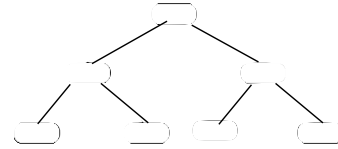


# Lecture 15: Binary Trees

CS 62  
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Kim Bruce & Kevin Coogan

## Definition

- Def: A tree is either
  - empty or
  - consists of a node, called the root node, together with a collection of trees, called its subtrees. These trees are disjoint from each other and the root.

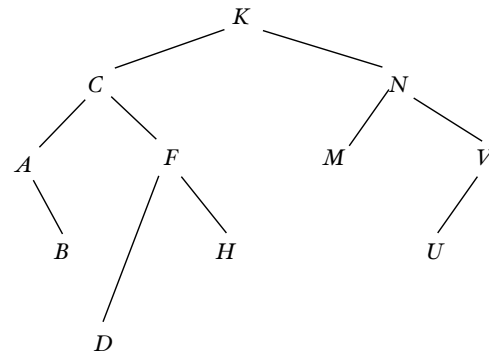


## More Defs

- An *edge* connects a node to its subtrees.
- The roots of the subtrees of a node are said to be the *successors* or *descendants* of the node.
- Nodes without successors are called *leaves*. The others are called *interior nodes*.
- All nodes except root have unique predecessor.
- A collection of trees is called a *forest*.

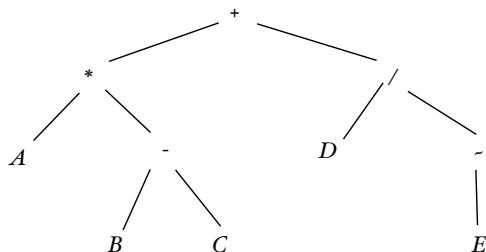
## Example: Binary Search Tree

K, C, A, N, B, V, F, U, D, H, M



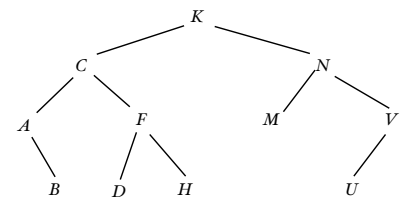
## Expression Tree

$[A*(B-C)]+(D/-E)$



## Family Tree Terminology

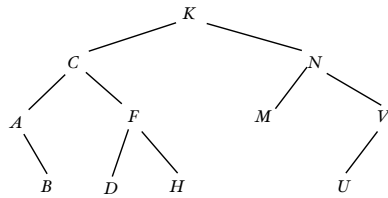
- *Parent* node is directly above *child* node: K is parent to C, N.
- *Sibling* node has same parent: A, F
- K is ancestor of B
- B is descendant of K



Node plus all descendants gives subtree

## More Terminology

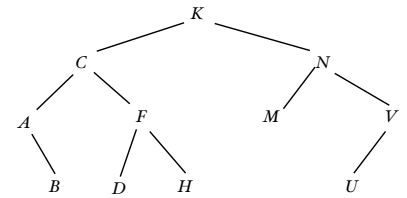
- Simple path is series of distinct nodes s.t. there is edge between successive nodes.
- Path length = # edges in path



Height of node = length of longest path to a leaf  
 Height of tree = height of root  
 Depth of node is length of path from root to that node  
 Degree of node is # of children

## More Terminology

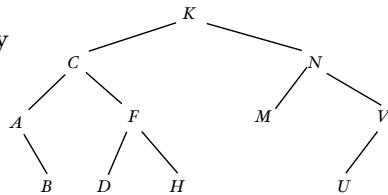
- Level of node defined recursively:
  - Root is at level 0
  - Level of any other node is one greater than level of parent
- Level of node is also length of path from root to the node.



Binary tree has all nodes of degree  $\leq 2$ .

## Counting

- Lemma: If T is a binary tree then at level k, T has  $\leq 2^k$  nodes.



- Theorem: If T has height h, then # nodes in T  $\leq 2^{h+1} - 1$ .
- Equivalently, if T has n nodes then  $n - 1 \geq h \geq \log(n+1) - 1$