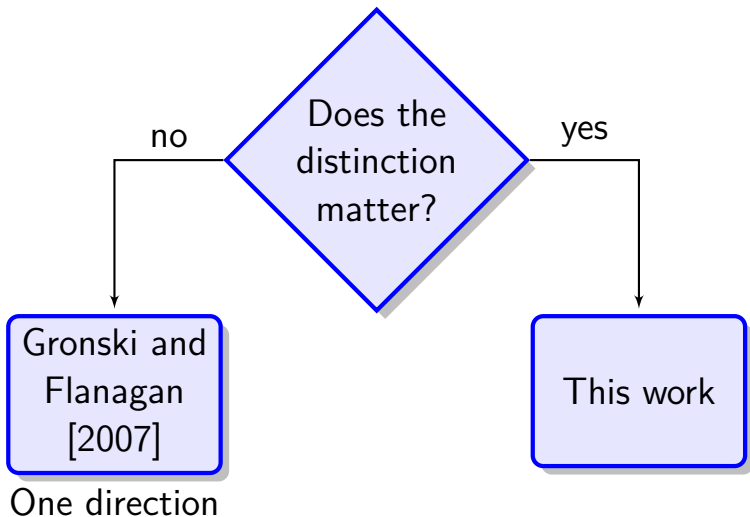
# Contracts for the $\lambda$ -calculus



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# Blame assignment

$$f : \text{Int} \rightarrow (\text{Int} \rightarrow \text{Int}) \rightarrow \text{Int}$$
$$f = \lambda n. \lambda g. (g\ n)$$

If we give  $f$  the contract

$$\text{Int} \mapsto (\{x:\text{Int} \mid x > 0\} \mapsto \{y:\text{Int} \mid y > 0\}) \mapsto \text{Int}$$

How does  $(f\ 0)\ \lambda x. 1$  evaluate?



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How does  $(f\ 0)\ \lambda x. 1$  evaluate?

What about  $(f\ 1)\ \lambda x. 0$ ?

What about  $(f\ 0)\ \lambda x. 0$ ?

# Latent contracts

According to Findler and Felleisen [2002]

$c ::= \{x:B \mid t\}$       base contracts  
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Can't in general *decide* whether a function is, e.g.  $\text{Pos} \mapsto \text{Pos}$

Instead, defer checking to **runtime**

Check that argument, result satisfy contracts

# Higher-order contracts

Let Pos mean  $\{x:\text{Int} \mid x > 0\}$

$$\begin{aligned} \langle \text{Pos} \rangle^{l,l'} 1 &\longrightarrow^* 1 \\ \langle \text{Pos} \rangle^{l,l'} 0 &\longrightarrow^* \uparrow / \end{aligned}$$

$$\begin{aligned} (\langle \text{Pos} \mapsto \text{Pos} \rangle^{l_{\text{fun}}, l_{\text{arg}}} \lambda x:\text{Int}. x) 1 &\longrightarrow^* 1 \\ (\langle \text{Pos} \mapsto \text{Pos} \rangle^{l_{\text{fun}}, l_{\text{arg}}} \lambda x:\text{Int}. x) 0 &\longrightarrow^* \uparrow /_{\text{arg}} \\ (\langle \text{Pos} \mapsto \text{Pos} \rangle^{l_{\text{fun}}, l_{\text{arg}}} \lambda x:\text{Int}. x - 1) 1 &\longrightarrow^* \uparrow /_{\text{fun}} \end{aligned}$$

# Function contracts

$$\begin{array}{l} (\langle \text{Pos} \mapsto \text{Pos} \rangle^{f_{\text{fun}}, l_{\text{arg}}} \lambda x:\text{Int}. x) 0 \quad \longrightarrow \\ \langle \text{Pos} \rangle^{f_{\text{fun}}, l_{\text{arg}}} ((\lambda x:\text{Int}. x) (\langle \text{Pos} \rangle^{l_{\text{arg}}, f_{\text{fun}}} 0)) \quad \longrightarrow^* \\ \langle \text{Pos} \rangle^{f_{\text{fun}}, l_{\text{arg}}} ((\lambda x:\text{Int}. x) \uparrow^{l_{\text{arg}}}) \quad \longrightarrow^* \quad \uparrow^{l_{\text{arg}}} \end{array}$$

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$$\begin{array}{lcl}
 (\langle \text{Pos} \mapsto \text{Pos} \rangle^{f_{\text{fun}}, l_{\text{arg}}} \lambda x:\text{Int}. x) 0 & \longrightarrow & \\
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 \\
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 \langle \text{Pos} \rangle^{l_{\text{fun}}, l_{\text{arg}}} 0 & \longrightarrow^* & \uparrow^{l_{\text{fun}}}
 \end{array}$$

## Function contract obligations

$$(\langle c_1 \mapsto c_2 \rangle^{l, l'} v_1) v_2 \longrightarrow \langle c_2 \rangle^{l, l'} (v_1 (\langle c_1 \rangle^{l', l} v_2))$$



# Dependency

## Nondependent

$$(\langle c_1 \mapsto c_2 \rangle^{l,l'} v_1) v_2 \longrightarrow \langle c_2 \rangle^{l,l'} (v_1 (\langle c_1 \rangle^{l',l} v_2))$$

## Dependent

$$\begin{array}{l} \langle c_2 \{x := v_2\} \rangle^{l,l'} (v_1 (\langle c_1 \rangle^{l',l} v_2)) \\ \nearrow \\ (\langle x:c_1 \mapsto c_2 \rangle^{l,l'} v_1) v_2 \\ \searrow \\ \langle c_2 \{x := \langle c_1 \rangle^{l',l} v_2\} \rangle^{l,l'} (v_1 (\langle c_1 \rangle^{l',l} v_2)) \end{array} \begin{array}{l} \text{lax} \\ \\ \text{picky} \end{array}$$

# Dependency

$$f\ n = \langle g:(\text{Pos} \mapsto \text{Pos}) \mapsto \{z:\text{Int} \mid z = g\ 0\} \rangle^{l_f, l_g} \\ (\lambda g:(\text{Int} \rightarrow \text{Int}).\ g\ n)$$

$(f\ 1)\ \lambda x:\text{Int}.\ 1$

→

$$\langle g:(\text{Pos} \mapsto \text{Pos}) \mapsto \{z:\text{Int} \mid z = g\ 0\} \rangle^{l_f, l_g} \\ (\lambda g:(\text{Int} \rightarrow \text{Int}).\ g\ 1)\ \lambda x:\text{Int}.\ 1$$

$g := ?$

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$g := ?$

→

$$\langle \{z:\text{Int} \mid z = (\lambda x:\text{Int}.\ 1)\ 0\} \rangle^{l_f, l_g}$$

$\lambda x$

$$((\lambda g:\text{Int} \rightarrow \text{Int}.\ g\ 1)\ (\langle \text{Pos} \mapsto \text{Pos} \rangle^{l_g, l_f}\ \lambda x:\text{Int}.\ 1))$$

→\* 1

# Dependency

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$$(f\ 1)\ \lambda x:\text{Int}.\ 1 \longrightarrow \\ (\langle g:(\text{Pos} \mapsto \text{Pos}) \mapsto \{z:\text{Int} \mid z = g\ 0\} \rangle^{l_f, l_g} \\ (\lambda g:(\text{Int} \rightarrow \text{Int}).\ g\ 1))\ \lambda x:\text{Int}.\ 1 \longrightarrow \quad g := ?$$

$$\langle \{z:\text{Int} \mid z = (\lambda x:\text{Int}.\ 1)\ 0\} \rangle^{l_f, l_g} \quad \text{lax} \\ ((\lambda g:\text{Int} \rightarrow \text{Int}.\ g\ 1)\ (\langle \text{Pos} \mapsto \text{Pos} \rangle^{l_g, l_f}\ \lambda x:\text{Int}.\ 1)) \longrightarrow^* 1$$

$$\langle \{z:\text{Int} \mid z = (\langle \text{Pos} \mapsto \text{Pos} \rangle^{l_g, l_f}\ \lambda x:\text{Int}.\ 1)\ 0\} \rangle^{l_f, l_g} \quad \text{picky} \\ ((\lambda g:\text{Int} \rightarrow \text{Int}.\ g\ 1)\ (\langle \text{Pos} \mapsto \text{Pos} \rangle^{l_g, l_f}\ \lambda x:\text{Int}.\ 1)) \longrightarrow^* \uparrow l_f$$

# Abusive contracts

An abusive contract

$$g: (\text{Pos} \mapsto \text{Pos}) \mapsto \{z:\text{Int} \mid z = g\ 0\}$$

Picky checking detects abusive contracts

Lax checking doesn't

Only higher-order contracts can be abusive

# Contracts, made manifest

Based on Flanagan [2006]

## Contracts = Types

$S ::= \{x:B \mid s\}$       refinements of base type  
|  $x:S_1 \rightarrow S_2$       function contracts

# Contracts, made manifest

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$S ::= \{x:B \mid s\}$       refinements of base type  
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$s ::= \dots$   
|  $\langle S_1 \Rightarrow S_2 \rangle'$       casts  
|  $\uparrow'$       blame

# Contracts, made manifest

Based on Flanagan [2006]

Contracts = Types

Unfold function casts **contravariantly**; **semi-picky**

Choice forced by the type system

Complicated metatheory

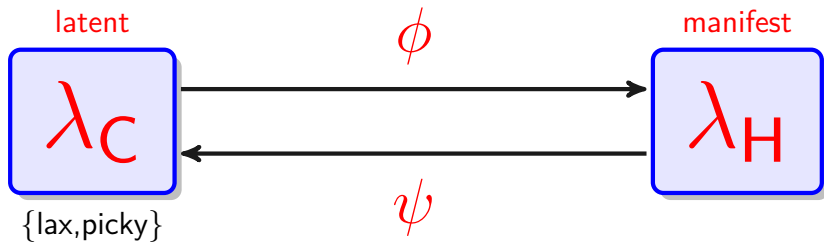
Particularly in dependent case



# Our question

Does the distinction  
between **latent** and **manifest**  
matter?

# Our Work



# Comparing latent calculi

## Latent calculi

	FF02	BM06	HJL06	GF07 $\lambda_c$	our $\lambda_c$
dependency	✓ lax	✓ $\perp$	✓ picky	×	✓ either
eval order	CBV	CBV	lazy	CBV	CBV
blame	$\uparrow!$	$\uparrow!$ or $\perp$	$\uparrow!$	$\uparrow!$	$\uparrow!$
checking	if	active	if	$\bigcirc$	active
typing	✓	n/a	✓	✓	✓
arb. con.	✓	✓	✓	✓	✓

## Legend

dependency	Dependent function contracts?	FF02	Findler and Felleisen [2002]
blame	How are failures indicated?	BM06	Blume and McAllester [2006]
checking	How are refinements checked?	HJL06	Hinze, Jeuring, and Löh [2006]
typing	Type system well-defined?	GF07	Gronski and Flanagan [2007]
arb. con.	Arbitrary user-defined contracts?		

# Comparing manifest calculi

## Manifest calculi

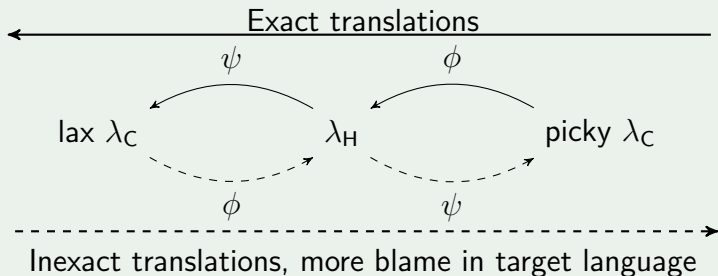
	OTMW04	F06	GF07 $\lambda_H$	KF09	WF09	our $\lambda_H$
dependency	✓	✓	×	✓	×	✓
eval order	CBV	NDCBN	CBV	full $\beta$	CBV	CBV
blame	$\uparrow$	stuck	$\uparrow!$	stuck	$\uparrow!$	$\uparrow!$
checking	if	○	○	active	active	active
typing	✓	×	×	✓	✓	✓
arb. con.	×	✓	✓	✓	✓	✓

## Legend

dependency	Dependent function contracts?	OTMW04	Ou, Tan, Mandelbaum, and Walker [2004]
blame	How are failures indicated?	F06	Flanagan [2006]
checking	How are refinements checked?	GF07	Gronski and Flanagan [2007]
typing	Type system well-defined?	KF09	Knowles and Flanagan [2009]
arb. con.	Arbitrary user-defined contracts?	WF09	Wadler and Findler [2009]

# Our answer

## The axis of blame



Inexactitude due to treatment of abusive contracts

# Correspondence

Nondependent	Dependent	
	First-order	Higher-order

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Exact!		

No lax/picky distinction in  $\lambda_C$

# Correspondence

Nondependent	Dependent	
	First-order	Higher-order
Exact!	Exact!	

No abusive contracts



# Correspondence

Nondependent	Dependent	
	First-order	Higher-order
Exact!	Exact!	Inexact

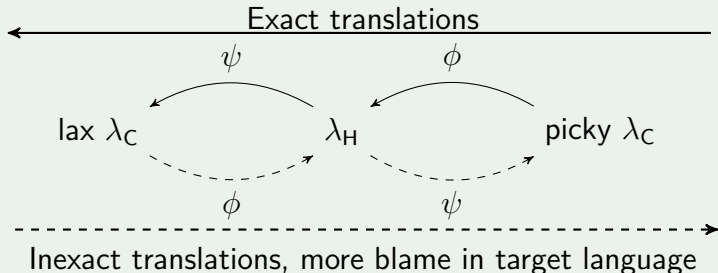
Due to **abusive** contracts...

# Exactitude

in the higher-order dependent case

Can *add* checks to be **pickier**

The axis of blame



Can't *remove* checks to be **laxer**

None of the languages inter-translate *exactly*

# Conclusion

Lax  $\lambda_C$ ,  $\lambda_H$ , and picky  $\lambda_C$  are **all subtly different**

Not entirely clear which is the “right” one

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	Latent	Manifest
Implemented		
Language	✓	×
Library	✓	N/A
Extensible	✓	×
Intuitive <small>(to Michael Greenberg)</small>		
Op. Beh.	✓	×
Meaning	✓	✓
Blame	?	?

# Outlook

What is the **surface language**?

Different for latent and manifest?

How does **blame** compare in the two approaches?

What does a **high-performance** implementation of manifest contracts look like?