INTRODUCTION TO MACHINE LEARNING

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CS 51A – Spring 2025

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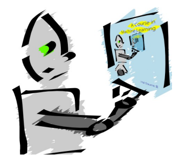
Machine Learning is…

Machine learning is about predicting the future based on the past.
-- Hal Daume III



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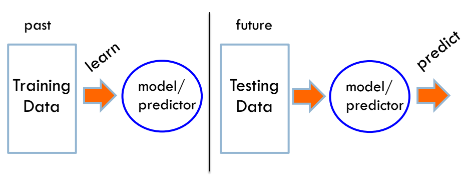

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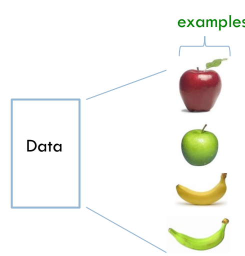
Data



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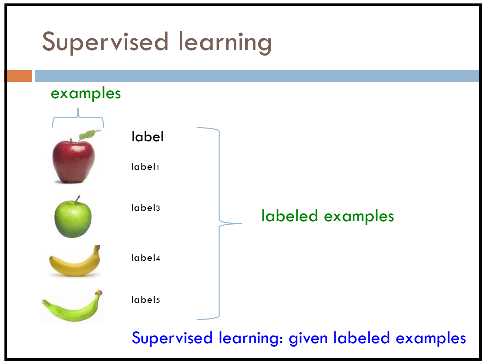
Supervised learning
examples
label
label1
label3
label4
label5
labeled examples
Supervised learning: given labeled examples

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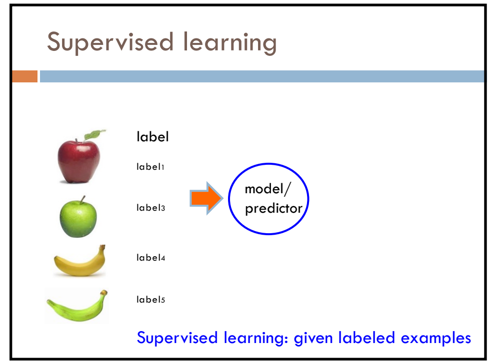
Supervised learning
label
label1
label3
label4
label5
model/
predictor
Supervised learning: given labeled examples

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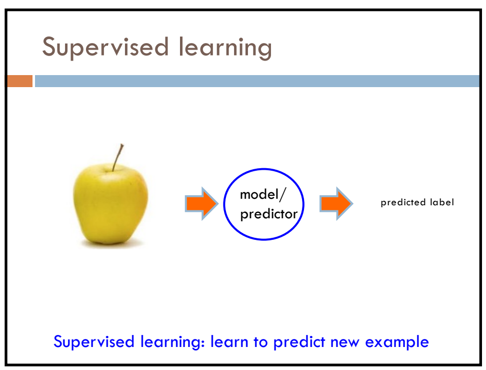
Supervised learning
model/
predictor
predicted label
Supervised learning: learn to predict new example

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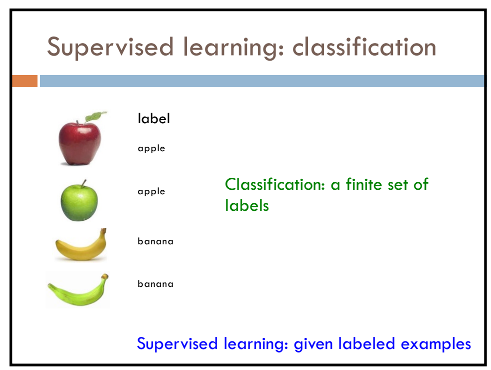
Supervised learning: classification
label
apple
apple
banana
banana
Classification: a finite set of labels
Supervised learning: given labeled examples

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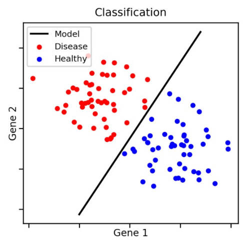
Classification Example



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Classification Applications

Face recognition

Character recognition

Spam detection

Medical diagnosis: From symptoms to illnesses

Biometrics: Recognition/authentication using physical and/or behavioral characteristics: Face, iris, signature, etc

...

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Supervised learning: regression

label

-4.5

10.1

3.2

4.3

Regression: label is real-valued

Supervised learning: given labeled examples

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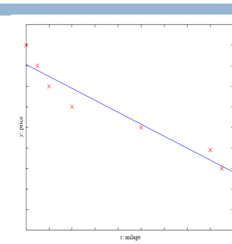
Regression Example

Price of a used car

x : car attributes (e.g. mileage)

y : price



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Regression Applications

Economics/Finance: predict the value of a stock

Epidemiology

Car/plane navigation: angle of the steering wheel, acceleration, …

Temporal trends: weather over time

…

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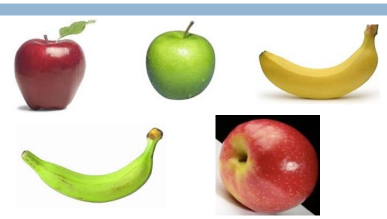
Unsupervised learning



Unpervised learning: given data, i.e. examples, but no labels

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Unsupervised learning applications

learn clusters/groups without any label

customer segmentation (i.e. grouping)

image compression

bioinformatics: learn motifs

…

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Reinforcement learning

left, right, straight, left, left, left, straight	GOOD
left, straight, straight, left, right, straight, straight	BAD

left, right, straight, left, left, left, straight	18.5
left, straight, straight, left, right, straight, straight	-3

Given a *sequence* of examples/states and a *reward* after completing that sequence, learn to predict the action to take in for an individual example/state

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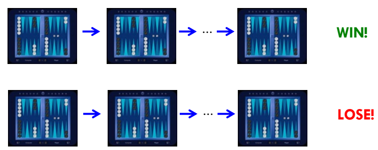
Reinforcement learning example

Backgammon



Given sequences of moves and whether or not the player won at the end, learn to make good moves

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Other learning variations

What data is available:

- Supervised, unsupervised, reinforcement learning
- semi-supervised, active learning, …

How are we getting the data:

- online vs. offline learning

Type of model:

- generative vs. discriminative
- parametric vs. non-parametric

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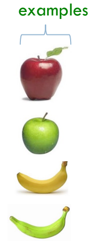
Representing examples

examples



What is an example?
How is it represented?

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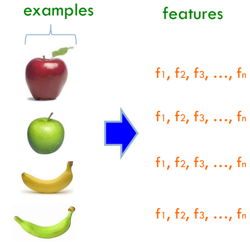
Features

examples

features



How our algorithms actually "view" the data

Features are the questions we can ask about the examples

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Features

examples

features

red, round, leaf, 3oz, …

green, round, no leaf, 4oz, …

yellow, curved, no leaf, 8oz, …

green, curved, no leaf, 7oz, …

How our algorithms actually "view" the data

Features are the questions we can ask about the examples

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Classification revisited

examples	label
red, round, leaf, 3oz, …	apple
green, round, no leaf, 4oz, …	apple
yellow, curved, no leaf, 8oz, …	banana
green, curved, no leaf, 7oz, …	banana

learn → model/ classifier

During learning/training/induction, learn a model of what distinguishes apples and bananas *based on the features*

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Classification revisited

red, round, no leaf, 4oz, … → model/ classifier → predict → Apple or banana?

The model can then classify a new example *based on the features*

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Classification revisited

red, round, no leaf, 4oz, … → model/ classifier → predict → Apple

Why?

The model can then classify a new example *based on the features*

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Classification revisited

Training data		Test set
examples	label	
red, round, leaf, 3oz, …	apple	
green, round, no leaf, 4oz, …	apple	red, round, no leaf, 4oz, … ?
yellow, curved, no leaf, 4oz, …	banana	
green, curved, no leaf, 5oz, …	banana	

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Classification revisited

Training data		Test set
examples	label	
red, round, leaf, 3oz, …	apple	
green, round, no leaf, 4oz, …	apple	red, round, no leaf, 4oz, … ?
yellow, curved, no leaf, 4oz, …	banana	Learning is about ***generalizing*** from the training data
green, curved, no leaf, 5oz, …	banana	

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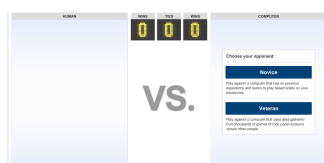
Rock, paper, scissors

https://archive.nytimes.com/www.nytimes.com/interactive/science/rock-paper-scissors.html



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models

model/
classifier

We have many, many different options for the model

They have different characteristics and perform differently (accuracy, speed, etc.)

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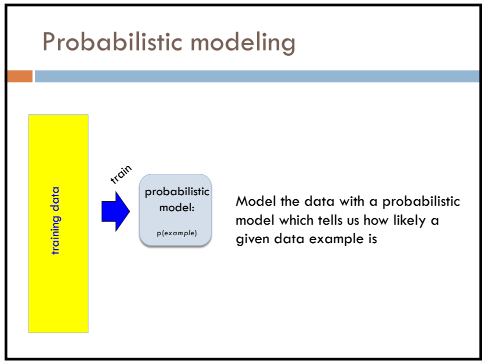
Probabilistic modeling
training data
train
probabilistic model:
p(example)
Model the data with a probabilistic model which tells us how likely a given data example is

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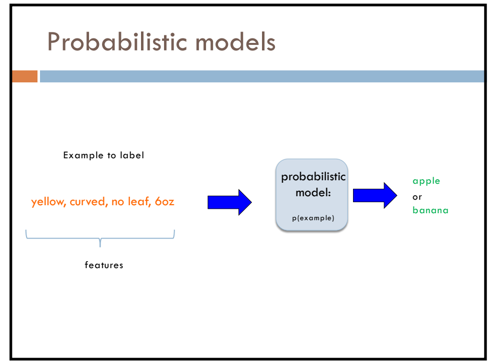
Probabilistic models
Example to label
yellow, curved, no leaf, 6oz
features
probabilistic model:
p(example)
apple
or
banana

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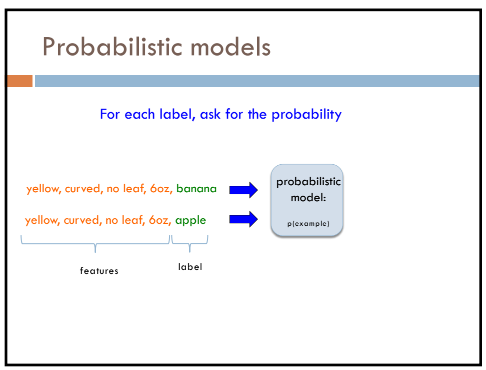
Probabilistic models
For each label, ask for the probability
yellow, curved, no leaf, 6oz, banana
yellow, curved, no leaf, 6oz, apple
features
label
probabilistic model:
p(example)

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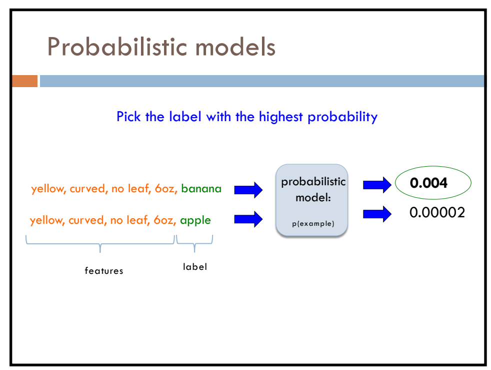
Probabilistic models
Pick the label with the highest probability
yellow, curved, no leaf, 6oz, banana
yellow, curved, no leaf, 6oz, apple
features
label
probabilistic model:
p(example)
0.004
0.00002

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Probability basics

A probability distribution gives the probabilities of all possible values of an event

For example, say we flip a coin three times. We can define the probability of the number of time the coin came up heads.

P(num heads)
P(3) = ?
P(2) = ?
P(1) = ?
P(0) = ?

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Probability distributions

What are the possible outcomes of three flips (hint, there are eight of them)?

T T T
T T H
T H T
T H H
H T T
H T H
H H T
H H H

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Probability distributions

Assuming the coin is fair, what are our probabilities?

$$\text{probability} = \frac{\text{number of times it happens}}{\text{total number of cases}}$$

T T T
T T H
T H T
T H H
H T T
H T H
H H T
H H H

P(num heads)
P(3) = ?
P(2) = ?
P(1) = ?
P(0) = ?

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Probability distributions

Assuming the coin is fair, what are our probabilities?

$$\text{probability} = \frac{\text{number of times it happens}}{\text{total number of cases}}$$

T T T
T T H
T H T
T H H
H T T
H T H
H H T
H H H

P(num heads)
P(3) = ?
P(2) = ?
P(1) = ?
P(0) = ?

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Probability distributions

Assuming the coin is fair, what are our probabilities?

probability = $\frac{\text{number of times it happens}}{\text{total number of cases}}$

T T T
T T H
T H T
T H H
H T T
H T H
H H T
H H H

P(num heads)
P(3) = 1/8
P(2) = ?
P(1) = ?
P(0) = ?

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Probability distributions

Assuming the coin is fair, what are our probabilities?

probability = $\frac{\text{number of times it happens}}{\text{total number of cases}}$

T T T
T T H
T H T
T H H
H T T
H T H
H H T
H H H

P(num heads)
P(3) = 1/8
P(2) = ?
P(1) = ?
P(0) = ?

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Probability distributions

Assuming the coin is fair, what are our probabilities?

probability = $\frac{\text{number of times it happens}}{\text{total number of cases}}$

T T T
T T H
T H T
T H H
H T T
H T H
H H T
H H H

P(num heads)
P(3) = 1/8
P(2) = 3/8
P(1) = ?
P(0) = ?

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Probability distributions

Assuming the coin is fair, what are our probabilities?

probability = $\frac{\text{number of times it happens}}{\text{total number of cases}}$

T T T
T T H
T H T
T H H
H T T
H T H
H H T
H H H

P(num heads)
P(3) = 1/8
P(2) = 3/8
P(1) = 3/8
P(0) = 1/8

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Probability distribution

A probability distribution assigns probability values to *all possible values*

Probabilities are between 0 and 1, inclusive

The sum of all probabilities in a distribution must be 1

P(num heads)
P(3) = 1/8
P(2) = 3/8
P(1) = 3/8
P(0) = 1/8

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Probability distribution

A probability distribution assigns probability values to *all possible values*

Probabilities are between 0 and 1, inclusive

The sum of all probabilities in a distribution must be 1

P
P(3) = 1/2
P(2) = 1/2
P(1) = 1/2
P(0) = 1/2

P
P(3) = -1
P(2) = 2
P(1) = 0
P(0) = 0

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Some example probability distributions

probability of heads
(distribution options: heads, tails)

probability of passing class
(distribution options: pass, fail)

probability of rain today
(distribution options: rain or no rain)

probability of getting an 'A'
(distribution options: A, B, C, D, F)

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Conditional probability distributions

Sometimes we may know extra information about the world that may change our probability distribution

P(X|Y) captures this (read "probability of X *given* Y")

- Given some information (Y) what does our probability distribution look like
- Note that this is still just a normal probability distribution

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Conditional probability example

P(pass 51a)
P(pass) = 0.9
P(not pass) = 0.1

Unconditional probability distribution

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Conditional probability example

P(pass 51a)
P(pass) = 0.9
P(not pass) = 0.1

P(pass 51a | don't study)
P(pass) = 0.5
P(not pass) = 0.5

P(pass 51a | do study)
P(pass) = 0.95
P(not pass) = 0.05

Still probability distributions over passing 51A

Conditional probability distributions

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Conditional probability example

P(rain in LA)
P(rain) = 0.05
P(no rain) = 0.95

Unconditional probability distribution

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Conditional probability example

P(rain in LA)
P(rain) = 0.05
P(no rain) = 0.95

P(rain in LA | March)
P(rain) = 0.2
P(no rain) = 0.8

P(rain in LA | not March)
P(pass) = 0.03
P(not pass) = 0.97

Still probability distributions over passing rain in LA

Conditional probability distributions

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Joint distribution

Probability over two events: P(X,Y)

Has probabilities for all possible combinations over the two events

51Pass, EngPass	P(51Pass, EngPass)
true, true	.88
true, false	.01
false, true	.04
false, false	.07

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Joint distribution

Still a probability distribution

All questions/probabilities that we might want to ask about these two things can be calculated from the joing distribution

51Pass, EngPass	P(51Pass, EngPass)
true, true	.88
true, false	.01
false, true	.04
false, false	.07

What is P(51pass = true)?

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Joint distribution

51Pass, EngPass	P(51Pass, EngPass)
true, true	.88
true, false	.01
false, true	.04
false, false	.07

There are two ways that a person can pass 51:
they can do it while passing or not passing English

P(51Pass=true) = P(true, true) + P(true, false) = 0.89

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Relationship between distributions

$$P(X,Y) = P(Y) * P(X|Y)$$

joint distribution

unconditional distribution

conditional distribution

Can think of it as describing the two events happening in two steps:

The likelihood of X and Y happening:
1. How likely it is that Y happened?
2. Given that Y happened, how likely is it that X happened?

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Relationship between distributions

$$P(51Pass, EngPass) = P(EngPass) * P(51Pass|EngPass)$$

The probability of passing CS51 and English is:

1. Probability of passing English *
2. Probability of passing CS51 **given** that you passed English

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Relationship between distributions

$$P(51Pass, EngPass) = P(51Pass) * P(EngPass|51Pass)$$

The probability of passing CS51 and English is:

1. Probability of passing CS51 *
2. Probability of passing English **given** that you passed CS51

Can also view it with the other event happening first

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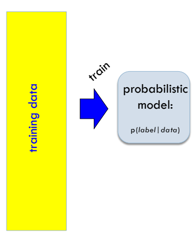
Back to probabilistic modeling



Build a model of the conditional distribution:

P(label | data)

How likely is a label given the data

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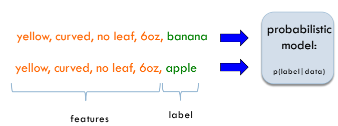
Back to probabilistic models

For each label, calculate the probability of the label given the data



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Back to probabilistic models

Pick the label with the highest probability

yellow, curved, no leaf, 6oz, banana → probabilistic model: p(label | data) → 0.004

yellow, curved, no leaf, 6oz, apple → 0.00002

features label

MAX

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Naïve Bayes model

Two parallel ways of breaking down the joint distribution

$$P(data, label) = P(label) * P(data|label)$$
$$P(data, label) = P(data) * P(label|data)$$

$$P(label) * P(data|label) \ = P(data) * P(label|data)$$

What is P(label | data)?

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Naïve Bayes

$$P(label) * P(data|label) \ = P(data) * P(label|data)$$

$$\Downarrow$$

$$P(label|data) = \frac{P(label) * P(data|label)}{P(data)}$$

(This is called Bayes' rule!)

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Naïve Bayes

$$P(label|data) = \frac{P(label) * P(data|label)}{P(data)}$$

probabilistic model: p(label | data)

$$\frac{P(positive) * P(data|positive)}{P(data)}$$

$$\frac{P(negative) * P(data|negative)}{P(data)}$$

MAX

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One observation

$$\frac{P(positive) * P(data|positive)}{P(data)}$$

MAX

$$\frac{P(negative) * P(data|negative)}{P(data)}$$

For picking the largest P(data) doesn't matter!

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One observation

$P(positive) * P(data|positive)$

MAX

$P(negative) * P(data|negative)$

For picking the largest P(data) doesn't matter!

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A simplifying assumption (for this class)

$P(positive) * P(data|positive)$

MAX

$P(negative) * P(data|negative)$

If we assume P(positive) = P(negative) then:

$P(data|positive)$

MAX

$P(data|negative)$

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Naïve Bayes Assumption

$$P(data|label) = P(f_1, f_2, \ldots, fn|label)$$

$$\approx P(f_1|label) * P(f_2|label) * \ldots P(f_n|label)$$

This is generally not true!

However…, it makes our life easier.

This is why the model is called **Naïve** Bayes

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Naïve Bayes

$P(f_1|positive) * P(f_2|positive) * \ldots * P(f_n|positive)$

MAX

$P(f_1|negative) * P(f_2|negative) * \ldots * P(f_n|negative)$

Where do these come from?

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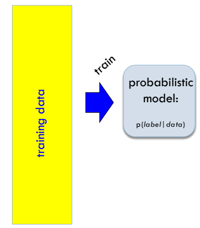
Training Naïve Bayes



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An aside: P(heads)

What is the P(heads) on a fair coin?

- 0.5

What if you didn't know that, but had a coin to experiment with?

- Flip it a bunch of times and count how many times it comes up heads

$$P(heads) = \frac{\text{number of times heads came up}}{\text{total number of coin tosses}}$$

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Try it out…

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P(feature | label)

$$P(heads) = \frac{number\ of\ times\ heads\ came\ up}{total\ number\ of\ coin\ tosses}$$

Can we do the same thing here? What is the probability of a feature given positive, i.e. the probability of a feature occurring in in the positive label?

$$P(feature|positive) = ?$$

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P(feature | label)

$$P(heads) = \frac{number\ of\ times\ heads\ came\ up}{total\ number\ of\ coin\ tosses}$$

Can we do the same thing here? What is the probability of a feature given positive, i.e. the probability of a feature occurring in in the positive label?

$$P(feature|positive) = \frac{number\ of\ positive\ examples\ with\ that\ feature}{total\ number\ of\ positive\ examples}$$

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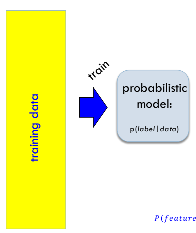
Training Naïve Bayes



1. Count how many examples have each label
2. For all examples with a particular label, count how many times each feature occurs
3. Calculate the conditional probabilities of each feature for all labels:

$$P(feature|label) = \frac{number\ of\ ``label"\ examples\ with\ that\ feature}{total\ number\ of\ examples\ with\ that\ label}$$

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Classifying with Naïve Bayes

For each label, calculate the product of p(feature | label) for each label

yellow, curved, no leaf, 6oz

P(yellow | banana)*…*P(6oz | banana)

P(yellow | apple)*…*P(6oz | apple)

MAX

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Naïve Bayes Text Classification

Positive	Negative
I loved it	I hated it
I loved that movie	I hated that movie
I hated that I loved it	I loved that I hated it

Given examples of text in different categories, learn to predict the category of new examples

Sentiment classification: given positive/negative examples of text (sentences), learn to predict whether new text is positive/negative

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Text classification training

Positive	Negative
I loved it	I hated it
I loved that movie	I hated that movie
I hated that I loved it	I loved that I hated it

We'll assume words just occur once in any given sentence

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Text classification training

Positive	Negative
I loved it	I hated it
I loved that movie	I hated that movie
I hated that loved it	I loved that hated it

We'll assume words just occur once in any given sentence

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Training the model

Positive	Negative
I loved it	I hated it
I loved that movie	I hated that movie
I hated that loved it	I loved that hated it

For each word and each label, learn:

p(word | label)

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Training the model

Positive	Negative
I loved it	I hated it
I loved that movie	I hated that movie
I hated that loved it	I loved that hated it

P(I | positive) = ?

$$P(word|label) = \frac{number\ of\ times\ word\ occured\ in\ "label"\ examples}{total\ number\ of\ examples\ with\ that\ label}$$

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Training the model

Positive	Negative
I loved it	I hated it
I loved that movie	I hated that movie
I hated that loved it	I loved that hated it

P(I | positive) = 3/3 = 1.0

$$P(word|label) = \frac{number\ of\ times\ word\ occured\ in\ "label"\ examples}{total\ number\ of\ examples\ with\ that\ label}$$

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Training the model

Positive	Negative
I loved it	I hated it
I loved that movie	I hated that movie
I hated that loved it	I loved that hated it

P(I | positive) = 1.0
P(loved | positive) = ?

$$P(word|label) = \frac{number\ of\ times\ word\ occured\ in\ "label"\ examples}{total\ number\ of\ examples\ with\ that\ label}$$

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Training the model

Positive	Negative
I loved it	I hated it
I loved that movie	I hated that movie
I hated that loved it	I loved that hated it

P(I | positive) = 1.0
P(loved | positive) = 3/3

$$P(word|label) = \frac{number\ of\ times\ word\ occured\ in\ "label"\ examples}{total\ number\ of\ examples\ with\ that\ label}$$

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Training the model

Positive	Negative
I loved it	I hated it
I loved that movie	I hated that movie
I hated that loved it	I loved that hated it

P(I | positive) = 1.0
P(loved | positive) = 3/3
P(hated | positive) = ?

$$P(word|label) = \frac{\text{number of times word occured in "label" examples}}{\text{total number of examples with that label}}$$

85

Training the model

Positive	Negative
I loved it	I hated it
I loved that movie	I hated that movie
I hated that loved it	I loved that hated it

P(I | positive) = 1.0
P(loved | positive) = 2/3
P(hated | positive) = 1/3
…

P(I | negative) = ?

$$P(word|label) = \frac{\text{number of times word occured in "label" examples}}{\text{total number of examples with that label}}$$

86

Training the model

Positive	Negative
I loved it	I hated it
I loved that movie	I hated that movie
I hated that loved it	I loved that hated it

P(I | positive) = 1.0
P(loved | positive) = 2/3
P(hated | positive) = 1/3
…

P(I | negative) = 1.0

$$P(word|label) = \frac{\text{number of times word occured in "label" examples}}{\text{total number of examples with that label}}$$

87

Training the model

Positive	Negative
I loved it	I hated it
I loved that movie	I hated that movie
I hated that loved it	I loved that hated it

P(I | positive) = 1.0
P(loved | positive) = 2/3
P(hated | positive) = 1/3
…

P(I | negative) = 1.0
P(movie | negative) = ?

$$P(word|label) = \frac{\text{number of times word occured in "label" examples}}{\text{total number of examples with that label}}$$

88

Training the model

Positive	Negative
I loved it	I hated it
I loved that movie	I hated that movie
I hated that loved it	I loved that hated it

P(I | positive) = 1.0
P(loved | positive) = 2/3
P(hated | positive) = 1/3
…

P(I | negative) = 1.0
P(movie | negative) = 1/3
…

$$P(word|label) = \frac{number\ of\ times\ word\ occured\ in\ "label"\ examples}{total\ number\ of\ examples\ with\ that\ label}$$

89

Classifying

P(I | positive)	= 1.0	P(I | negative)	= 1.0
P(loved | positive)	= 1.0	p(hated | negative)	= 1.0
p(it | positive)	= 2/3	p(that | negative)	= 2/3
p(that | positive)	= 2/3	P(movie | negative)	= 1/3
p(movie | positive)	= 1/3	p(it | negative)	= 2/3
P(hated | positive)	= 1/3	p(loved | negative)	= 1/3

Notice that each label has its own probability distribution

P(loved | positive)
P(loved | positive) = 2/3
P(no loved | positive) = 1/3

90

Trained model

P(I | positive)	= 1.0	P(I | negative)	= 1.0
P(loved | positive)	= 2/3	p(hated | negative)	= 1.0
p(it | positive)	= 2/3	p(that | negative)	= 2/3
p(that | positive)	= 2/3	P(movie | negative)	= 1/3
p(movie | positive)	= 1/3	p(it | negative)	= 2/3
P(hated | positive)	= 1/3	p(loved | negative)	= 1/3

How would we classify: "I hated movie"?

91

Trained model

P(I | positive)	= 1.0	P(I | negative)	= 1.0
P(loved | positive)	= 2/3	p(hated | negative)	= 1.0
p(it | positive)	= 2/3	p(that | negative)	= 2/3
p(that | positive)	= 2/3	P(movie | negative)	= 1/3
p(movie | positive)	= 1/3	p(it | negative)	= 2/3
P(hated | positive)	= 1/3	p(loved | negative)	= 1/3

P(I | positive) * P(hated | positive) * P(movie | positive) = 1.0 * 1/3 * 1/3 = 1/9

P(I | negative) * P(hated | negative) * P(movie | negative) = 1.0 * 1.0 * 1/3 = 1/3

92

Trained model

P(I | positive)	= 1.0	P(I | negative)	= 1.0
P(loved | positive)	= 2/3	p(hated | negative)	= 1.0
p(it | positive)	= 2/3	p(that | negative)	= 2/3
p(that | positive)	= 2/3	P(movie | negative)	= 1/3
p(movie | positive)	= 1/3	p(it | negative)	= 2/3
P(hated | positive)	= 1/3	p(loved | negative)	= 1/3

How would we classify: "I hated the movie"?

93

Trained model

P(I | positive)	= 1.0	P(I | negative)	= 1.0
P(loved | positive)	= 2/3	p(hated | negative)	= 1.0
p(it | positive)	= 2/3	p(that | negative)	= 2/3
p(that | positive)	= 2/3	P(movie | negative)	= 1/3
p(movie | positive)	= 1/3	p(it | negative)	= 2/3
P(hated | positive)	= 1/3	p(loved | negative)	= 1/3

P(I | positive) * P(hated | positive) * P(the | positive) * P(movie | positive) =

P(I | negative) * P(hated | negative) * P(the | negative) * P(movie | negative) =

94

Trained model

P(I | positive)	= 1.0	P(I | negative)	= 1.0
P(loved | positive)	= 2/3	p(hated | negative)	= 1.0
p(it | positive)	= 2/3	p(that | negative)	= 2/3
p(that | positive)	= 2/3	P(movie | negative)	= 1/3
p(movie | positive)	= 1/3	p(it | negative)	= 2/3
P(hated | positive)	= 1/3	p(loved | negative)	= 1/3

P(I | positive) * P(hated | positive) * P(the | positive) * P(movie | positive) =

P(I | negative) * P(hated | negative) * P(the | negative) * P(movie | negative) =

What are these?

95

Trained model

P(I | positive)	= 1.0	P(I | negative)	= 1.0
P(loved | positive)	= 2/3	p(hated | negative)	= 1.0
p(it | positive)	= 2/3	p(that | negative)	= 2/3
p(that | positive)	= 2/3	P(movie | negative)	= 1/3
p(movie | positive)	= 1/3	p(it | negative)	= 2/3
P(hated | positive)	= 1/3	p(loved | negative)	= 1/3

P(I | positive) * P(hated | positive) * P(the | positive) * P(movie | positive) =

P(I | negative) * P(hated | negative) * P(the | negative) * P(movie | negative) =

0. Is this a problem?

96

Trained model

P(I | positive)	= 1.0	P(I | negative)	= 1.0
P(loved | positive)	= 2/3	p(hated | negative)	= 1.0
p(it | positive)	= 2/3	p(that | negative)	= 2/3
p(that | positive)	= 2/3	P(movie | negative)	= 1/3
p(movie|positive)	= 1/3	p(it | negative)	= 2/3
P(hated | positive)	= 1/3	p(loved | negative)	= 1/3

P(I | positive) * P(hated | positive) * P(the | positive) * P(movie | positive) =

P(I | negative) * P(hated | negative) * P(the | negative) * P(movie | negative) =

Yes. They make the entire product go to 0 !

97

Trained model

P(I | positive)	= 1.0	P(I | negative)	= 1.0
P(loved | positive)	= 2/3	p(hated | negative)	= 1.0
p(it | positive)	= 2/3	p(that | negative)	= 2/3
p(that | positive)	= 2/3	P(movie | negative)	= 1/3
p(movie|positive)	= 1/3	p(it | negative)	= 2/3
P(hated | positive)	= 1/3	p(loved | negative)	= 1/3

P(I | positive) * P(hated | positive) * P(the | positive) * P(movie | positive) =

P(I | negative) * P(hated | negative) * P(the | negative) * P(movie | negative) =

Our solution: assume any unseen word has a small, fixed probability, e.g. in this example 1/10

98

Trained model

P(I | positive)	= 1.0	P(I | negative)	= 1.0
P(loved | positive)	= 2/3	p(hated | negative)	= 1.0
p(it | positive)	= 2/3	p(that | negative)	= 2/3
p(that | positive)	= 2/3	P(movie | negative)	= 1/3
p(movie|positive)	= 1/3	p(it | negative)	= 2/3
P(hated | positive)	= 1/3	p(loved | negative)	= 1/3

P(I | positive) * P(hated | positive) * P(the | positive) * P(movie | positive) = 1/90

P(I | negative) * P(hated | negative) * P(the | negative) * P(movie | negative) = 1/30

Our solution: assume any unseen word has a small, fixed probability, e.g. in this example 1/10

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Full disclaimer

I've fudged a few things on the Naïve Bayes model for simplicity

Our approach is very close, but it takes a few liberties that aren't technically correct, but it will work just fine ☺

If you're curious, I'd be happy to talk to you offline

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