CS054: Truth tables

The goal of this worksheet is to give you practice with truth tables: what are truth tables and how do you construct them? It's not for a grade—no need to turn it in! I'll post solutions, but you'll get the most out of it if you don't peek.

For all of these questions, I'll use programmatic notation—`&&` for `andb` a/k/a `&&` in Coq, `||` for `orb` a/k/a `||` in Coq, and `!` for `negb`. We write t for `true` and f for `false`; you can also use T and F or $\top$ and $\bot$. The questions are asking about the definitions we have in `Basics.v`.

I expect you to simply *know* the truth tables for `||`, `&&`, `!`, and $\Rightarrow$.

1. **Sample:** Consider the boolean expression `!a || b` given booleans a and b.

   (a) How many rows will the truth table have? **Answer:** four, because there are two variables, a and b, and we must consider each value they consider.

   (b) How many columns will the truth table have and what are they? **Answer:** four; one each for a and b, one for `!a`, and one for the whole expression.

   (c) What is the truth table? **Answer:**

a	b	`!a`	`!a || b`
t	t	f	t
t	f	f	f
f	t	t	t
f	f	t	t

   (d) Can you rephrase this expression in terms of other boolean operators? That is, can you find a smaller boolean expression that has an equivalent truth table? **Answer:** The expression $a \Rightarrow b$ (i.e., `impb a b`) has the same truth table.

2. Consider the boolean expression p `||` $(p$ `&&` $q)$.

   (a) How many rows will the truth table have?

   (b) How many columns will the truth table have?

   (c) What is the truth table?

   (d) Can you rephrase this expression in terms of other boolean operators? That is, can you find a smaller boolean expression that has an equivalent truth table?

3. Consider the boolean expression x && $(x \mathbin{||} y)$.

   (a) How many rows will the truth table have?

   (b) How many columns will the truth table have?

   (c) What is the truth table?

   (d) Can you rephrase this expression in terms of other boolean operators? That is, can you find a smaller boolean expression that has an equivalent truth table?

4. Consider the boolean expression $(x$ && $y) \mathbin{||} (x$ && $z)$.

   (a) How many rows will the truth table have?

   (b) How many columns will the truth table have?

   (c) What is the truth table?

   (d) Can you rephrase this expression in terms of other boolean operators? That is, can you find a smaller boolean expression that has an equivalent truth table?

5. Consider the boolean expression `!`x `&&` `!`y.

    (a) How many rows will the truth table have?

    (b) How many columns will the truth table have?

    (c) What is the truth table?

    (d) Can you rephrase this expression in terms of other boolean operators? That is, can you find a smaller boolean expression that has an equivalent truth table?

6. Consider the boolean expression `!`a `||` `!`b.

    (a) How many rows will the truth table have?

    (b) How many columns will the truth table have?

    (c) What is the truth table?

    (d) Can you rephrase this expression in terms of other boolean operators? That is, can you find a smaller boolean expression that has an equivalent truth table?

7. Use a truth table to prove that || is commutative, i.e., $p \ || \ q$ is the same as $q \ || \ p$.

8. Use a truth table to prove that || is associative, i.e., $p \ || \ (q \ || \ r)$ is the same as $(p \ || \ q) \ || \ r$.

9. Write a logical formula that's equivalent to the following truth table over the variables a and b.

a	b	???
t	t	f
t	f	f
f	t	f
f	f	t

What's the formula? If it were a boolean operator, what might you name it?

Other good practice exercises (for which no solutions will be provided):

- Use truth tables to prove that && and $\otimes$ (a/k/a xorb) are commutative and associative.
- Use truth tables to prove that ! is involutive, i.e., ! (! b) is equivalent to b.
- Is $\Rightarrow$ (a/k/a impb) commutative or associative? Use truth tables to prove or disprove it.