

BINARY TREES

David Kauchak
CS 62 – Fall 2026

1

Admin

OnDiskSort

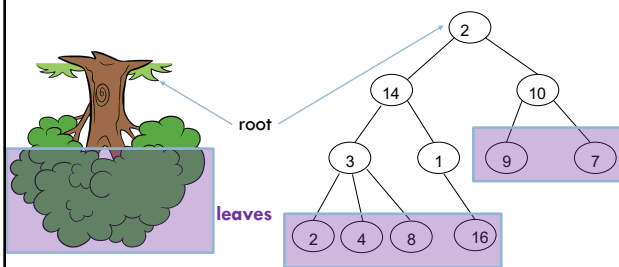
Lab tomorrow

- ▣ Quiz at the beginning
- ▣ Review

Autocomplete assignment out tomorrow (start early)

2

Trees

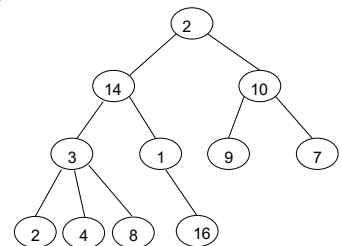


3

Trees

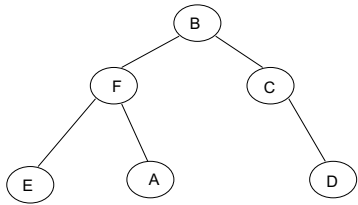
A set of nodes based on a parent-child relationship

- ▣ Each node has one parent
- ▣ Root has no parent



4

Another example



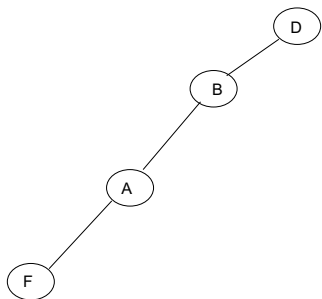
5

Another example: the lone wolf



6

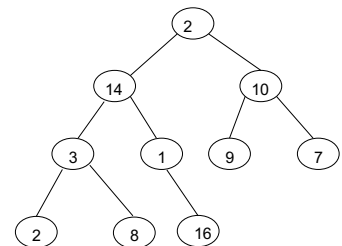
Another example: the twig



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Binary tree

Each parent has at most 2 children

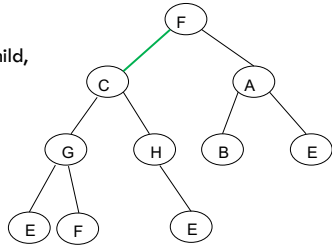


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Tree terminology

Edge:

- one of the lines
- Defined by a parent and child, **(C,F)**

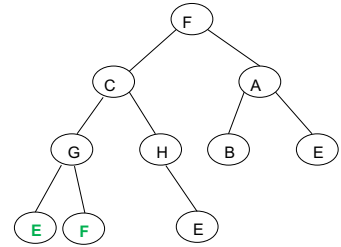


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Tree terminology

Sibling: two nodes that share the same parent

E and **F** are siblings

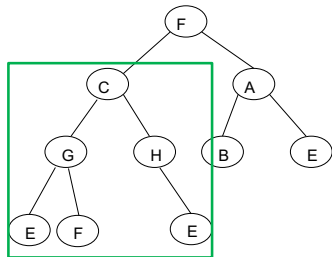


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Tree terminology

Subtree: a node and all of the nodes below it

subtree rooted at **C**

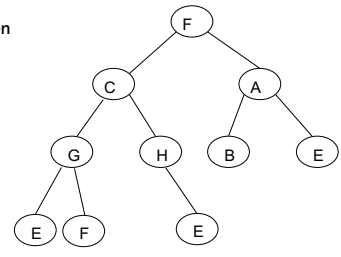


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Tree terminology

Leaf: node without any children

Internal node: non-leaf node

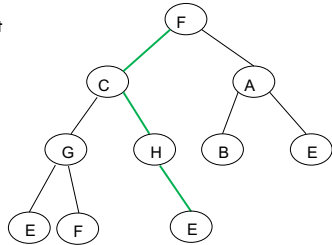


12

Tree terminology

Simple path: a series of distinct nodes with edges between successive nodes

F-C-H-E



13

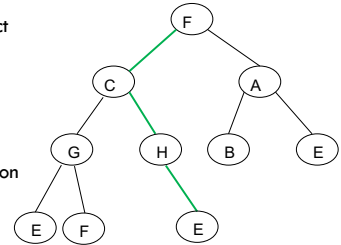
Tree terminology

Simple path: a series of distinct nodes with edges between successive nodes

F-C-H-E

Path length: number of edges on path

How long is this path?



14

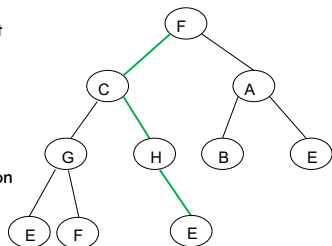
Tree terminology

Simple path: a series of distinct nodes with edges between successive nodes

F-C-H-E

Path length: number of edges on path

3: (F,C), (C,H), (H,E)

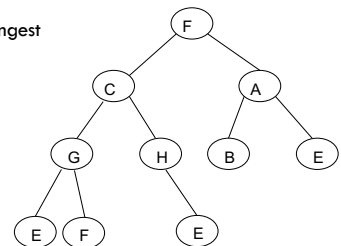


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Tree terminology

Height of a node: length of longest path from the node to a leaf

What is the height of C?



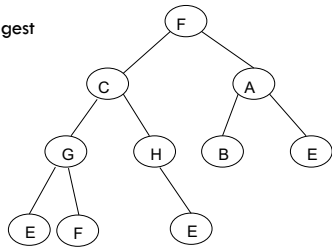
16

Tree terminology

Height of a node: length of longest path from the node to a leaf

What is the height of C?

2

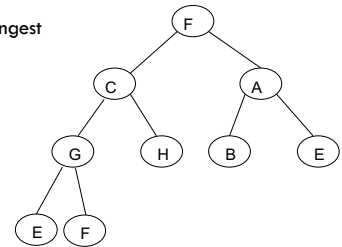


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Tree terminology

Height of a node: length of longest path from the node to a leaf

What is the height of C?



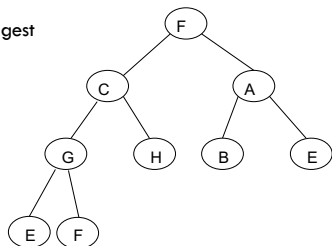
18

Tree terminology

Height of a node: length of longest path from the node to a leaf

What is the height of C?

2

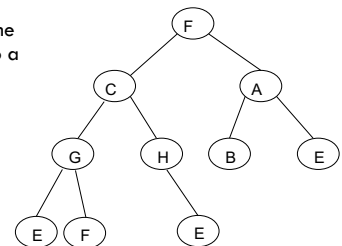


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Tree terminology

Height of the tree: height of the root (longest path from root to a leaf)

What is the height of the tree?



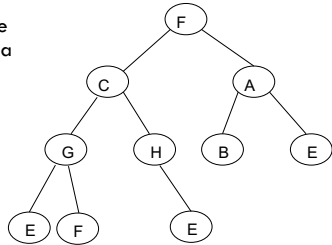
20

Tree terminology

Height of the tree: height of the root (longest path from root to a leaf)

What is the height of the tree?

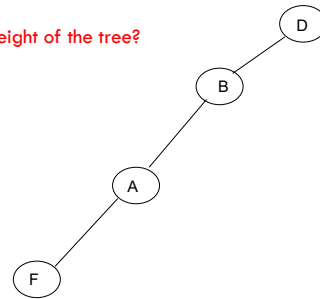
3



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Tree terminology

What is the height of the tree?

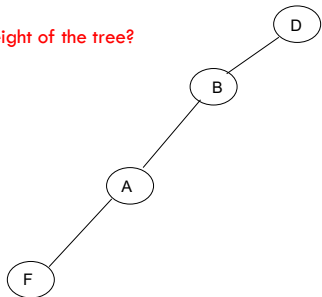


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Tree terminology

What is the height of the tree?

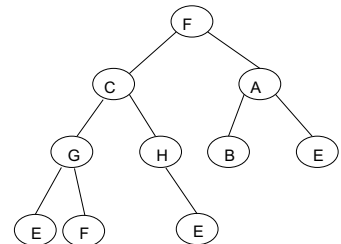
3



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Tree terminology

Ancestors: the "ancestors" of a node are any nodes along the path from that node to the root (including the root)

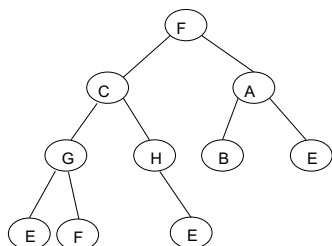


The ancestors of G: C and F

24

Tree terminology

Ancestors: the “ancestors” of a node are any nodes along the path from that node to the root (including the root)

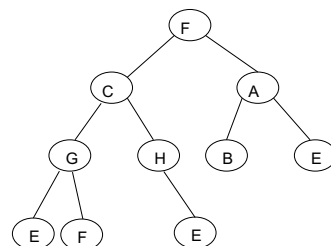


What are the ancestors of B?

25

Tree terminology

Ancestors: the “ancestors” of a node are any nodes along the path from that node to the root (including the root)

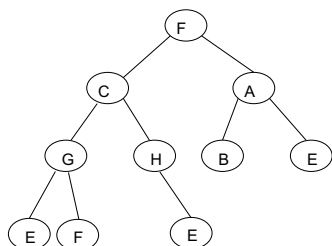


The ancestors of B: A and F

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Tree terminology

Ancestors: the “ancestors” of a node are any nodes along the path from that node to the root (including the root)

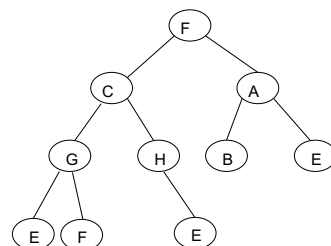


What are the ancestors of F?

27

Tree terminology

Ancestors: the “ancestors” of a node are any nodes along the path from that node to the root (including the root)

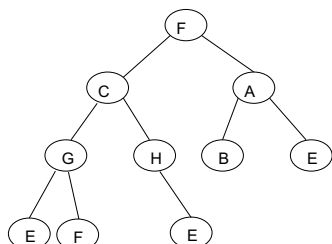


The ancestors of F: none

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Tree terminology

Descendant: the descendants of a node X are all nodes that have X as an ancestor (i.e., all nodes below a given node)



The descendants of C: E, G, F, H, E

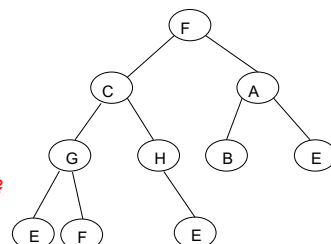
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Tree terminology

Degree of node: number of children

Degree of tree (arity): max degree of any of the nodes

What is the degree of the tree?



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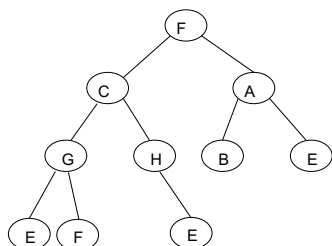
Tree terminology

Degree of node: number of children

Degree of tree (arity): max degree of any of the nodes

What is the degree of the tree?

2



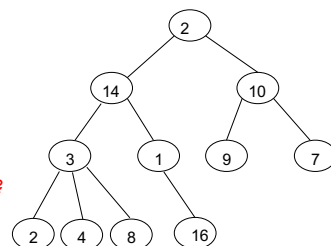
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Tree terminology

Degree of node: number of children

Degree of tree (arity): max degree of any of the nodes

What is the degree of the tree?



32

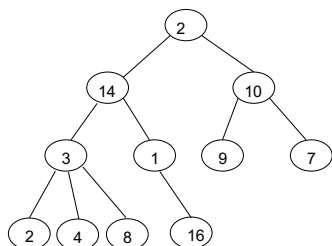
Tree terminology

Degree of node: number of children

Degree of tree (arity): max degree of any of the nodes

What is the degree of the tree?

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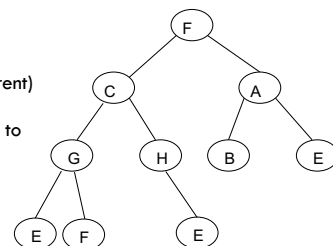
Tree terminology

Level/depth of node:

- Root is 0
- $\text{Level}(\text{child}) = 1 + \text{level}(\text{parent})$

(also: length of path from root to node)

What is the depth of G?



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Tree terminology

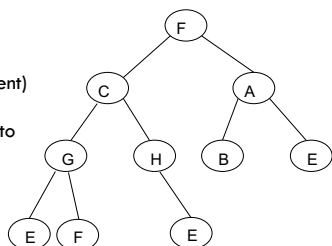
Level/depth of node:

- Root is 0
- $\text{Level}(\text{child}) = 1 + \text{level}(\text{parent})$

(also: length of path from root to node)

What is the depth of G?

2



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Worksheet problem #1

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Tree terminology (almost there!)

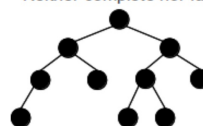
Full tree: a binary tree where every node has 0 or 2 children

Complete: All levels except the last are completely filled and all nodes on the last level are on the left

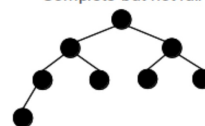
37

Full + Complete

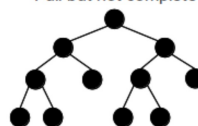
Neither complete nor full



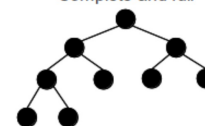
Complete but not full



Full but not complete



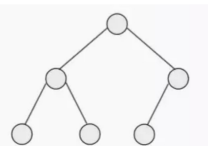
Complete and full



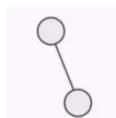
<http://code.cloudkaka.org/binary-tree/types-binary-tree>

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Full + Complete: worksheet #2



12

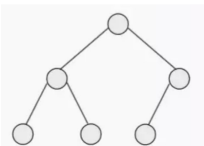


Full tree: a binary tree where every node has 0 or 2 children

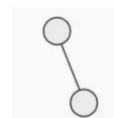
Complete: All levels except the last are completely filled and all nodes on the last level are on the left

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Full + Complete?



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Complete

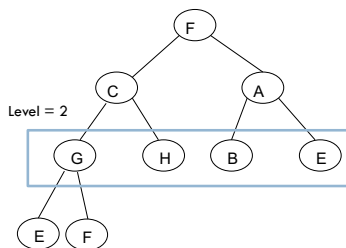
Full + Complete

Neither

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Nodes in a binary tree

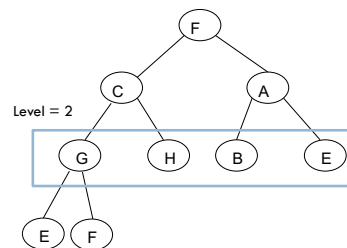
What is the most nodes we can have at a level k ?



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Nodes in a binary tree

What is the most nodes we can have at a level k ?

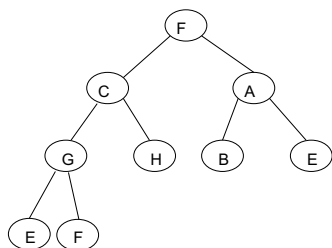


At most 2^k nodes (when every node above it has two children)

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Nodes in a binary tree

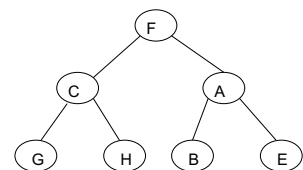
What is the most nodes we can have in a tree of height h ?



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Nodes in a binary tree

What is the most nodes we can have in a tree of height h ?



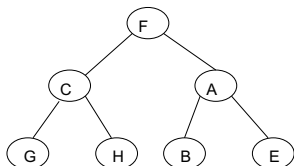
$$1 + 2 + 4 + 8 + \dots + 2^h = 2^{h+1} - 1$$

When the tree is full and complete!

44

Bounding the nodes in a binary tree

If we have n nodes in the tree, what is the smallest height we can have?



$$1 + 2 + 4 + 8 + \dots + 2^h = 2^{h+1} - 1$$

When the tree is full and complete!

45

Bounding the nodes in a binary tree

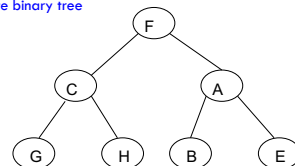
$$n \leq 2^{h+1} - 1 \quad \text{full+ complete binary tree}$$

$$n + 1 \leq 2^{h+1}$$

$$2^{h+1} \geq n + 1$$

$$h + 1 \geq \log(n + 1)$$

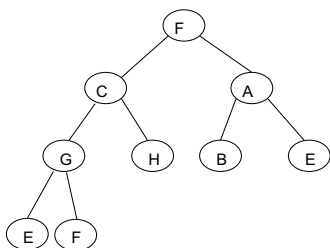
$$h \geq \log(n + 1) - 1$$



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Nodes in a binary tree

What is the smallest number of nodes we can have in a tree of height h ?

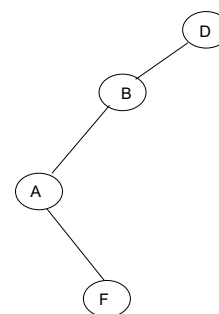


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Nodes in a binary tree

For a tree of height h , what is the fewest nodes we can have in the tree?

$h+1$ (the twig!)



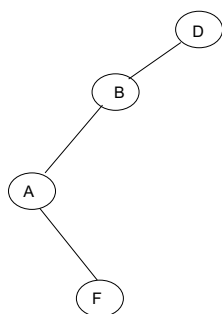
48

Nodes in a binary tree

Given a tree of n nodes,
what is the largest h can
be?

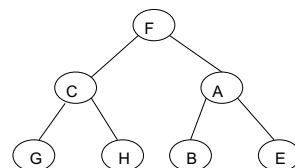
$$n \geq h + 1$$

$$h \leq n - 1$$



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Nodes in a binary tree



$$\log(n + 1) - 1 \leq h \leq n - 1$$

Height is somewhere between $\log(n)$ of nodes and n nodes

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Implementing a binary tree

How do we do it?

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Implementing a binary tree

```

public class BinaryTree<E> {
    private Node root;

    private class Node {
        private E value;
        private Node left;
        private Node right;

        public Node(Node left, Node right, E value) {
            this.left = left;
            this.right = right;
            this.value = value;
        }
    }
}
  
```

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Recursive data structure!

```

public class BinaryTree<E> {
    private Node root;

    private class Node {
        private E value;
        private Node left;
        private Node right;

        public Node(Node left, Node right, E value) {
            this.left = left;
            this.right = right;
            this.value = value;
        }
    }
}

public class LinkedList<E> {
    private Node head;

    private class Node {
        private E value;
        private Node next;

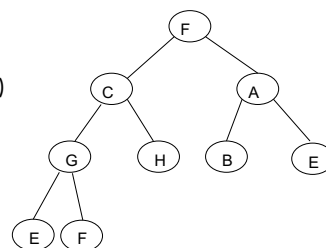
        public Node(Node next, E value) {
            this.next = next;
            this.value = value;
        }
    }
}

```

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Tree traversals

Inorder(node):
 Inorder(left)
 visit node (e.g., print)
 Inorder(right)

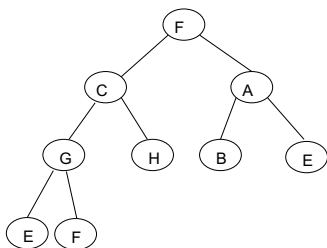


What would be printed out?

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Tree traversals

Inorder(node):
 Inorder(left)
 visit node (e.g., print)
 Inorder(right)

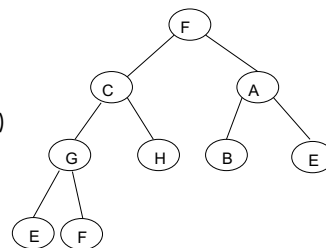


EGFCHFB AE

55

Tree traversals

Postorder(node):
 Postorder(left)
 Postorder(right)
 visit node (e.g., print)

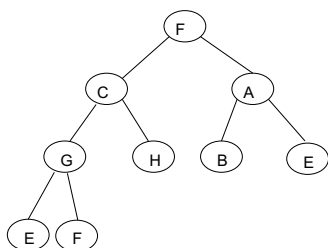


What would be printed out?

56

Tree traversals

Postorder(node):
 Postorder(left)
 Postorder(right)
 visit node (e.g., print)

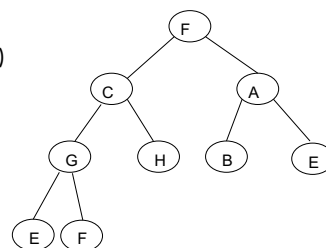


EFGHCBEAF

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Tree traversals

Preorder(node):
 visit node (e.g., print)
 Preorder(left)
 Preorder(right)

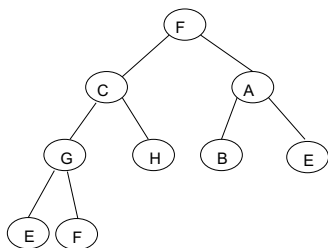


What would be printed out?

58

Tree traversals

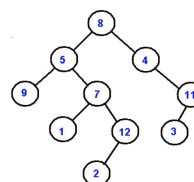
Preorder(node):
 visit node (e.g., print)
 Preorder(left)
 Preorder(right)



FCGEFHABE

59

in-order, post-order, pre-order



Inorder(node):
 Inorder(left)
 visit node (e.g., print)
 Inorder(right)

Postorder(node):
 Postorder(left)
 Postorder(right)
 visit node (e.g., print)

Preorder(node):
 visit node (e.g., print)
 Preorder(left)
 Preorder(right)

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