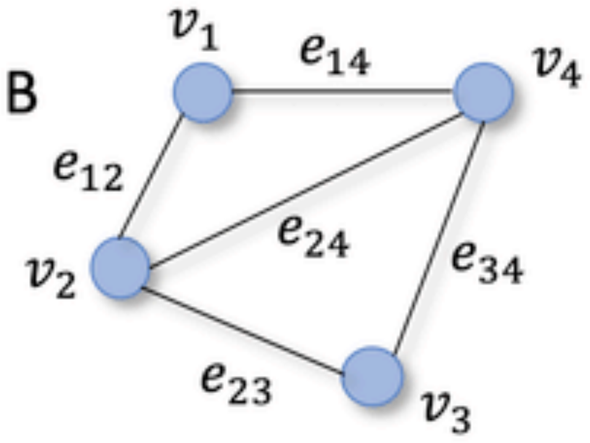
CS62 Class 20: Graphs (intro, BFS/DFS)

Graphs

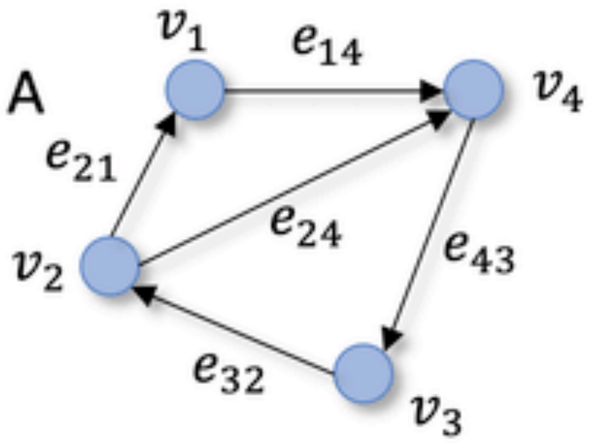
Undirected graph G(V,E)

B



Directed graph G(V,E)

A



adjacency matrix

F

	v_1	v_2	v_3	v_4
v_1	0	1	0	1
v_2	1	0	1	1
v_3	0	1	0	1
v_4	1	1	1	0

E

	v_1	v_2	v_3	v_4
v_1	0	0	0	1
v_2	1	0	0	1
v_3	0	1	0	0
v_4	0	0	1	0

Agenda

- Last time: .equals()
- Undirected graphs
  - Depth-first search
  - Breadth-first search
- Directed graphs
  - Depth-first search
  - Breadth-first search

.equals()

If 2 objects are “equal”, they should have the same hashCode

- Requirement: If `x.equals(y)` then it should be `x.hashCode()==y.hashCode()`.
- Ideally (but not necessarily): If `!x.equals(y)` then it should be `x.hashCode()!=y.hashCode()`.
- Need to override *both* `equals()` and `hashCode()` for custom types.
  - Already done for us for `Integer`, `Double`, etc.

Bottom line: If your class override `equals`, you should also override `hashCode` in a consistent manner.

Equality test in Java

- If you don’t override it, the default implementation of .equals() checks whether `x` and `y` refer to **the same object in memory.** i.e. == (== checks references, i.e. memory location)
- Equality requirements: For any objects `x`, `y`, and `z`.
  - Reflexive: `x.equals(x)` is `true`.
  - Symmetric: `x.equals(y)` iff `y.equals(x)`.
  - Transitive: if `x.equals(y)` and `y.equals(z)` then `x.equals(z)`.
  - Non-null: if `x.equals(null)` is `false`.

General equality test recipe in Java: `x.equals(y)`

- Optimization for reference equality.
  - `if (y == this) {return true;}`
- Check against null.
  - `if (y == null) {return false;}`
- Check that two objects are of the same type.
  - `if (y.getClass() != this.getClass()) {return false;}`
- Cast them.
  - `Date that = (Date) y;`
- Compare each significant field (i.e. instance variable).
  - `return (this.day == that.day && this.month == that.month && this.year == that.year);`
  - If a field is a primitive type, use ==.
  - If a field is an object, use equals().
  - If field is an array of primitives, use Arrays.equals().
  - If field is an area of objects, use Arrays.deepEquals().
    - But make sure the objects are immutable!

Overriding `equals()` for user-defined types

```
public class Date {
     private int month;
     private int day;
     private int year;
     …
     public boolean equals(Object y) {
         if (y == this){ return true;}
         if (y == null){ return false;}
         if (y.getClass() != this.getClass()){ return false;}
         Date that = (Date) y;
         return (this.day == that.day &&
                 this.month == that.month &&
                 this.year == that.year);
     }
}
```

signature: public boolean equals(Object objToCompare)

same memory location

non-null requirement

same class

cast the obj to be compared as the same class

and compare specific attributes
(of primitive types)

Graphs

Why study graphs?

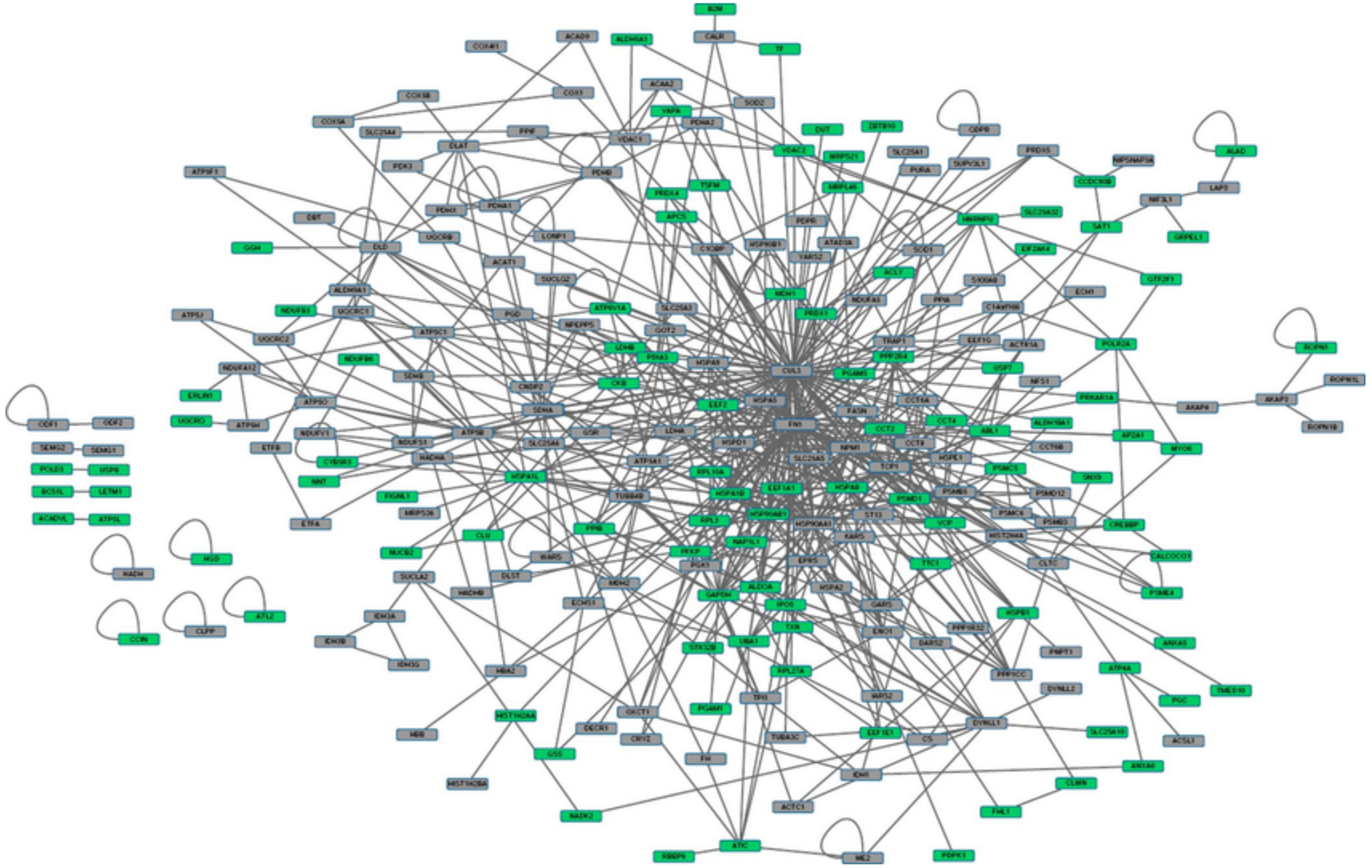
- Thousands of practical applications.
- Hundreds of graph algorithms known.
- Interesting and broadly useful abstraction.
- Challenging branch of theoretical computer science.

Undirected graphs

Undirected Graphs

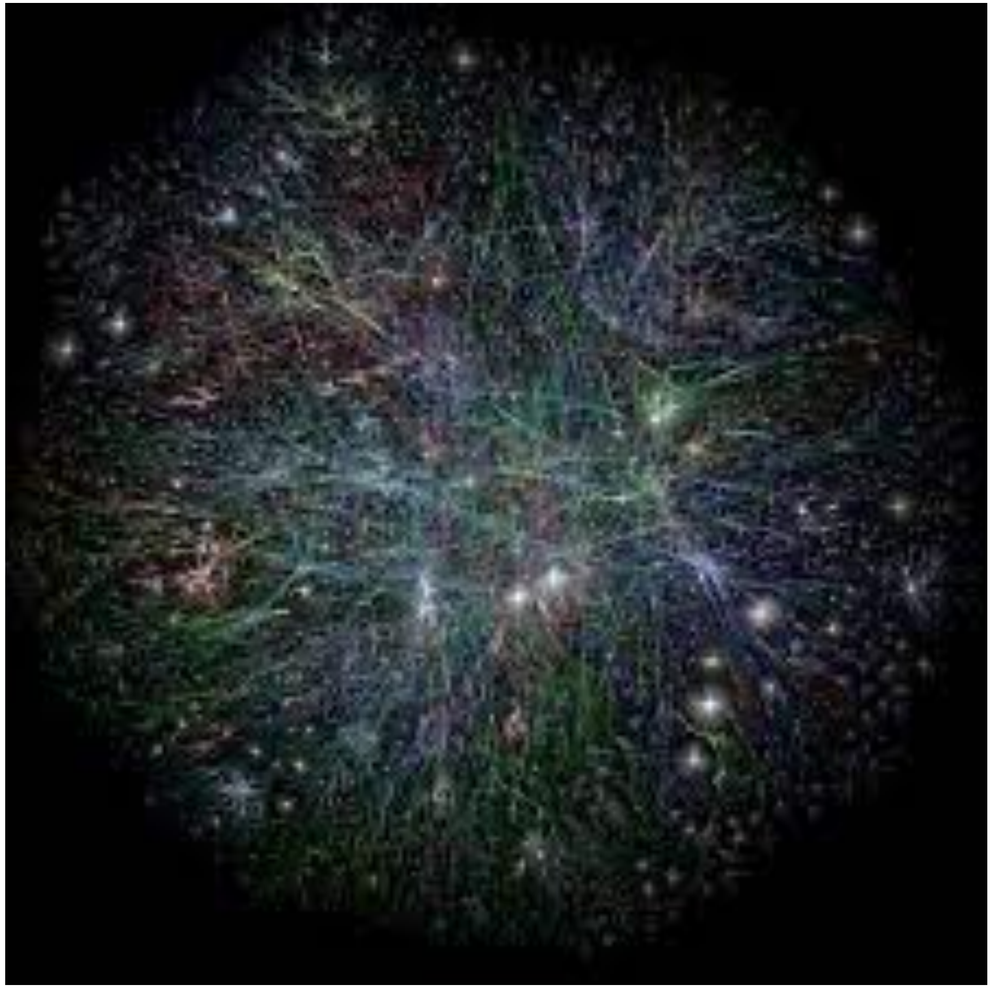
- Graph: A set of *vertices* connected pairwise by *edges*.
- Undirected graph: The edges do not point in a specific direction

Protein-protein interaction graph



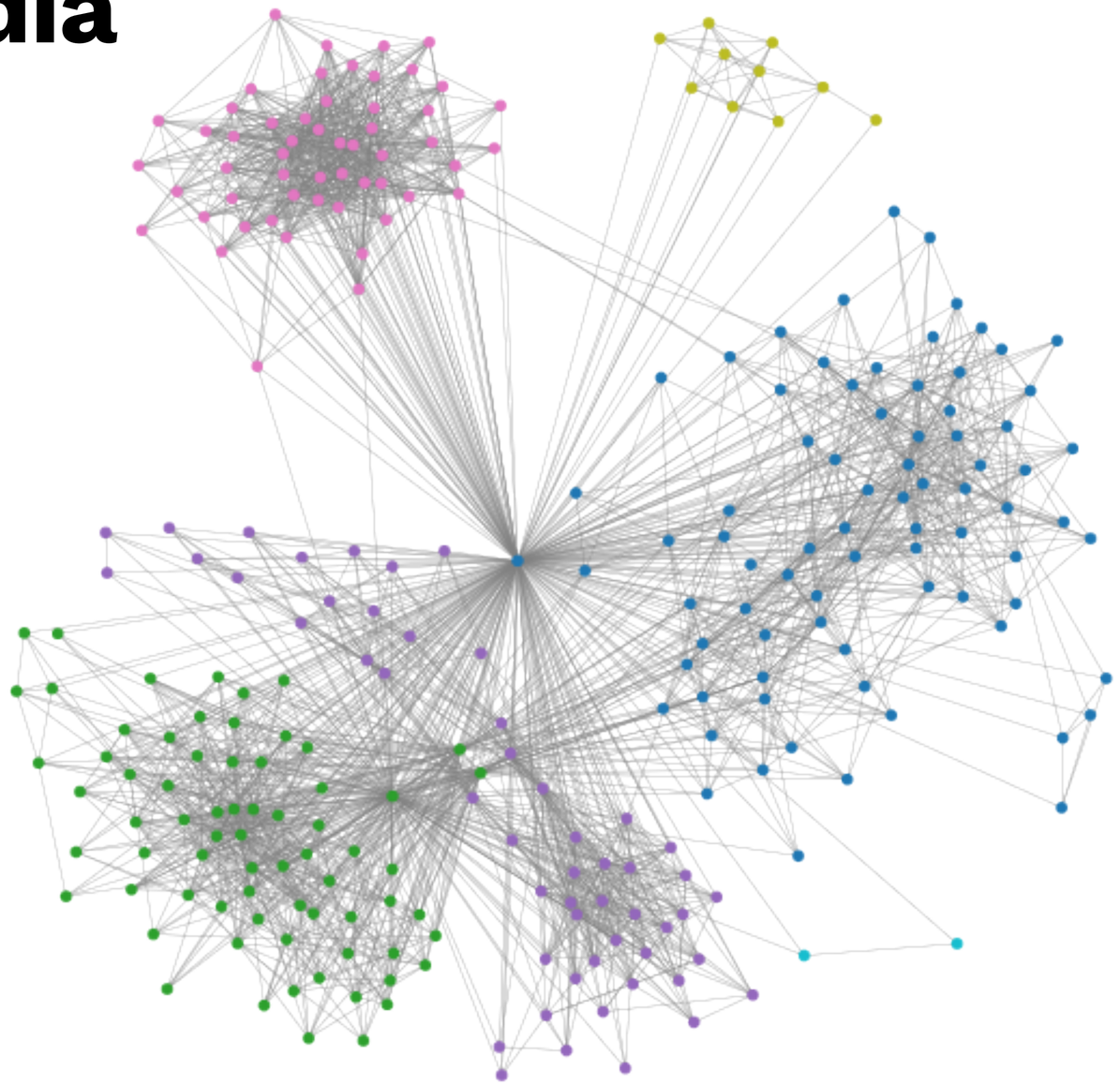
https://www.researchgate.net/figure/Network-graph-of-the-protein-protein-interactions-Green-color-represents-proteins_fig4_272297002

The Internet



https://www.opte.org/the-internet

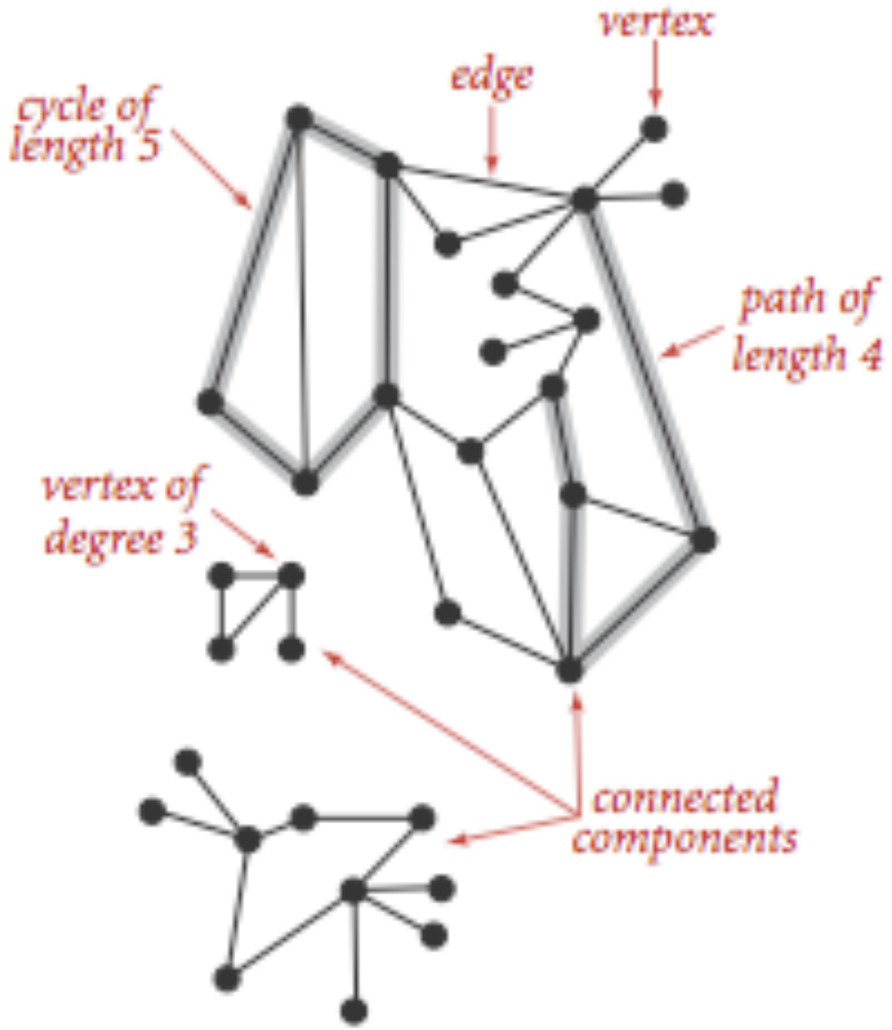
Social media



https://www.databentobox.com/2019/07/28/facebook-friend-graph/

Graph terminology

- Path: Sequence of vertices connected by edges
- Cycle: Path whose first and last vertices are the same
- Two vertices are connected if there is a path between them



Anatomy of a graph

Examples of graph-processing problems

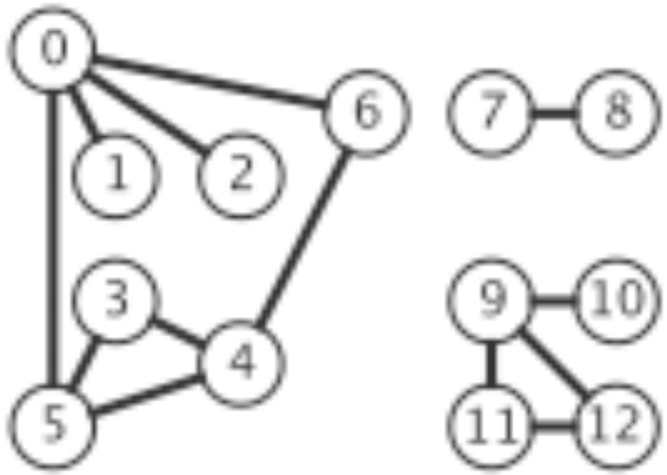
- Is there a path between vertex s and t?
  - What is the shortest path between s and t?
- Is there a cycle in the graph?
  - Euler Tour: Is there a cycle that uses each edge exactly once?
  - Hamilton Tour: Is there a cycle that uses each vertex exactly once?
- Is there a way to connect all vertices?
  - What is the shortest way to connect all vertices?
- Is there a vertex whose removal disconnects the graph?

Graph representation

- Vertex representation: integers between 0 and V-1 (but can be generalized to any type, e.g., custom Nodes).

0 5 means there's an edge between vertices 0 and 5

```
0  5
4  3
0  1
9  12
6  4
5  4
0  2
11  12
9  10
0  6
7  8
9  11
5  3
```



Basic Graph API

`public class Graph`

- `Graph(int V)`: create an empty graph with V vertices.
- `void addEdge(int v, int w)`: add an edge v-w.
- `Iterable<Integer> adj(int v)`: return vertices adjacent to v.
- `int V()`: number of vertices.
- `int E()`: number of edges.

Example of how to use the Graph API to process the graph

The degree of a vertex v is the number of vertices connected to v (i.e., the number of edges.)

```
public static int degree(Graph g, int v){
    int count = 0;
    for(int w : g.adj(v))
        count++;
    return count;
}
```

Graph density

|V| = number of vertices

- In a simple graph (no parallel edges or loops), if $|V| = n$, then:
  - minimum number of edges is 0 and
  - maximum number of edges is $n(n-1)/2$.

O(n^2) - all vertices are connected to each other

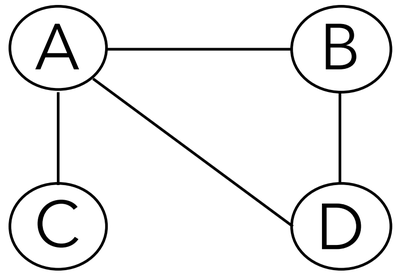
- Dense graph -> edges closer to maximum.
- Sparse graph -> edges closer to minimum.

Graph representation: adjacency matrix

- Maintain a $|V|$-by-$|V|$ boolean array; for each edge `v-w`:
  - `adj[v][w] = adj[w][v] = true;`
- Good for dense graphs (edges close to $|V|^2$).
- Constant time for lookup of an edge.
- Constant time for adding an edge.
- $|V|$ time for iterating over vertices adjacent to v.
- Symmetric, therefore wastes space in undirected graphs ($|V|^2$).
- Not widely used in practice.

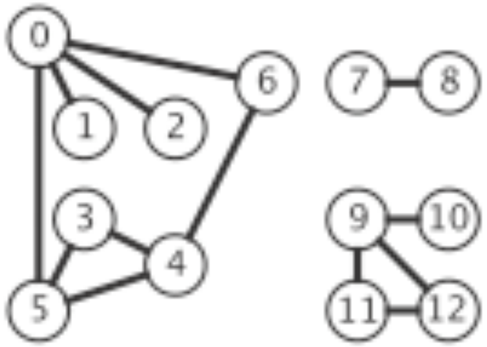
For undirected graphs, adjacency matrices are always symmetric along the diagonal

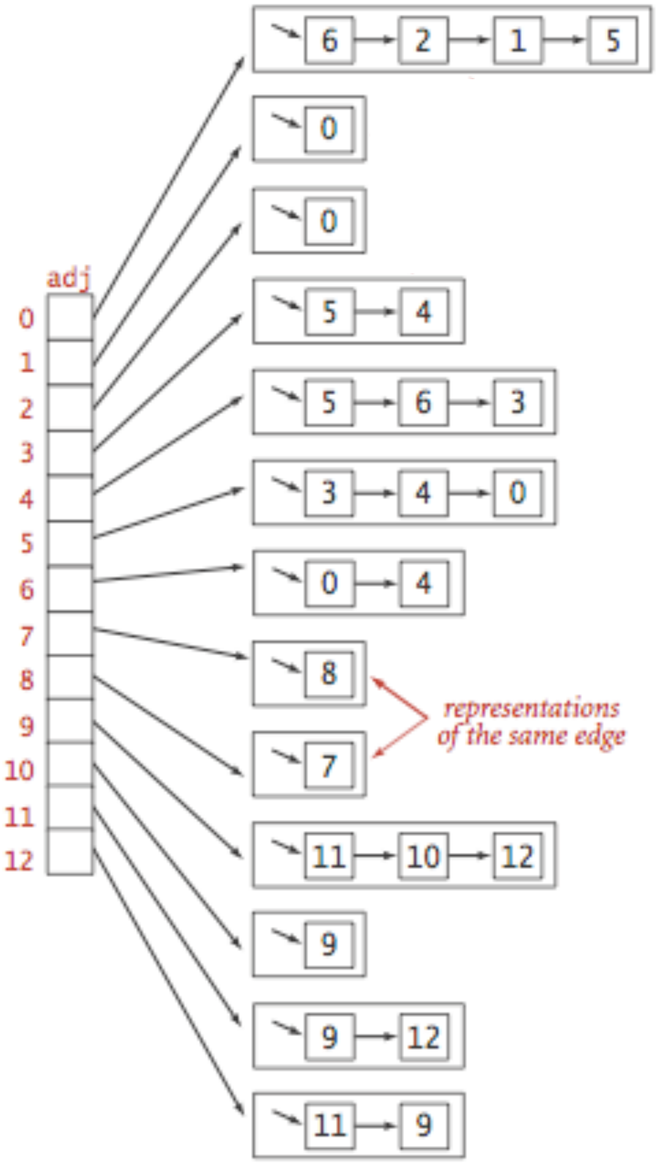
	A	B	C	D
A	0	1	1	1
B	1	0	0	1
C	1	0	0	0
D	1	1	0	0



Graph representation: adjacency list

- Maintain vertex-indexed array of lists. The list stores vertices adjacent to v.
- Good for sparse graphs (edges proportional to $|V|$) which are much more common in the real world.
- Space efficient ($|E| + |V|$).
- Constant time for adding an edge.
- Lookup of an edge or iterating over vertices adjacent to v is $degree(v)$.





Adjacency-list graph representation in Java

```
9  public class Graph {
10     private final int V;  // number of vertices
11     private int E;        // number of edges
12     private final List<Integer>[] adj;  // adjacency lists
13
14     //init empty graph with V vertices and 0 edges
15     @SuppressWarnings("unchecked")
16     public Graph(int V) {
17         this.V = V;
18         this.E = 0;
19         adj = (List<Integer>[]) new List[V];
20         for (int v = 0; v < V; v++) {
21             adj[v] = new ArrayList<>();
22         }
23     }
```

Q: What if we want to add another vertex? Can we still use a List<Integer>?

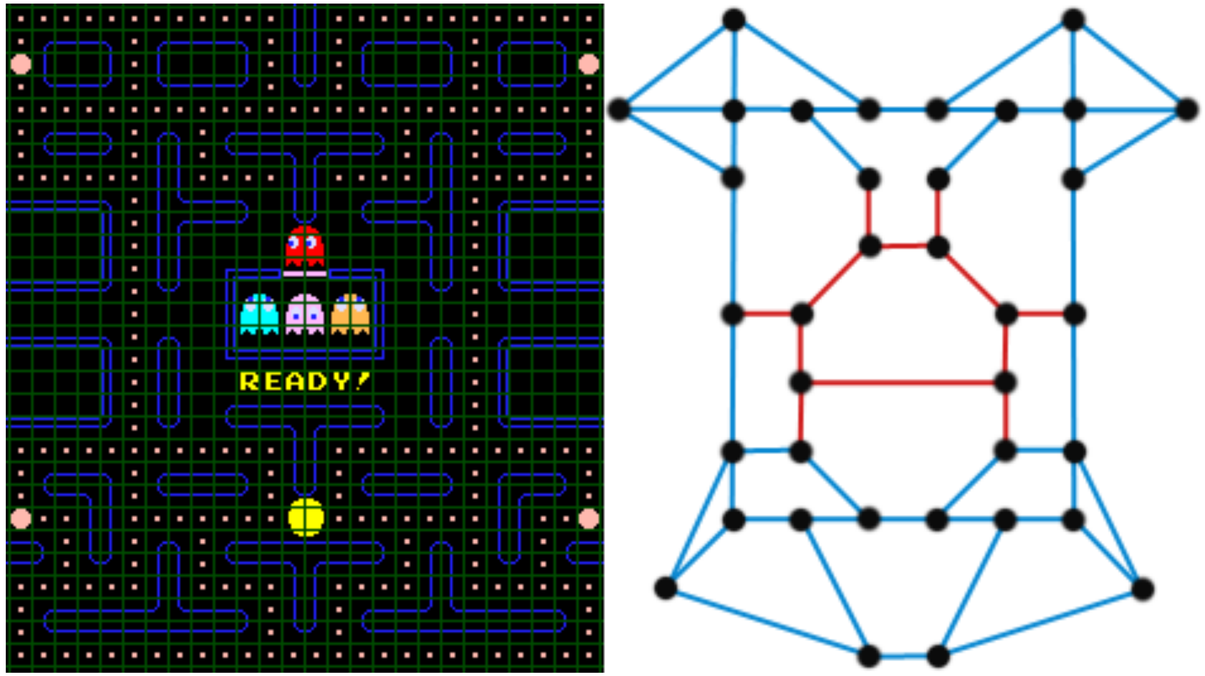
A: No, lists aren't resizable, so we can use an ArrayList instead (or make a brand new list with size V+1…)

```
24     //adds undirected edge v-w to graph. parallel edges and
       self-loops allowed
25     public void addEdge(int v, int w) {
26         E++;
27         adj[v].add(w);
28         adj[w].add(v);
29     }
30     //returns vertices adjacent to vertex v
31     public Iterable<Integer> adj(int v) {
32         return adj[v];
33     }
```

Depth-first search

Mazes as graphs

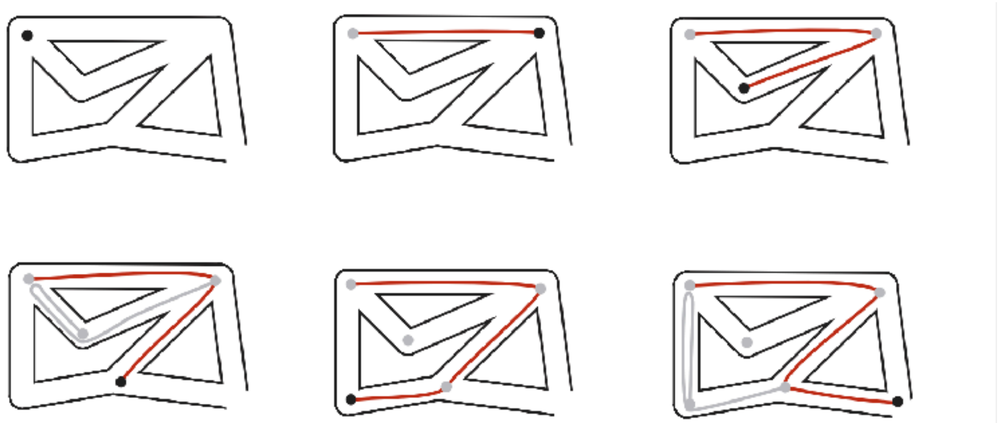
- Vertex = intersection; edge = passage

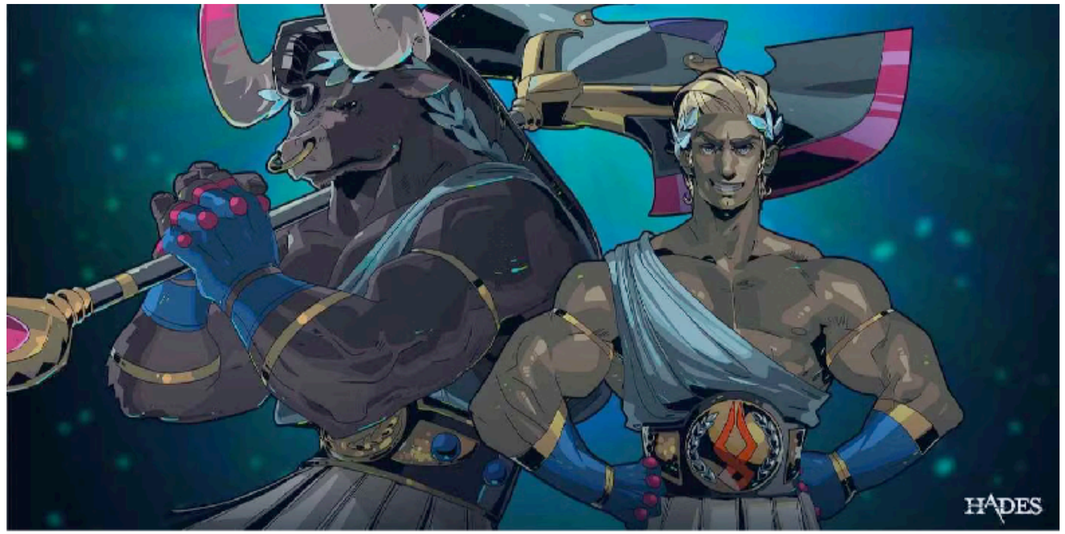


http://oatzy.blogspot.com/2011/09/playing-with-pac-man.html

How to survive a maze: a lesson from a Greek myth

- Theseus escaped from the labyrinth after killing the Minotaur with the following strategy instructed by Ariadne:
  - Unroll a ball of string behind you.
  - Mark each newly discovered intersection and passage.
  - Retrace steps when no unmarked options.
- Also known as the Trémaux algorithm.





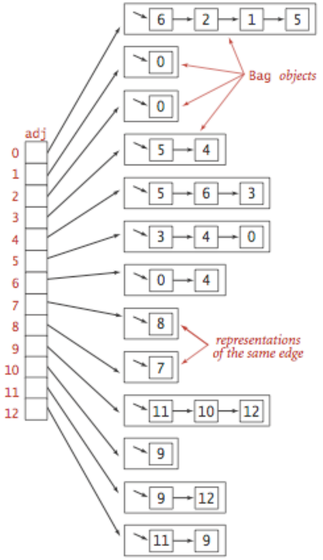
Depth-first search

- Basic idea: Go deep in a graph until you can't anymore, visiting all vertices. Then retrace your steps.
- Goal: Systematically traverse a graph.
- DFS (to visit a vertex v)
  - Mark vertex v.
  - Recursively visit all unmarked vertices w adjacent to v.

- Typical applications:
  - Find all vertices connected to a given vertex.
  - Find a path between two vertices.

http://algs4.cs.princeton.edu

4.1 DEPTH-FIRST SEARCH DEMO



Order visited: 0, 6, 4, 5, 3, 2, 1

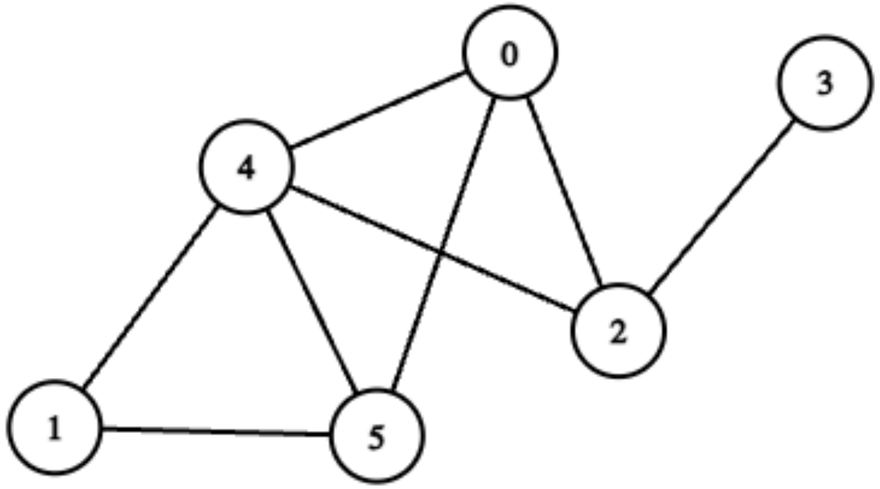
Depth-first search

- Goal: Find all vertices connected to s (and a corresponding path).
- Idea: Mimic maze exploration.
- Algorithm:
  - Use recursion (ball of string).
  - Mark each visited vertex (and keep track of edge taken to visit it).
  - Return (retrace steps) when no unvisited options.

- When started at vertex *s*, DFS marks all vertices connected to s (and no other).

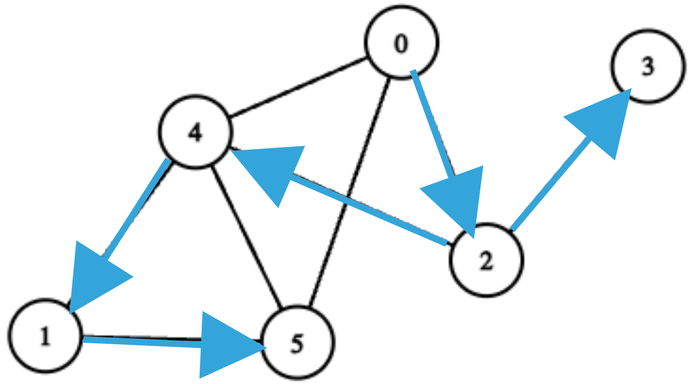
Worksheet time!

- Run DFS on the following graph starting at vertex 0 and return the **vertices in the order of being marked.** Assume that the adjacent vertices are returned in increasing numerical order (i.e. visit smaller ones first, opposite of the demo).



Worksheet answers

- Vertices marked as visited: 0, 2, 3, 4, 1, 5



V	marked	edgeTo
0	T	-
1	T	4
2	T	0
3	T	2
4	T	2
5	T	1

Implementation of depth-first search in Java

```
public void dfs(int s) {
    boolean[] marked = new boolean[V]; //marked[v] - is there an s-v path?
    int[] edgeTo = new int[V]; //edgeTo[v] = previous vertex on path from s to v
    int[] distTo = new int[V]; //distTo[v] - distance from s to v

    for (int i = 0; i < V; i++) {
        distTo[i] = -1;  // initialize distances to -1
    }

    marked[s] = true;
    distTo[s] = 0;
    dfsHelper(s, marked, edgeTo, distTo);
```

```
 private void dfsHelper(int v, boolean[] marked, int[] edgeTo, int[] distTo) {
     for (int w : adj[v]) {
         if (!marked[w]) {
             marked[w] = true;
             edgeTo[w] = v;
             distTo[w] = distTo[v] + 1;
             dfsHelper(w, marked, edgeTo, distTo);
         }
     }
 }
```

for each adjacent vertex, mark it and call DFS on it

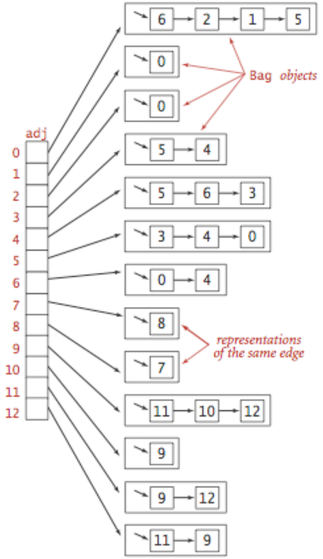
Depth-first search analysis

- DFS marks all vertices connected to `s` in time proportional to $|V| + |E|$ in the worst case.
  - Initializing arrays `marked` and `edgeTo` takes time proportional to $|V|$.
  - Each adjacency-list entry is examined exactly once and there are $2|E|$ such entries (two for each edge in an undirected graph).
- Once we run DFS, we can check if vertex `v` is connected to `s` in constant time (look into the marked array). We can also find the `v`-`s` path (if it exists) in time proportional to its length (follow the edgeTo array).

Breadth-first search

Breadth-first search

- Basic idea: BFS traverses vertices in order of distance from s. (All of s's adjacent vertices get seen first, then the ones 2 away, then the ones 3 away…)
- BFS (from source vertex s)
  - Put s on a queue and mark it as visited.
  - Repeat until the queue is empty:
    - Dequeue vertex v.
    - Enqueue each of v's unmarked neighbors and mark them.
  - When we enqueue a vertex is when we mark it as visited/



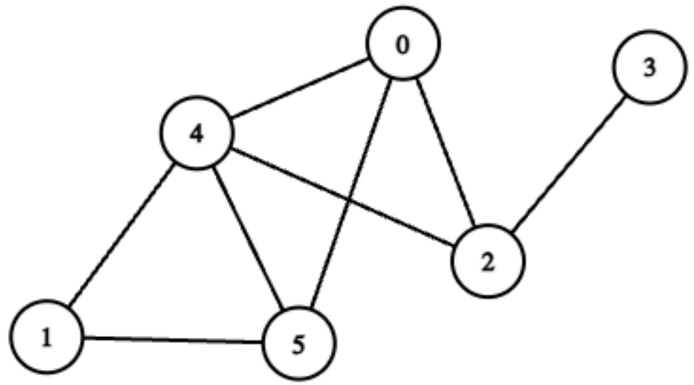
http://algs4.cs.princeton.edu

4.1 BREADTH-FIRST SEARCH DEMO

Order visited: 0, 2, 1, 5, 3, 4

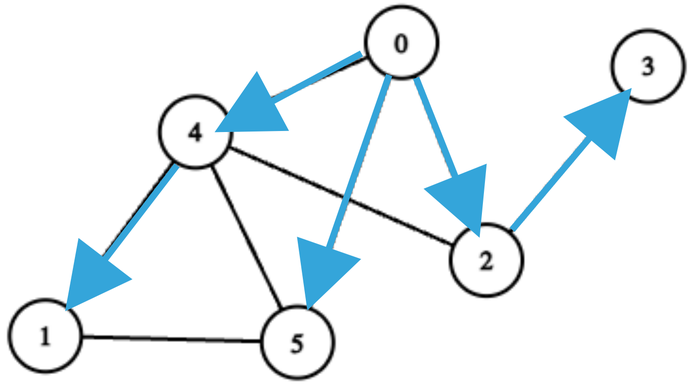
Worksheet time!

- Run the BFS on the following graph starting at vertex 0 and return the vertices in the order of being marked. Assume the adjacent vertices are returned in increasing numerical order (i.e. you visit smaller numbers first).



Worksheet answers

Vertices marked as visited: 0, 2, 4, 5, 3, 1



V	marked	edgeTo	distTo
0	T	-	0
1	T	4	2
2	T	0	1
3	T	2	2
4	T	0	1
5	T	0	1

Breadth-first search in Java

```
public void bfs(int s) {
    boolean[] marked = new boolean[V];
    int[] edgeTo = new int[V];
    int[] distTo = new int[V];

    Queue<Integer> queue = new LinkedList<>();
    marked[s] = true;
    distTo[s] = 0;
    queue.add(s);

    while (!queue.isEmpty()) {
        int v = queue.remove();
        for (int w : adj[v]) {
            if (!marked[w]) {
                marked[w] = true;
                edgeTo[w] = v;
                distTo[w] = distTo[v] + 1;
                queue.add(w);
            }
        }
    }
}
```

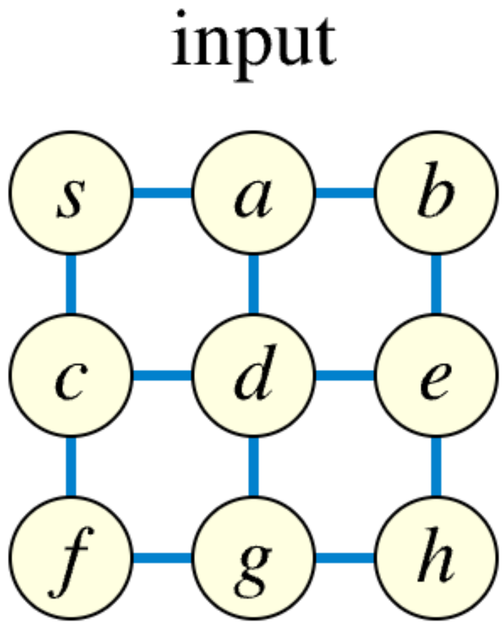
enqueue s

dequeue v

enqueue adjacent vertices, w

Worksheet time!

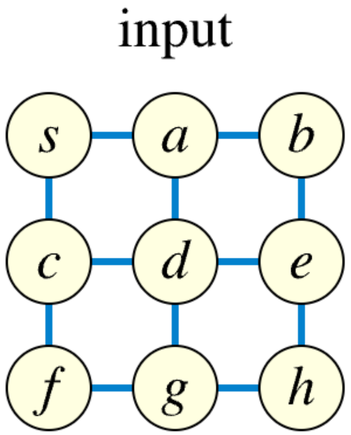
- Run DFS and BFS on the following graph starting at vertex s. Assume the adjacent vertices are returned in lexicographic (i.e., alphabetical) order.

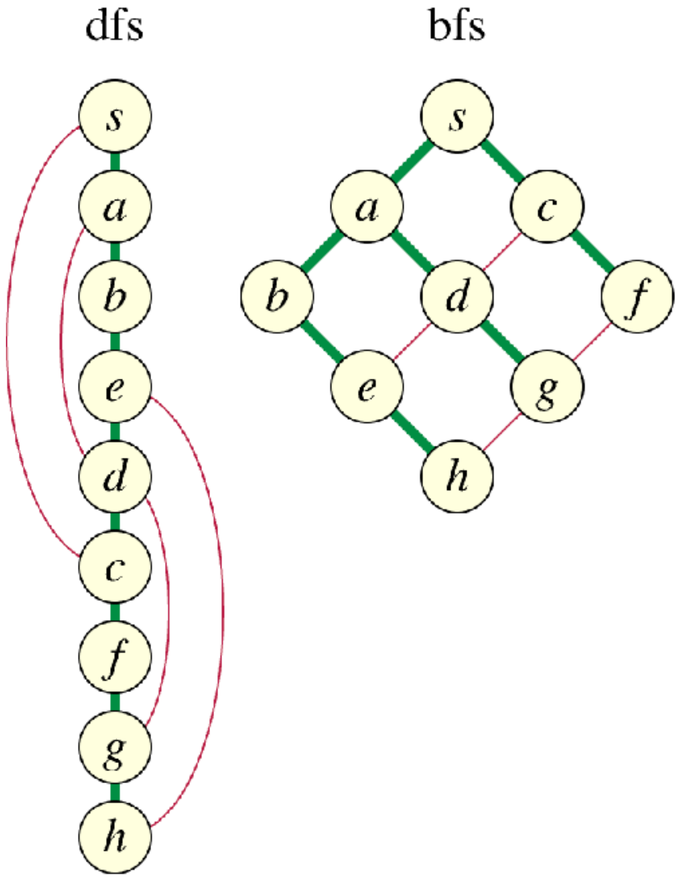


https://11011110.github.io/blog/2013/12/17/stack-based-graph-traversal.html

Worksheet answer

- Run DFS and BFS on the following graph starting at vertex s. Assume that the adj method returns back the adjacent vertices in lexicographic order.
- DFS: s->a->b->e->d->c->f->g->h
- BFS: s->a->c->b->d->f->e->g->h





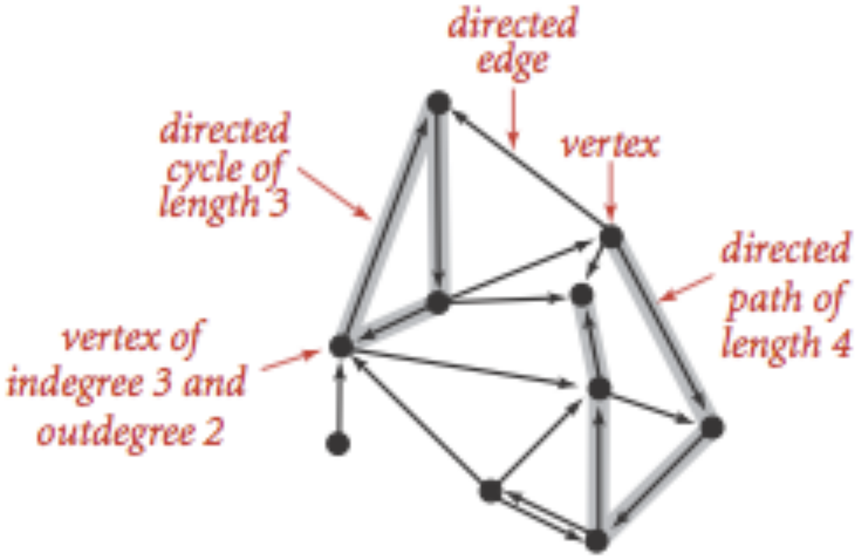
Summary

- DFS: Uses recursion.
- BFS: Put unvisited vertices on a queue.
- Shortest path problem: Find path from `s` to `t` that uses the fewest number of edges.
  - E.g., calculate the fewest numbers of hops in a communication network.
  - E.g., calculate the Kevin Bacon number or Erdös number.
- **BFS computes shortest paths** from `s` to all vertices in a graph in time proportional to $|E| + |V|$
  - The queue always consists of zero or more vertices of distance `k` from `s`, followed by zero or more vertices of `k+1`.
  - DFS, on the other hand, will find *a path*, but it's not guaranteed to be the shortest one.

Directed graphs

Directed Graph Terminology

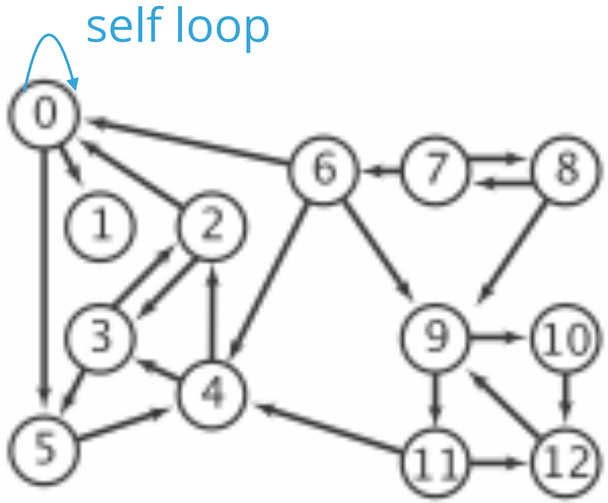
- Directed Graph (digraph) : a set of vertices V connected pairwise by a set of directed edges E.
- Directed path: a sequence of vertices in which there is a directed edge pointing from each vertex in the sequence to its successor in the sequence, with no repeated edges. (Basically just a path in the graph.)
  - A simple directed path is a directed path with no repeated vertices.
- Directed cycle: Directed path with at least one edge whose first and last vertices are the same.
  - A simple directed cycle is a directed cycle with no repeated vertices (other than the first and last).
- The length of a cycle or a path is its number of edges.



Anatomy of a digraph

Directed Graph Terminology

- Self-loop: an edge that connects a vertex to itself.
- Two edges are parallel if they connect the same pair of vertices.
- The outdegree of a vertex is the number of edges pointing from it.
- The indegree of a vertex is the number of edges pointing to it.
- A vertex w is reachable from a vertex v if there is a directed path from v to w.
- Two vertices v and w are strongly connected if they are mutually reachable.

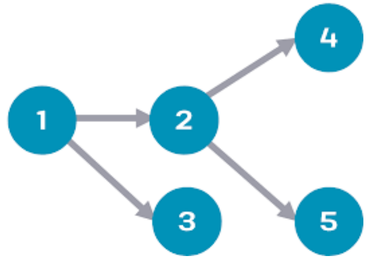


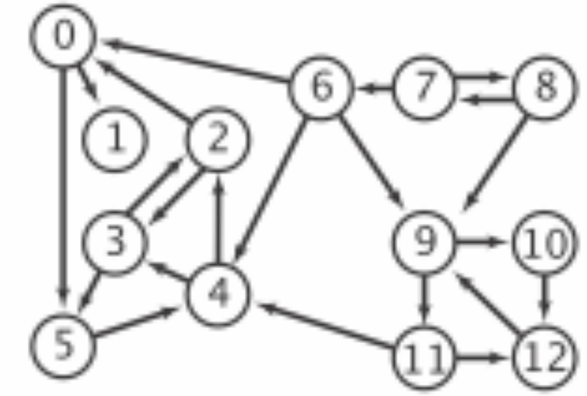
V = {0,1,2,3,4,5,6,7,8,9,10,11,12 }

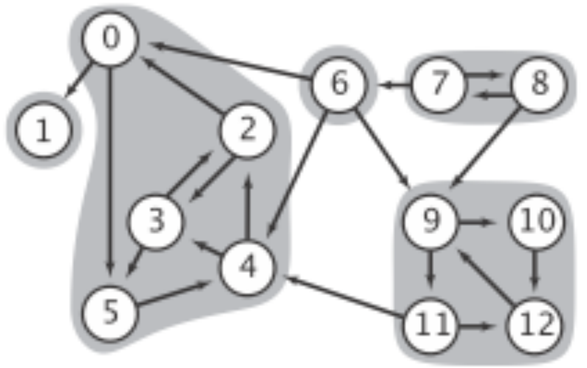
E = {{0,0}, {0,1}, {0,5}, {2,0}, {2,3},{3,2},{3,5}, {4,2},{4,3},{5,4},{6,0}, {6,4},{6,9},{7,6}{7,8},{8,7}, {8,9},{9,10},{9,11},{10,12}, {11,4},{11,12},{12,9}}.

Directed Graph Terminology

- A digraph is strongly connected if there is a directed path from every vertex to every other vertex.
- A digraph that is not strongly connected consists of a set of strongly connected components, which are maximal strongly connected subgraphs.
- A directed acyclic graph (DAG) is a digraph with no directed cycles.







A digraph and its strong components

Digraph Applications

Digraph	Vertex	Edge
Web	Web page	Link
Cell phone	Person	Placed call
Financial	Bank	Transaction
Transportation	Intersection	One-way street
Game	Board	Legal move
Citation	Article	Citation
Infectious Diseases	Person	Infection
Food web	Species	Predator-prey relationship

Popular digraph problems

Problem	Description
s->t path	Is there a path from s to t?
Shortest s->t path	What is the shortest path from s to t?
Directed cycle	Is there a directed cycle in the digraph?
Topological sort	Can vertices be sorted so all edges point from earlier to later vertices?
Strong connectivity	Is there a directed path between every pair of vertices?

Basic Graph API

```
public class Digraph
```

Digraph(int V): create an empty digraph with V vertices.

void addEdge(int v, int w): add an edge v->w.

Iterable<Integer> adj(int v): return vertices adjacent from v.

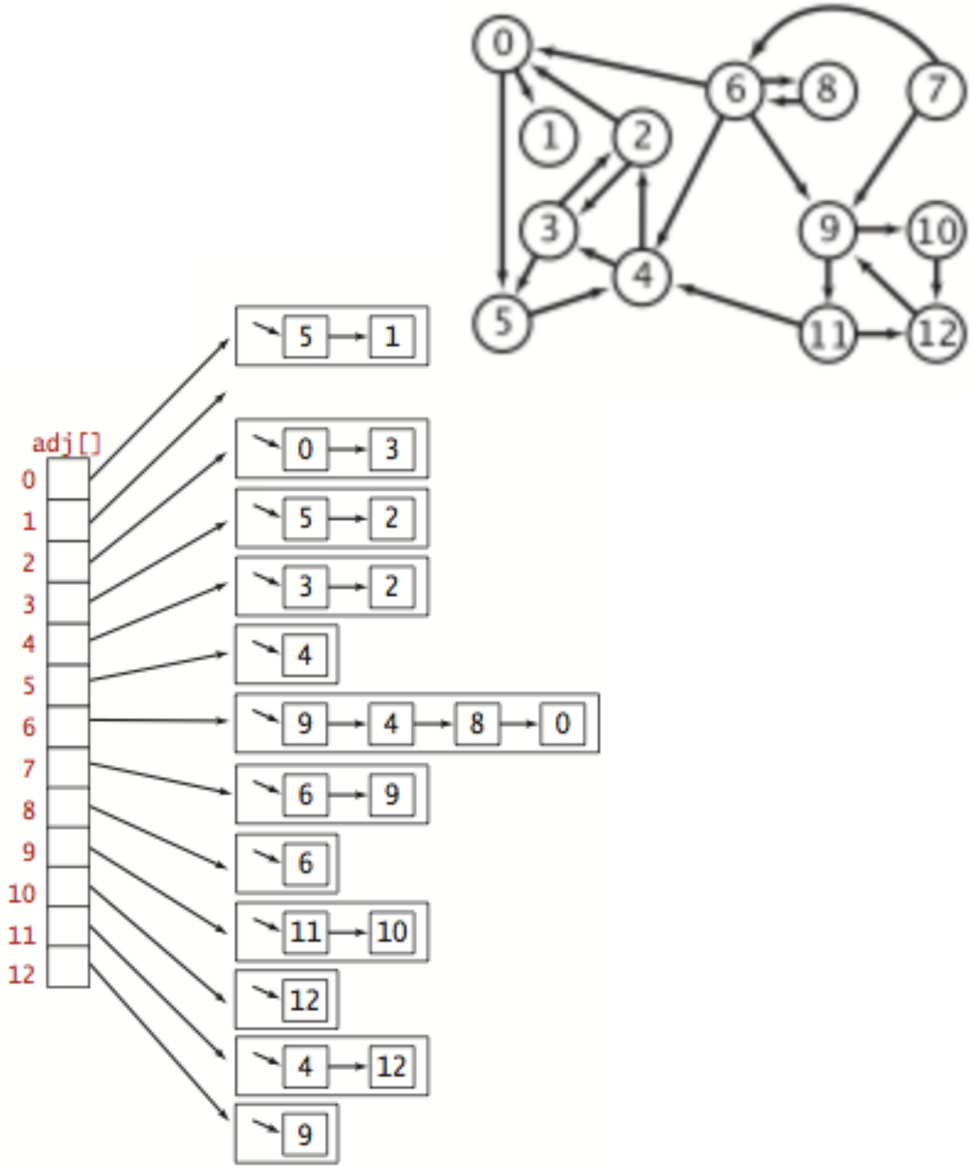
int V(): number of vertices.

int E(): number of edges.

Digraph reverse(): reverse edges of digraph.

Digraph representation: adjacency list

- Maintain vertex-indexed array of lists.
- Good for sparse graphs (edges proportional to $|V|$) which are much more common in the real world.
- Algorithms based on iterating over vertices adjacent from v.
- Space efficient ($|E| + |V|$).
- Constant time for adding a directed edge.
- **New difference:** Lookup of a directed edge or iterating over vertices adjacent from v is $outdegree(v)$**.**



Adjacency-list digraph representation in Java

```java
public class DirectedGraph {
    private final int V;
    private int E;
    private final List<Integer>[] adj;

    @SuppressWarnings("unchecked")
    public DirectedGraph(int V) {
        this.V = V;
        this.E = 0;
        adj = (List<Integer>[]) new List[V];
        for (int v = 0; v < V; v++) {
            adj[v] = new ArrayList<>();
        }
    }

    public void addEdge(int v, int w) {
        E++;
        adj[v].add(w); // Directed edge from v to w
    }

    public Iterable<Integer> adj(int v) {
        return adj[v];
    }

```

Very similar to undirected graph implementation, main change is adding directed edges (1 edge, not 2)

```java
//adds undirected edge v-w to graph. parallel edges and
self-loops allowed
public void addEdge(int v, int w) {
    E++;
    adj[v].add(w);
    adj[w].add(v);
}
```

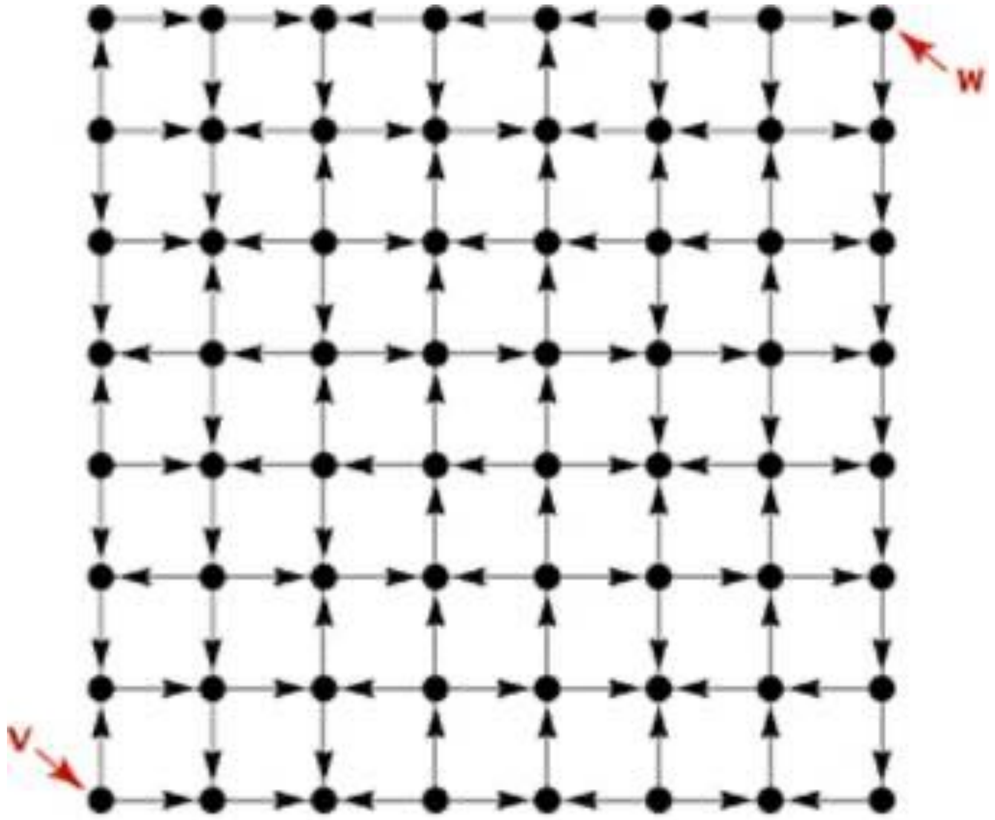
undirected implementation

DFS in Directed graphs

Reachability

Problem	Description
s->t path	Is there a path from s to t?

- Find all vertices reachable from s along a directed path.

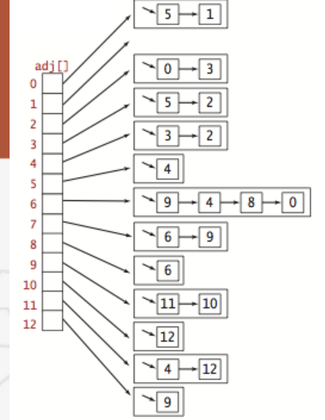


Is w reachable from v in this digraph?

https://apprize.info/science/algorithms_2/2.html

Depth-first search in digraphs

- **Same method as for undirected graphs.**
  - Every undirected graph is a digraph with edges in both directions.
  - Maximum number of edges in a simple digraph is $n(n-1)$.
- DFS (to visit a vertex v)
  - Mark vertex v.
  - Recursively visit all unmarked vertices w adjacent from v.
- Typical applications:
  - Find a directed path from source vertex s to a given target vertex v.
  - Topological sort (sort so dependencies are ordered, e.g. for fulfilling course pre-reqs).
  - Directed cycle detection.

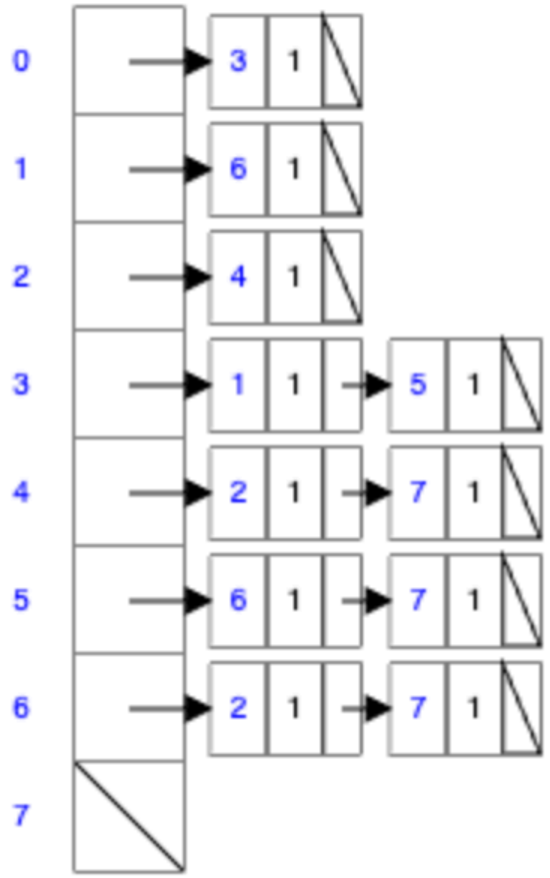


http://algs4.cs.princeton.edu

4.2 DIRECTED DFS DEMO

Worksheet time!

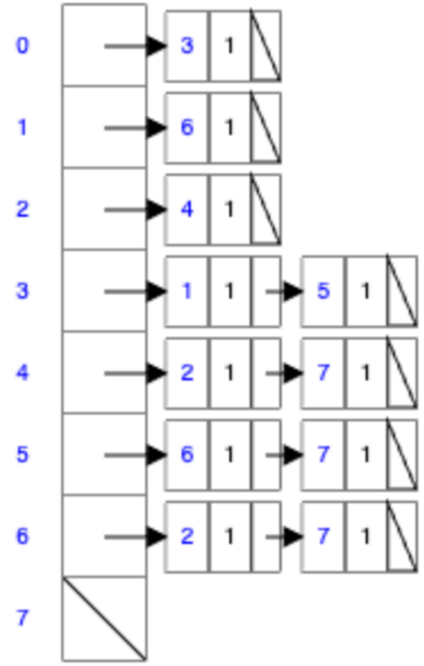
- Given the following adjacency list, visualize the resulting digraph and run DFS on it starting at vertex 0. In what order did you visit the vertices?
- Is every vertex reachable from 0?

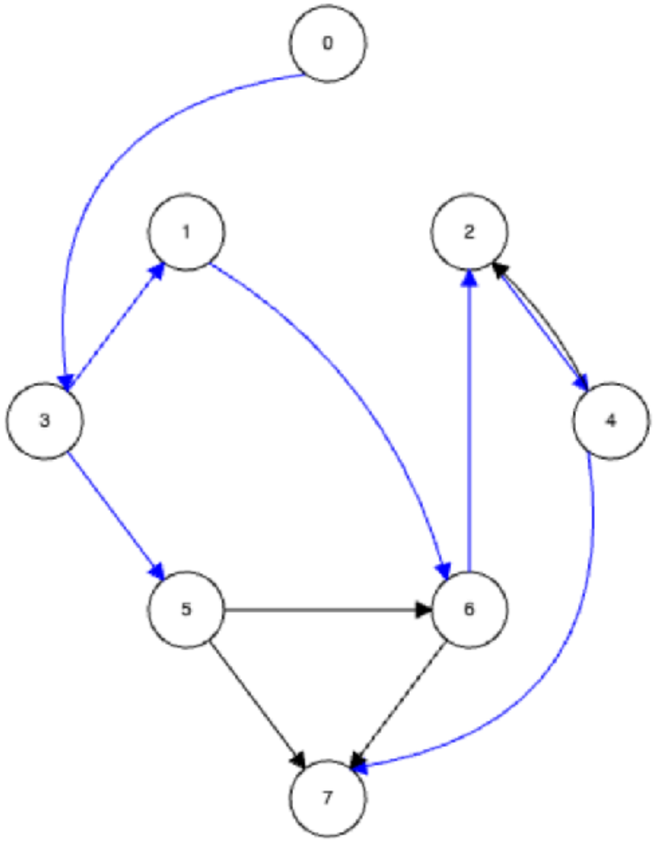


Note: Ignore the “1” value

Worksheet answer

- Given the following adjacency list, visualize the resulting digraph and run DFS on it starting at vertex 0.





Order: 0, 3, 1, 6, 2, 4, 7, 5

Yes, every vertex reachable from 0

V	marked	edgeTo
0	T	-
1	T	3
2	T	6
3	T	0
4	T	2
5	T	3
6	T	1
7	T	4

Depth-first search analysis

- DFS marks all vertices reachable from `s` in time proportional to $|V| + |E|$ in the worst case.
  - Initializing arrays `marked` takes time proportional to $|V|$.
  - Each adjacency-list entry is examined exactly once and there are E such edges (different than undirected graphs, which have 2|E| edges).
- Once we run DFS, we can check if vertex `v` is reachable from `s` in constant time (look into the marked array). We can also find the `s->v` path (if it exists) in time proportional to its length.
- Basically, same as DFS analysis on undirected graphs, but E instead of 2E edges

BFS in Directed graphs

Breadth-first search

- **Same method as for undirected graphs.**
  - Every undirected graph is a digraph with edges in both directions.
- BFS (from source vertex s)
  - Put s on queue and mark s as visited.
  - Repeat until the queue is empty:
    - Dequeue vertex v.
    - Enqueue all unmarked vertices adjacent from v, and mark them.
- Typical applications:
  - Find the shortest (in terms of number of edges) directed path between two vertices in time proportional to $|E| + |V|$.

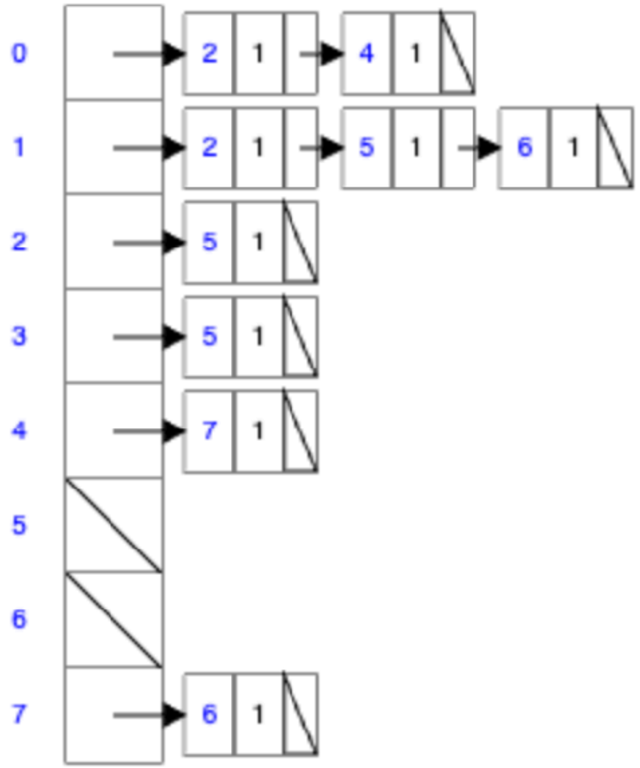
http://algs4.cs.princeton.edu

4.2 DIRECTED BFS DEMO

Order visited: 0, 2, 1, 4, 3, 5

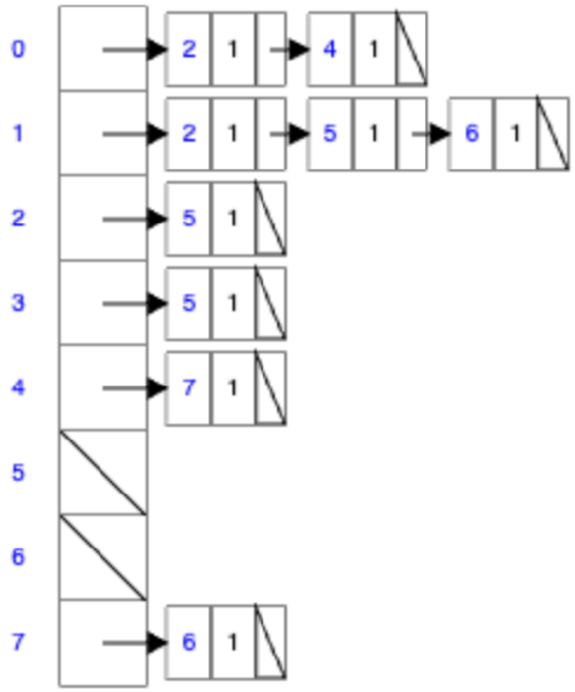
Worksheet time!

- Given the following adjacency list, visualize the resulting digraph and run BFS on it starting at vertex 0. In what order did you visit the vertices? Is every vertex reachable from 0?



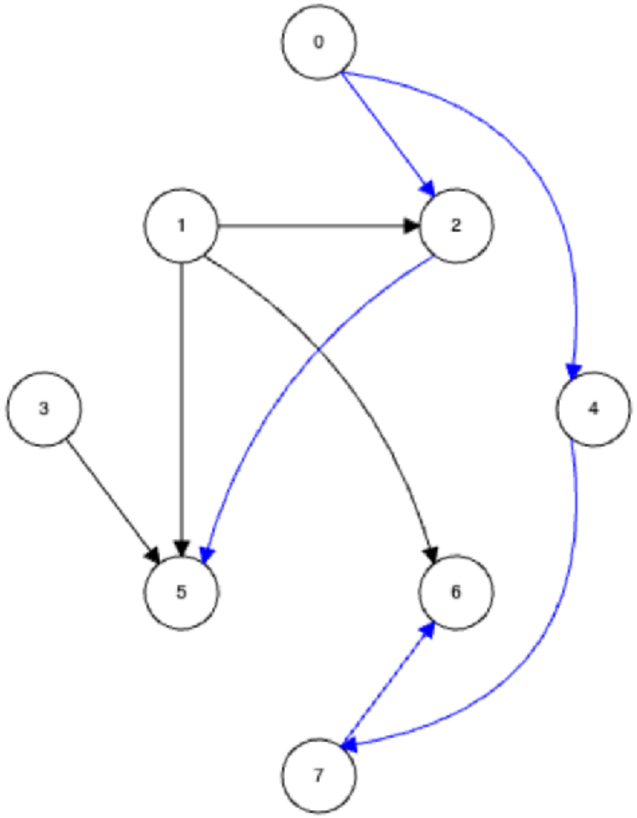
Worksheet answer

- Given the following adjacency list, visualize the resulting digraph and run BFS on it starting at vertex 0. In what order did you visit the vertices?



- 0, 2, 4, 5, 7, 6

No, we never reach 1 or 3 from 0



V	marked	edgeTo	distTo
0	T	-	0
1	F		
2	T	0	1
3	F		
4	T	0	1
5	T	2	2
6	T	7	3
7	T	4	2

Summary

- Single-source reachability in a digraph: DFS/BFS.
- Shortest path in a digraph: BFS.

Lecture 20 wrap-up

- No lab tonight
- Checkpoint corrections due Thu 11:59pm
- Final proj part 1 due next Fri 11/21
- HW10: Graphs due Tues 11/25 (day before Thanksgiving break) 11:59pm; released by end of week

Resources

- Recommended Textbook: Chapter 4.1 (Pages 522-556), Chapter 4.2 (Pages 566-594)
- Website: https://algs4.cs.princeton.edu/41graph/, https://algs4.cs.princeton.edu/42digraph/
- Visualization: https://visualgo.net/en/dfsbfs
- Practice problems (3!) behind this slide

Problem 1

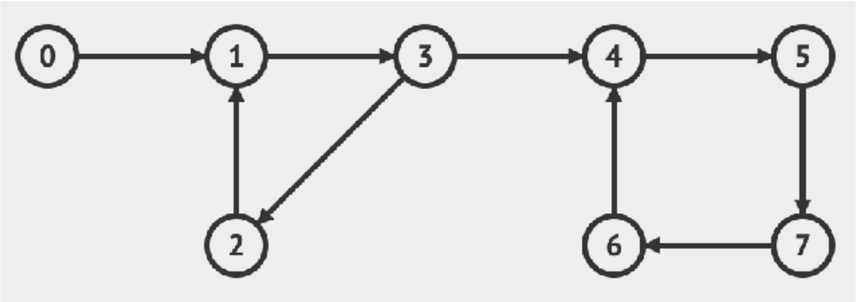
- What is the maximum number of edges in an undirected graph with V vertices and no parallel edges?
- What is the minimum number of edges in an undirected graph with V vertices, none of which are isolated (have degree 0)?
- What is the maximum number of edges in a digraph with V vertices and no parallel edges?
- What is the minimum number of edges in a digraph with V vertices, none of which are isolated?

Problem 2

- Assume you are given the following 16 edges of an undirected graph with 12 vertices, inserted in an adjacency list in this order:
- 8-4
- 2-3
- 1-11
- 0-6
- 3-6
- 10-3
- 7-11
- 7-8
- …
  - 11-8
  - 2-0
  - 6-2
  - 5-2
  - 5-10
  - 5-0
  - 8-1
  - 4-1

Problem 3

- Run DFS and BFS on the following digraph starting at vertex 0.



Answer 1

- What is the maximum number of edges in an undirected graph with V vertices and no parallel edges?
  - $n(n-1)/2$, where $n = |V|$.
- What is the minimum number of edges in an undirected graph with V vertices, none of which are isolated (have degree 0)?
  - $n-1$.
- What is the maximum number of edges in a digraph with V vertices and no parallel edges?
  - $n(n-1)$, where $n = |V|$.
- What is the minimum number of edges in a digraph with V vertices, none of which are isolated?
  - $n-1$.

Answer 2

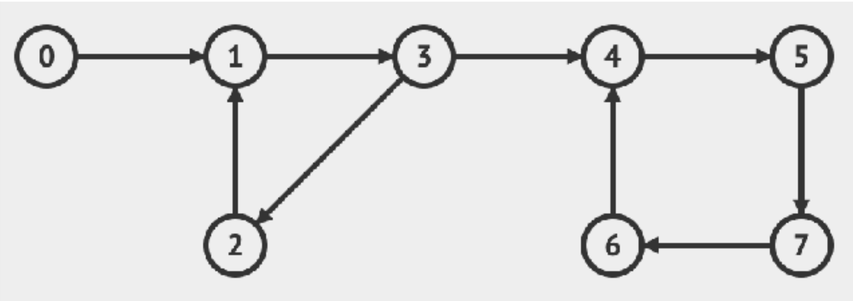
- Assume you are given the following 16 edges of an undirected graph with 12 vertices, inserted in an adjacency list in this order:
- 8-4
- 2-3
- 1-11
- 0-6
- 3-6
- 10-3
- 7-11
- 7-8
- …

- 11-8
- 2-0
- 6-2
- 5-2
- 5-10
- 5-0
- 8-1
- 4-1

- 0 -> 5 -> 2 -> 6
- 1 -> 4 -> 8 -> 11
- 2 -> 5 -> 6 -> 0 -> 3
- 3 -> 10 -> 6 -> 2
- 4 -> 1 -> 8
- 5 -> 0 -> 10 -> 2
- 6 -> 2 -> 3 -> 0
- 7 -> 8 -> 11
- 8 -> 1 -> 11 -> 7 -> 4
- 9 ->
- 10 -> 5 -> 3
- 11 -> 8 -> 7 -> 1

Answer 3

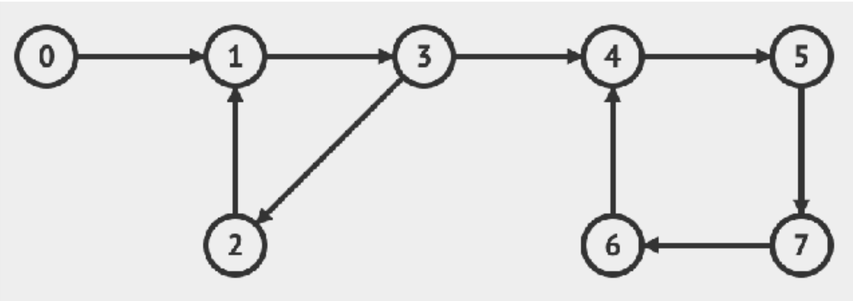
- DFS - Order of visit: 0, 1, 3, 2, 4, 5, 7, 6



V	marked	edgeTo
0	T	-
1	T	0
2	T	3
3	T	1
4	T	3
5	T	4
6	T	7
7	T	5

Answer 3

- BFS - Order of visit: 0, 1, 3, 2 4, 5, 7, 6



V	marked	edgeTo	distTo
0	T	-	0
1	T	1	1
2	T	3	2
3	T	1	2
4	T	3	3
5	T	4	4
6	T	7	6
7	T	5	5